# Intrinsic chiral-spin glass and spin glass transitions in compacted Ni<sub>5</sub>Al<sub>3</sub>/NiO core/shell nanoparticles

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We report the first synthesis of the intermetallic compound  $Ni_5Al_3$  in the nanocrystalline form. At the nanometer length scale, a stable Ni<sub>5</sub>Al<sub>3</sub> compound (core) does not form without a NiO shell. Detailed structural and microstructural characterization as well as the compositional analysis indicate the presence of Ni<sub>5</sub>Al<sub>3</sub>/NiO core/shell nanoparticles and log-normal crystallite size distribution with mean size  $d \simeq 6$  nm. "Zero-field" (dc magnetic field, H = 0 linear ( $\chi_1$ ) and nonlinear ( $\chi_n$  with n = 2, 3, 4, 5) ac-magnetic susceptibilities have been measured as functions of temperature at various ac driving field amplitudes over three decades of frequency on the nanocrystalline samples  $S_1$  and  $S_2$  of composition  $Ni_{5+x}Al_{3-x}/NiO$  (x = 0.014) and  $Ni_{5-x}Al_{3+x}/NiO$ (x = 0.038), respectively.  $\chi_1(T)$  and  $\chi_n(T)$  are shown to provide conclusive evidence for the existence of two spin glass (SG) thermodynamic phase transitions: one at  $T_i(H = 0) \simeq 138$  K and the other at a lower temperature  $T_p(H=0) \simeq 108$  K in both S<sub>1</sub> and S<sub>2</sub>. Linear ac-susceptibility in superposed dc magnetic fields demonstrates that the thermodynamic nature of these transitions is preserved in finite fields and the H-T phase diagrams for the samples  $S_1$  and  $S_2$  conform very well only with the H-T phase diagram predicted by the chirality-driven spin glass (SG) ordering model for a three-dimensional nearest-neighbor Heisenberg SG system with weak random anisotropy. A detailed comparison between theory and experiment permits us to unambiguously identify various "zero-field" and "in-field" SG phase transitions as (i) the simultaneous paramagnetic (PM): chiral-spin glass (CG) and PM-SG phase transitions at  $T_i(H)$  and (ii) the replica symmetry-breaking SG transition at  $T_p(H)$ .

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# I. INTRODUCTION

Transition-metal (TM = Fe, Co, Ni) aluminides (TM<sub>x</sub>Al<sub>y</sub>) exhibit a rich variety of physical properties due to the dramatic changes in the density of states  $N(E_F)$  at the Fermi level  $E_F$  caused by the choice of the TM atom, alloy composition (stoichiometric or otherwise), chemical pressure (doping) and local defects (vacancies and antisite disorder) [1,2]. Nickel aluminides (Ni-Al) stand out because a number of stable crystallographic phases have finite homogeneity ranges (i.e., the single phase regions have finite widths) around stoichiometric compositions, spanning Ni concentrations as large as  $\simeq 14 at. \%$  Ni. Existence of a homogeneity range provides a unique opportunity to study the effects of off-stoichiometry. This is so because the deviations from the stoichiometric composition either result in the substitution of excess nickel atoms onto aluminum sites in the Ni-rich compositions or in the formation of vacancies on the nickel sites in the Al-rich compositions. The crystallographic phase diagram [3] for the Ni-Al binary alloy system asserts that there are five stable equilibrium phases: Ni<sub>3</sub>Al, Ni<sub>5</sub>Al<sub>3</sub>, NiAl, Ni<sub>2</sub>Al<sub>3</sub>, and NiAl<sub>3</sub>. Among these phases, Ni<sub>3</sub>Al and NiAl [1,3] have attracted maximum attention for the reasons stated below.

The intermetallic compound Ni<sub>3</sub>Al has the ordered  $L1_2$ (fcc) structure and is, by far, the most extensively studied system [4–6] because it exhibits many fascinating novel physical phenomena, which include: weak itinerant-electron magnetism [4,7–15], pressure-induced [16] or compositional and/or site disorder-induced [17–19] non-Fermi liquid (NFL) behavior in bulk  $Ni_{75\pm x}Al_{25\mp x}$ , thermoelastic martensitic transformation and ferromagnetic shape memory effects in bulk Fe-doped Ni<sub>3</sub>Al [20,21]. In sharp contrast, depending upon the average crystallite size d, nanocrystalline Ni<sub>3</sub>Al exhibits exchange-enhanced Pauli spin paramagnetism for d = 25 nm [22], antiferromagnetic spin fluctuation-mediated superconductivity and NFL behavior [23] close to the quantum critical point [24] when  $d \simeq 50$  nm and chirality-driven intrinsic spin-glass ordering [25] when d ranges from 5 to 50 nm.

