# Fermi surface topology and anisotropic superconducting gap in electron-doped hydride compounds at high pressure

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The recent theoretical discovery of the predicted high-temperature superconductivity (superconducting transition temperature  $T_c \sim 351$  K at 300 GPa) in clathrate Li<sub>2</sub>MgH<sub>16</sub> is an important advance toward room-temperature superconductors. Here we use first-principle approaches to identify a new ternary hydride of clathrate structure of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> by Rb-doped MgH<sub>16</sub> at 300 GPa. We first verified the thermal and dynamic stability for the  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> through the formation enthalpy and phonon calculations, respectively. Next, we showcased that  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> has two Fermi surface (FS) sheets, which are both dominated by the H2 *s* orbital. Using the Eliashberg formalism, the  $T_c$  of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> has four FS sheets with mixed H1 *s* and H2 *s* orbital character and reaches the  $T_c$  of 352 K at 300 GPa. Compared with  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub>,  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub> produces high-frequency phonon softening, leading to the enhancement of  $T_c$ . Moreover, it is demonstrated that Rb substitution for Li produces the reduction of contribution of H orbitals to the FS sheets and the decrease in FS sheets in number, which tend to decrease the electron-phonon coupling strength and  $T_c$ . Our observed FS sheets and their associated superconducting gap will provide key insights into the understanding of the superconductivity mechanism in these and related systems.

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# I. INTRODUCTION

An important approach of looking for high-temperature superconductors in hydrogen-rich hydrides has been proposed due to the discovery of a superconducting transition temperature ( $T_c$ ) of 8 K in Th<sub>4</sub>H<sub>15</sub> at ambient pressure in 1970 [1]. However, this strategy was not widely accepted until Ashcroft proposed that hydrogen-rich compounds containing main group elements might exhibit superconductivity at lower pressures compared with metallic hydrogen as the hydrogen in these hydrogen-rich compounds may be considered "chemically precompressed" [2]. Following this idea, it is necessary to carry out extensive theoretical and experimental studies to find high-temperature superconductors in hydride compounds [3–8].

Recently, hydrogen-rich materials as the main candidate for high- $T_c$  superconductors have attracted more and more attention because the theoretical studies predicted sodalitelike clathrate LaH<sub>10</sub> to have a  $T_c$  of ~280 K at 200 GPa [9,10]. Some subsequent experiments confirmed that the theoretically predicted, the synthesized structure of LaH<sub>10</sub> was measured to have a high superconductivities of  $T_c \sim 250$ K at pressures above 170 GPa, setting a record high  $T_c$  for superconductors [11–13]. Another key advance is that the theoretically predicted cubic SH<sub>3</sub>s are confirmed by experimental studies to have high-temperature superconductivity with a  $T_c$  of ~204 K at 200 GPa [14–17]. In these observations, the theoretical studies are always meaningful for material science innovation to design novel superconducting materials [18,19].

Nearly all binary hydrides have been investigated by structure searching simulations [4,5,7], among which, simple hydrides with the highest predicted  $T_c$  are MgH<sub>6</sub> (271 K at 300 GPa) [20], CaH<sub>6</sub> (235 K at 150 GPa) [21], and YH<sub>6</sub> (264 K at 120 GPa) [22]. These studies stimulate more efforts to look for high-temperature superconductors in binary hydrides as well as ternary hydrides [23–25]. Encouragingly, an experimental work has measured the C-S-H system to have room-temperature superconductivities with a high  $T_c$  of 290 K at 270 GPa [26]. Although structural information is unknown of this ternary system, this result still clearly indicates that it is a feasible way of looking for high-temperature superconductors in ternary hydrogen-rich compounds. Very recently, a metastable phase  $Li_2MgH_{16}$  is predicted with a  $T_c$  of 351 K at 300 GPa via Li-doped MgH<sub>16</sub>. Li acts as a donor, which dopes extra electrons into the system to enhance the superconductivity of the MgH<sub>16</sub> system ( $T_c \sim 73$  K at 300 GPa) [27]. This result indicates that metal doping is an effective way to design new high-T<sub>c</sub> superconductors in hydrogen-rich hydrides. This

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is reminiscent of Rb having a low electronegativity among the alkali metals. It is natural to wonder if the extra electrons introduced via Rb-doped  $MgH_{16}$  may effectively tune the crystal structures and improve the superconductivity for the  $MgH_{16}$  system [21].

In this paper, we identify a new ternary hydride of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> by Rb-doped MgH<sub>16</sub> at 300 GPa based on the density functional theory [28]. First of all, the thermal and dynamical stabilities were verified by the formation enthalpy and phonon calculations. Next, through systematic first-principles calculations, we demonstrated that Fd3m-Rb<sub>2</sub>MgH<sub>16</sub> has two Fermi surface (FS) sheets, which are both dominated by the H2 s orbital. The superconductivity calculations show that the  $T_c$  of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> reaches 130 K at 300 GPa. We find, however, that Rb substitution for Li generates a substantial change in structure and electronic properties in this system, leading to the reduction of  $T_c$  of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub>, compared with Li<sub>2</sub>MgH<sub>16</sub>. These studies emphasize the importance of the chemical tuning structural stability and superconductivity via metal doping, which provide a new angle for designing ternary hydride high- $T_{\rm c}$  superconductors.

#### **II. METHOD**

The Vienna *ab initio* simulation package (VASP) was used to calculate structure optimizations, enthalpies, and electronic properties [29]. The exchange correlation functional was parameterized by the Perdew-Burke-Ernzerhof (PBE) model to describe the electron interactions, and the projector augmented wave (PAW) method was chosen to describe electro-ion interactions [30,31]. The Rb  $4s^24p^65s^1$  and Li  $1s^22s^12p^0$  and Mg  $3s^23p^0$  and H  $1s^1$  orbitals were included as valence electrons. We set the threshold value for the forces of atomic positions relaxation with 1 meV/Å.

To illustrate the dynamical stability of  $A_2MgH_{16}$  (A = Li, Na, K, Rb, and Cs), based on density functional perturbation theory (DFPT), the dispersion calculation was performed using the educible set of a regular  $4 \times 4 \times 4$  *q*-point meshes with a kinetic-energy cutoff of 220 Ry, and the Methfessel-Paxton smearing width of 0.03 Ry was used. The QUANTUM ESPRESSO code was used in our DFPT calculations [32]. Then, the superconducting properties of Rb<sub>2</sub>MgH<sub>16</sub> and Li<sub>2</sub>MgH<sub>16</sub> were calculated by solving the Migdal-Eliashberg formalism using the EPW code [33–35]. Recently, this method was used to study the superconductivity of crystal structure and the high anisotropy of FSs [27,36–39]. The electronic wave functions required for the Wannier interpolation [40,41] within EPW were calculated on uniform and unshifted k-point meshes of size  $8 \times 8 \times 8$  for Rb<sub>2</sub>MgH<sub>16</sub> and Li<sub>2</sub>MgH<sub>16</sub>. The optimized norm-conserving Vanderbilt (ONCV) pseudopotentials with the valence electrons of Li  $1s^22s^1$ , Na  $2s^22p^63s^1$ , K  $3s^23p^64s^1$ , Rb  $4s^24p^65s^1$ , Cs  $5s^25p^66s^1$ , Mg  $2s^22p^63s^2$ , and H  $1s^1$  orbitals were used [42]. For the Eliashberg equations, we use  $20 \times 20 \times 20$  k-point and  $20 \times 20 \times 20$  q-point meshes. The frequency cutoff was set to ten times the maximum phonon frequency of the system, and the electrons and phonons smearing in Dirac- $\delta$  functions were used with widths of 30 and 0.5 meV, respectively.

The Migdal-Eliashberg equations was solved to study the superconducting properties. The temperature dependence of energy distribution of anisotropic superconducting gap can be estimated along the imaginary axis at the fermion Matsubara frequencies using the anisotropic Migdal-Eliashberg equations,

$$Z_{nk}(i\omega_j) = 1 + \frac{\pi T}{N(\varepsilon_F)\omega_j} \sum_{mk'j'} \frac{\omega_{j'}}{\sqrt{\omega_{j'}^2 + \Delta_{mk'}^2 (i\omega_{j'})}} \times \lambda(nk, mk', \omega_j - \omega_{j'})\delta(\varepsilon_{mk'} - \varepsilon_F)$$
(1)