By comparison, the equiatomic NiAl phase with an ordered B2 (b.c.c) structure is reported to exhibit Pauli paramgnetic behavior [26]. A peculiar minimum in resistivity at  $x \simeq 50$  at. % Al in Ni<sub>100-x</sub>Al<sub>x</sub> alloys was observed in the resistivity-x plots [26]. High thermal conductivity, excellent oxidation-resistant property, low-density and high strength, compared to the other Ni-based superalloys, make this compound suitable for aerospace applications such as highpressure turbine blades [3].

Though the Ni<sub>5</sub>Al<sub>3</sub> phase [27–29] is stable at temperatures  $\simeq 973$  K within the range  $\sim 64-68$  at. % Ni, and falls in between the NiAl phase (extending from 45 to 59 at. % Ni)

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and the Ni<sub>3</sub>Al phase (occurring in the range 73–77 at. %Ni) in the Ni-Al phase diagram, hardly anything is known about the physical properties of this Ni-rich intermetallic compound. This is so because it is a formidable task to obtain Ni<sub>5</sub>Al<sub>3</sub> as a single phase in the bulk form [29,30]. To elucidate this point further, the formation of the Ni<sub>5</sub>Al<sub>3</sub> phase proceeds via a two-step process: the Ni-rich *B*2 NiAl (stable at high temperatures), with excess Ni atoms on Al sites, undergoes Bain distortion [28,31] and transforms into the *L*1<sub>0</sub>-NiAl martensite upon quenching from temperatures as high as 1523 K, and the subsequent aging at elevated temperatures (823 K for 15 days) causes ordering of the Ni atoms on the Al sublattice leading to the transformation of the martensite phase into Ni<sub>5</sub>Al<sub>3</sub>.

In this work,  $Ni_5Al_3$  has been synthesized, for the first time, in the nanocrystalline form, by inert gas condensation (IGC) technique. After a thorough structural, microstructural and compositional characterization, detailed investigation of the magnetic properties of nanocrystalline  $Ni_5Al_3$  has been undertaken with a view to unravel the true nature of the intrinsic magnetic ordering.

## **II. SYNTHESIS AND CHARACTERIZATION**

From several attempts to synthesize the intermetallic compound  $Ni_5Al_3$  in the nanocrystalline state by inert-gas condensation technique [32,33], we found that, at the nanometer length scale, the  $Ni_5Al_3$  compound could be formed only when a target composition richer in the Ni content than the stoitiometric composition is used and the residual oxygen level in the sample chamber is varied by evacuating the chamber to different pressures in the range  $10^{-7}$  to  $10^{-8}$ Torr prior to back-filling the chamber with a few mTorr inert gas (helium) and starting the thermal evaporation process. From this finding, we conclude that a NiO shell is necessary to stabilize the Ni<sub>5</sub>Al<sub>3</sub> crystalline core of the nanoparticles. The Ni<sub>5</sub>Al<sub>3</sub>/NiO core/shell nanoparticles, so synthesized, were compacted *in situ* up to pressures as high as 2 GPa to form discs (referred to as the samples S<sub>1</sub> and S<sub>2</sub>) of 8 mm diameter and 0.355 mm thickness. Rietveld analysis of room temperature x-ray diffraction (XRD) patterns, taken on S<sub>1</sub> and S<sub>2</sub>, revealed that a major Ni<sub>5</sub>Al<sub>3</sub> phase coexists with a minor NiO phase, as is evident from Fig. 1.

Ni<sub>5</sub>Al<sub>3</sub> compound has the Pt<sub>5</sub>Ga<sub>3</sub>-type orthorhombic unit cell structure with space group  $D_{2h}^{19}$  (*Cmmm*) [27,28], whereas NiO belongs to the Fm-3m (face-centered cubic) space group. Numerical estimates of the phase fractions, lattice parameters and lattice strains of the individual phases are given in Table I. Significant broadening of the diffraction peaks indicates that the samples under investigation are made up of tiny crystallites in the sub-micron range. From the XRD data, the volume-weighted mean crystallite size (d) has been determined as follows. The linewidth  $\beta_{hkl}$  of a Bragg peak of given (hkl), obtained from the Rietveld refinement of the XRD pattern, is the sum of size-  $(\beta_d)$  and strain-broadening  $(\beta_{\varepsilon})$ contributions. The size-strain plots [35], shown in the sub-Figs. 1(c) and 1(d), are based on the relation  $(\beta_{hkl}^*/d_{hkl}^*)^2 =$  $(1/d) \ (\beta_{hkl}^*/(d_{hkl}^*)^2) + (\varepsilon/2)^2$ , where  $\beta_{hkl}^* = \beta_{hkl} \ cos\theta/\lambda$  and  $d_{hkl}^* = 2d_{hkl} \sin\theta/\lambda$  and  $\varepsilon$  is the strain, yielded by the Halder-Wagner approximation [36]. Thus, intercept (on the ordinate) and slope of the linear  $(\beta_{hkl}^*/d_{hkl}^*)^2$  versus  $(\beta_{hkl}^*/(d_{hkl}^*)^2)$  plot