$$Z_{nk}(i\omega_j)\Delta_{nk}(i\omega_j) = 1 + \frac{\pi T}{N(\varepsilon_F)\omega_j} \sum_{mk'j'} \frac{\omega_{j'}}{\sqrt{\omega_{j'}^2 + \Delta_{mk'}^2 (i\omega_{j'})}} \times [\lambda(nk, mk', \omega_j - \omega_{j'}) - \mu_c^*] \times \delta(\varepsilon_{mk'} - \varepsilon_F), \qquad (2)$$

where  $Z_{nk}(i\omega_j)$  is the mass renormalization function,  $\Delta_{nk}(i\omega_j)$  is the superconducting gap function,  $N(\varepsilon_F)$  is the electronic density of states at the Fermi level,  $\mu_c^*$  is the effective Coulomb pseudopotential parameter, and  $\omega$  and  $\varepsilon$  are the vibrational frequency and eigenvalues, respectively. The anisotropic  $\lambda(nk, mk', \omega_j - \omega_{j'})$  to be used in the Migdal-Eliashberg equation is given by

$$\lambda(nk, mk', \omega_j - \omega_{j'})$$
  
=  $\int_0^\infty d\Omega \frac{2\Omega}{(\omega_j - \omega_{j'})^2 + \Omega^2} \alpha^2 F(nk, mk', \Omega).$  (3)

Here,  $\alpha^2 F$  is the isotropic Eliashberg spectral function. As a result,  $T_c$  is defined as the temperature at which the leading edge  $\Delta_{nk}(i\omega_i)$  superconducting gap disappears.

## **III. RESULTS AND DISCUSSION**

The  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub> was predicted to be a potential room-temperature ternary superconductor with a remarkably high estimated  $T_c$  of ~351 K at 300 GPa [27]. Similarly, we predicted a new clathrate structure of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> by substituting Rb for Li at 300 GPa. The crystal structure search for Rb<sub>2</sub>MgH<sub>16</sub> at 300 GPa was performed by the swarm intelligence-based CALYPSO method [27,43,44]. The structure search reached the convergence after the generation of 2000 structures, and the predicted structure parameters are summarized in Table S1 of the Supplemental Material [45]. Based on the results, we constructure the formation energy convex hull of the Rb-Mg-H system as shown in Fig. S1 of the Supplemental Material [45]. For a new structure, we further calculated the formation enthalpy with the VASP code [29]. The formation enthalpy for Rb<sub>2</sub>MgH<sub>16</sub> can be defined as [46]

$$\Delta H(\text{Rb}_{2}\text{MgH}_{16}) = H(\text{Rb}_{2}\text{MgH}_{16}) - \frac{2}{19}H(\text{Rb}) - \frac{1}{19}H(\text{Mg}) - \frac{16}{19}H(\text{H}), \quad (4)$$

where  $H(\text{Rb}_2\text{MgH}_{16})$  is the total enthalpy of  $\text{Rb}_2\text{MgH}_{16}$  per atom, and H(Rb), H(Mg), and H(H) are the enthalpy for the pure elements Rb, Mg, and *Cmca*-12-structured solid hydrogen at 300 GPa,  $\Delta H(\text{Rb}_2\text{MgH}_{16})$  is the formation enthalpy of

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 $Rb_2MgH_{16}$  per atom [47–49]. The *H* to be used in the enthalpy equation is given by

$$H = E + PV. \tag{5}$$

Here, *E* is the total energy of compound, *P* is the value of the pressure, and *V* is the volume of the compound. The zero-point energy (ZPE) is important to determine the phase stabilities of hydrogen-rich materials. The formation enthalpy  $\Delta H_{zero}$  with considering ZPE is defined as

$$\Delta H_{\rm zero} = \Delta H + \Delta ZPE, \tag{6}$$

where  $\Delta$ ZPE is the calculated ZPE of Rb<sub>2</sub>MgH<sub>16</sub>. The  $\Delta$ H<sub>zero</sub> value of Rb<sub>2</sub>MgH<sub>16</sub> is estimated to be -0.44 eV per atom (Table S2 of the Supplemental Material [45]). The results show that Rb<sub>2</sub>MgH<sub>16</sub> is thermally stable against decomposition into pure elements, but it is the metastable phase with respect to decomposition into the energetically most favorable binary phases. For the metastable phases of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub>, we also propose a possible synthetic route: RbH<sub>5</sub> + MgH<sub>4</sub> + H<sub>2</sub>  $\rightarrow$  Rb<sub>2</sub>MgH<sub>16</sub> with relative formation enthalpy at 300 GPa of -0.025 eV per atom.

The geometry of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> is a similar clathrate structure as  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub>, which consists of Mg-centered H<sub>28</sub> cages, Rb-centered H<sub>18</sub> cages, and no H<sub>2</sub> units (Fig. 1(a) and Fig. S2 of the Supplemental Material [45]). The H atoms have two different positions, which were denoted as H1 and H2, respectively. The H-H bond lengths are 0.97 and 1.08 Å, respectively, which is close to the H-H distance of  $\sim 0.98$  Å in atomic metallic hydrogen near 500 GPa [47]. To investigate the bonding nature, the electron localization function (ELF) was calculated as shown in Fig. 2. Here, the H-H bonds possess the feature of covalent bonds, whereas the bonds are ionic between H and Rb/Mg atoms. To analyze the ionicity, the Bader charges were calculated. The result shows that the charges transferring from Rb to H and Mg to H are 0.09 and 1.59 electrons per atom, respectively. It is clearly demonstrated that both Rb and Mg are electron donors in Rb<sub>2</sub>MgH<sub>16</sub>. We have also performed the electronic property calculations of the predicted structure. The electronic band structure and the PDOS are shown in Fig. 1(b). The PDOS reveal that electronic density of states is dominated by H2 atomic electrons at the Fermi level. The band dispersion shows the metallic character at 300 GPa. It is easy to find the two bands (denoted as n = 1, 2) crossing the Fermi level. The corresponding FS sheets for the two bands of n = 1, 2, which project their electronic states onto H1, H2, Rb, and Mg atoms at each FS sheets as shown in Fig. 1(c). The first FS sheet is topologically similar to a polyhedron centered on the  $\Gamma$  point, which is composed of the H2 s orbital (Fig. S3 of the Supplemental Material [45]). The second FS sheet with the cage shape around the  $\Gamma$  point is also mainly composed of the H2 *s* orbital (Fig. S3 of the Supplemental Material [45]). Furthermore, the large contribution of the H2 s orbital to the FS sheets results in the emergency of high  $T_c$  in Rb<sub>2</sub>MgH<sub>16</sub> as will be demonstrated later.

Before calculating superconducting properties, we calculated the phonon dispersion and projected phonon densities



FIG. 1. (a) Predicted crystal structure of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> at 300 GPa. (b) Calculated band structure and projected density of states (PDOS) of Rb<sub>2</sub>MgH<sub>16</sub>, and (c) the corresponding FS sheets for the two bands (n = 1, 2), which project their electronic states onto H1, H2, Rb, and Mg atoms at each FS sheet.



FIG. 2. ELF in (1 1 1) and (1  $\bar{1}$  4) sections for  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> at 300 GPa are plotted in the left and right panels, respectively.

of states (PHDOS) as shown in Fig. 3. There is no imaginary frequency in phonon calculation, implying that Rb<sub>2</sub>MgH<sub>16</sub> is dynamically stable. In this case, two regions can be distinguished in the PHDOS: a low-energy region below  $500 \text{ cm}^{-1}$ and a high-energy region in the range of  $500-2500 \text{ cm}^{-1}$ . The phonon modes in the low-energy region are associated with the vibration of Rb and Mg atoms. In the high-energy region, the phonon modes with a mixed character are associated with lattice vibration of H1 and H2 atoms, but H2 atoms are dominant, suggesting that the phonon modes of H2 atoms significantly contribute to the electron-phonon coupling (EPC) strength. Specifically, there are no high-frequency vibrations, indicating the absence of molecular H<sub>2</sub> units in Rb<sub>2</sub>MgH<sub>16</sub>. The frequencies of the highest optical phonon modes were calculated to be 2500 cm<sup>-1</sup>, similar to that of  $F d\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub>  $(2400 \text{ cm}^{-1})$  at 300 GPa and the  $I4_1$ /amd phase of metallic hydrogen (2600 cm<sup>-1</sup>) at 500 GPa [22,40]. In order to study superconducting properties of Rb<sub>2</sub>MgH<sub>16</sub>, the Eliashberg spectral function and the integrated frequency-dependent EPC strength were calculated (Fig. 3). Electron-coupling calculations for  $Rb_2MgH_{16}$  give a  $\lambda$  of 1.90. We can analyze that EPC strength  $\lambda$  increases with  $\omega$  increases in the region from 0 to 2400  $\text{cm}^{-1}$ , suggesting that the phonon modes from all energy regions participate in the increase in  $\lambda$ . The modes (above 500  $\text{cm}^{-1}$ ) of H1 and H2 atoms contribute approximately 50% of the total EPC strength ( $\lambda = 1.90$ ), and Rb



FIG. 3. Phonon spectrum, PHDOS, Eliashberg spectral function, and integrated frequency-dependent EPC strength for Rb<sub>2</sub>MgH<sub>16</sub>.

(below 500 cm<sup>-1</sup>) and Mg atoms (300–400 cm<sup>-1</sup>) account for approximately 45% and 5%, respectively. We displayed the normalized distribution of the anisotropic EPC strength on the FS sheets in Fig. S4 of the Supplemental Material [45]. It is noticeable that the distribution of  $\lambda$  on the second FS sheet represents the strong EPC of the n = 2 state stemming from the H2 *s* orbital.

In order to illustrate a relationship of the EPC and  $T_c$ of Rb<sub>2</sub>MgH<sub>16</sub>, we calculated the temperature dependence of energy distribution of anisotropic superconducting gap  $\Delta$ . In Figs. 4(a) and S5 of the Supplemental Material [45],  $T_c$  is estimated to be 130 K with a typical choice of Coulomb pseudopotential parameter  $\mu_c^* = 0.1$  at 300 GPa using the anisotropic Migdal-Eliashberg equations. We found a single superconducting gap with a wide-energy region from 17 to 28 meV at 20 K, which displays a strong anisotropic character. The superconducting gap at 20 K on the two FS sheets of  $Rb_2MgH_{16}$  is shown in Fig. 4(b). The distribution of EPC strength and superconducting gap on the two FS sheets are highly similar (Fig. S4 of the Supplemental Material [45]), and the EPC strength and superconducting gap on the two FS sheets are both dominated by the second FS (n = 2) sheets. Thus, we can conclude that superconducting gap  $\Delta$  correlates closely with the EPC strength on the individual FS sheets. In Fig. 4(a), the dashed line represents the calculated superconducting gap by solving the isotropic Migdal-Eliashberg formalism. The superconducting gap of Rb<sub>2</sub>MgH<sub>16</sub> closes at  $T_{\rm c} \sim 130$  K. The results demonstrated that the anisotropy of electron-phonon interactions in bands increases  $T_c$  by  $\sim 14$  K, similar to that of metallic hydrogen ( $\sim 10 \text{ K}$ ) and LaH<sub>10</sub> (~19 K) [36,50].

Through the calculations above, we found that the Rb substitution for Li reduces the  $T_c$  of Rb<sub>2</sub>MgH<sub>16</sub> compared with  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub> (~351 K) at 300 GPa. Thus, we investigated the electronic and superconducting properties of  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub> by analyzing FS sheets topology and its associated superconducting gap. The calculated electronic band structure and PDOS of  $Li_2MgH_{16}$  are shown in Fig. 5(a). By using the band-PDOS diagram, we found that the electronic density of states of H atoms near the Fermi level in Li<sub>2</sub>MgH<sub>16</sub> is higher than that in Rb<sub>2</sub>MgH<sub>16</sub>, and there are four bands (denoted as n = 1-4) crossing the Fermi level. In Fig. 5(b), we display the corresponding FS sheets and the electronic states of the four bands (n = 1-4) are projected onto H1, H2, Li, and Mg atoms at each FS sheets. The first FS sheet with mainly mixed H1 s and H2 s orbital characters is topologically composed of the six small cone shapes surrounding the  $\Gamma$ point (Fig. S6 of the Supplemental Material [45]). The second FS sheet is topologically similar with the complex shape around the  $\Gamma$  point, which is also composed of H1 s and H2 s orbitals (Fig. S6 of the Supplemental Material [45]). The shape of the third and fourth FS sheets are similar with the dish shape around the X point (Fig. S6 of the Supplemental Material [45]). However, these FS sheets have different orbital characters, the third FS sheet is composed of H1 s and H2 s orbitals, whereas the fourth FS sheet is mostly composed of the H2 s orbital.

In order to clarify the superconducting properties of  $Li_2MgH_{16}$ , the phonon dispersion, PHDOS, the Eliashberg spectral function, and the integrated frequency-dependent



FIG. 4. (a) Calculated temperature dependence of energy distribution of anisotropic superconducting gap of  $Rb_2MgH_{16}$ . The dashed line represents the calculated data based on isotropic Migdal-Eliashberg equations. (b) Calculated anisotropic superconducting gap  $\Delta_k$  at 20 K on different parts of the FS sheets.



FIG. 5. (a) Calculated band structure and PDOS of  $Li_2MgH_{16}$  and (b) the corresponding FS sheets for the four bands (n = 1-4), which project their electronic states onto H1, H2, Rb, and Mg atoms at each FS sheet.

EPC strength were calculated (Fig. S7 of the Supplemental Material [45]). The EPC strength ( $\lambda$ ) is estimated to be 3.47, which is contributed approximately 35% in the region from 0 to 700  $\text{cm}^{-1}$  by Li and Mg atoms. The H1 and H2 atoms account for 65% of the total coupling. For comparison with the Rb<sub>2</sub>MgH<sub>16</sub> system, the frequencies of the highest optical modes of  $Li_2MgH_{16}$  are down to 2350 cm<sup>-1</sup>. The results show that the high-frequency phonon modes produce softening, indicating that the enhancement of  $\lambda$  correlates with the softening of the high-frequency phonon modes. Furthermore, we show the normalized distribution of anisotropic EPC strength on the FS sheets in Fig. S8 of the Supplemental Material [45]. The distribution of  $\lambda$  on the four FS sheets represent the strong EPC of the n = 1-4 states stemming from H1 s and H2 s orbitals [Fig. 5(b)]. The results reveal that the larger contribution of H1 s and H2 s orbitals to the FS sheets in Li<sub>2</sub>MgH<sub>16</sub> leads to a considerable enhancement of EPC strength ( $\lambda = 3.47$ ) compared with Rb<sub>2</sub>MgH<sub>16</sub>. It is indicated that the high-temperature superconductivity of hydrides is determined by the contribution of H orbitals to the FS sheets. Thus, we can conclude that  $\lambda$  relates to the orbital characters of the electronic states on the FS sheets. Moreover, the increased number of FS sheets tend to enhance the EPC strength in similar hydride superconductors [36,51]. We subsequently calculated the temperature dependence of superconducting gap  $\Delta$  as shown in Fig. 6(a). T<sub>c</sub> is estimated to be 352 K in Fd3m-Li2MgH16 at 300 GPa with the Coulomb pseudopotential parameter  $\mu_c^* = 0.1$  by solving anisotropic Migdal-Eliashberg equations. For T < 240 K,  $\Delta$ is distributed from 67 to 84 meV. The calculated superconducting gap by solving isotropic Migdal-Eliashberg equations is represented by the dashed line in Fig. 6(a). The superconducting gap of Li<sub>2</sub>MgH<sub>16</sub> closes at  $T_c \sim 338$  K, indicating that the anisotropic electron-phonon interactions increase  $T_{\rm c}$ by  $\sim 14$  K. The single anisotropic superconducting gap of



FIG. 6. (a) Calculated temperature dependence of energy distribution of anisotropic superconducting gap of  $Li_2MgH_{16}$ . The dashed line represents the calculated data based on isotropic Migdal-Eliashberg equations. (b) Calculated anisotropic superconducting gap  $\Delta_k$  at 20 K on different parts of the FS sheets.

Li<sub>2</sub>MgH<sub>16</sub> at 20 K on the four FS sheets is shown in Fig. 6(b). It is noticeable that the four FS sheets have different orbital characters. Therefore, based on the results of  $\Delta$  and  $\lambda$  on the FS sheets, we can conclude that the strong anisotropic is associated with the different contribution of different orbital characters to the FS sheets.

Crystals containing heavier atoms typically host lower characteristic phonon frequencies, which tend to enhance the EPC strength [52]. However, our present calculations demonstrate that heavier Rb substitution for Li generates higher characteristic phonon frequencies. It is demonstrated that  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub> produces high-frequency phonon softening, leading to the increase in EPC strength. The EPC channels were reduced also due to the reduced number of FS sheets in  $Rb_2MgH_{16}$  compared with  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub>, resulting in the decrease in  $T_c$ . In addition, we also note that the contribution of H orbitals to the FS sheets is reduced in Rb<sub>2</sub>MgH<sub>16</sub> compared with  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub>. Therefore, it is demonstrated that the chemical tuning of the contribution of H orbitals to the FS sheets and the number of FS sheets through the electronic or hole doping is an effective strategic to search hydride high- $T_c$  superconductors [36,53].

In order to evaluate the reliability of our calculation results, we investigated the influence of spin-orbit coupling (SOC) on the bands near the Fermi energy as shown in Fig. S9 of the Supplemental Material [45]. We find that the bands with SOC near the Fermi energy have no obvious changes compared with the bands without SOC (Fig. 1), which show that relativistic effects have little impact on the electronic properties in the  $Rb_2MgH_{16}$  system. Therefore, we used the pseudopotential without SOC to analyze the properties of  $Rb_2MgH_{16}$ , which could attain the reasonable calculation results. Moreover, we substituted the Li atoms in  $Fd\bar{3}m$ -Li<sub>2</sub>MgH<sub>16</sub> with Na, K, and Cs at 300 GPa. We find that Na, K, and Cs substitution for Li produces substantial phonon softening in the system (Fig. S10 of the Supplemental Material [45]), which indicated that Na<sub>2</sub>MgH<sub>16</sub>, K<sub>2</sub>MgH<sub>16</sub> and Cs<sub>2</sub>MgH<sub>16</sub> are dynamically instable. The close similarity of the crystal structures  $A_2MgH_{16}$  (A = Li, Na, K, Rb, and Cs) allows an explanation of different dynamical stability: The degree of (in)stability of the alkali cation determines the stability of the compounds [54].

### **IV. CONCLUSION**

To summarize, we predicted a ternary hydride of the clathrate structure of  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> by Rb-doped MgH<sub>16</sub> at 300 GPa. The formation enthalpy calculations verified that  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> is a metastable phase compound, and the phonon calculations show that the  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> is dynamically stable. The first-principles calculations for  $Fd\bar{3}m$ -Rb<sub>2</sub>MgH<sub>16</sub> have shown that the constituent atoms on the two FS sheets are coupled with the phonon modes in the whole energy regions, indicating that phonon modes have a significant contribution to the EPC. Specifically, we displayed a single anisotropic superconducting gap on the two FS sheets of Rb<sub>2</sub>MgH<sub>16</sub>. It is, thus, revealed that the superconductivity of Rb<sub>2</sub>MgH<sub>16</sub> ( $T_c \sim 130$  K) is determined by the contribution of H2 s orbitals to the two FS sheets. In addition, the calculations revealed Li<sub>2</sub>MgH<sub>16</sub> has four FS sheets with mixed H1 s and H2 s orbital character and reaches the  $T_c$  of 352 K at 300 GPa. Compared with Rb<sub>2</sub>MgH<sub>16</sub>, Li<sub>2</sub>MgH<sub>16</sub> produces high-frequency phonon softening. Moreover, we find that Rb substitution for Li results in the reduction of the EPC channels and contribution of H orbitals to FS sheets, leading to a smaller  $T_c$  in Rb<sub>2</sub>MgH<sub>16</sub> compared with Li2MgH16. Our present findings may be helpful to find more hydrogen-rich high- $T_c$  superconductors in the extended compositional space, thereby identifying the dominant microscopic mechanisms of high-temperature superconductivity in these and related systems.

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 $Rb_2MgH_{16}$  with SOC, phonon spectrum relations, the predicted structural parameters of  $Rb_2MgH_{16}$  at 300 GPa, and the calculated energy, enthalpy, and the ZPE value for compounds at 300 GPa.

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