FIG. 1. [(a) and (b)] Room-temperature x-ray diffraction patterns along with the Rietveld refinement fits for the samples  $S_1$  and  $S_2$ . Note that, for the sake of clarity, the Miller indices are shown only for the prominent Bragg peaks. The linear Halder-Wagner plots, shown in the right panels (c) and (d), and based on the relation  $[36] (\beta_{hkl}^*/d_{hkl}^*)^2 = (1/d) (\beta_{hkl}^*/(d_{hkl}^*)^2) + (\varepsilon/2)^2$  (for definitions of the symbols, see text), yield the average crystallite size *d* as  $6 \pm 1$  nm for both  $S_1$  and  $S_2$ . Insets of (c) and (d) highlight the log-normal distribution of crystallite sizes (obtained from the TEM images) in both the samples.

Samples	Phases	Phase fractions (%)	lattice parameters (Å)	lattice strain
$\overline{S_1}$	Ni <sub>5</sub> Al <sub>3</sub> <sup>a</sup>	84.0(2.0)	a = 7.580(4), b = 6.561(4), c = 3.734(2)	$6.4 \times 10^{-4}$
	NiO <sup>a</sup>	16.0(1.0)	a = b = c = 4.185(3)	_
$S_2$	Ni <sub>5</sub> Al <sub>3</sub> <sup>a</sup>	79.4(2.7)	a = 7.211(4), b = 7.059(4), c = 3.658(2)	$2.1 \times 10^{-3}$
-	NiO <sup>a</sup>	20.6(1.1)	a = b = c = 4.120(2)	_
	Bulk-Ni <sub>5</sub> Al <sub>3</sub> <sup>b</sup>	100	a = 7.475, b = 6.727, c = 3.732	_
	Bulk-NiO <sup>c</sup>	100	a = b = c = 4.170(2)	_

TABLE I. Structural information from Rietveld refinement of the XRD patterns; for details, refer to Table S1 [34].

<sup>a</sup>Present work.

<sup>b</sup>ICDD-04-007-0381.

°ICDD-04-002-5335.

determine the average strain  $\varepsilon$  and average crystallite size d, respectively. The values of d, so determined, for the samples  $S_1$  and  $S_2$  are  $6.5 \pm 1.0$  and  $5.0 \pm 1.5$  nm. Compositional analysis of the samples by wavelength dispersion x-ray spectroscopy (WDS) yielded the composition of the samples as,  $S_1$ : 84.98% $Ni_{5+x}Al_{3-x}$  (x = 0.0144) + 15.02% NiO and S<sub>2</sub>: 79.80 % Ni<sub>5-x</sub>Al<sub>3+x</sub> (x = 0.0384) + 20.20% NiO. The phase fractions estimated from the detailed WDS analysis are in excellent agreement with those obtained from the Rietveld refinement of the XRD patterns (see Table I). The microstructure and crystallite size distribution were determined by high-resolution transmission electron microscopy (HR-TEM). Since the samples are brittle in nature, they could be powdered using a mortar and pestle. The powder was dispersed in ethanol by ultrasonic shaker. The dispersion was drop-cast onto a copper grid and dried, before the TEM examination. TEM images of agglomerates and individual crystallites as well as the selected-area electron diffraction (SAED) patterns, are shown in Fig. 2. TEM images in Fig. 2 indicate that Ni<sub>5</sub>Al<sub>3</sub> crystalline core of average size  $\simeq 6$  nm is surrounded by a structurally disordered NiO shell; the latter inference is drawn from the absence of lattice fringes corresponding to the shell. Disordered nature of the NiO shell is also supported by the unusually broad x-ray Bragg peaks of the NiO phase [Figs. 1(a) and 1(b)] and the diffuse rings in the SAED pictures (Fig. 2). Note that these rings and the bright spots on them correspond to the Bragg reflections of NiO and Ni<sub>5</sub>Al<sub>3</sub>, respectively. Furthermore, the spacing between the lattice fringes observed for the Ni<sub>5</sub>Al<sub>3</sub> core corresponds to the set of parallel (221) atomic planes.

A large number of crystallites of different sizes were visually counted from the TEM images taken from different regions of the sample and their sizes were determined by the ImageJ software package. The crystallite size histograms are displayed in the insets of Figs. 1(c) and 1(d). The size distribution is well represented by the log-normal distribution with a mean crystallite size (*d*) of 6.5(1.4) nm for S<sub>1</sub> and 5.0(1.5) nm for S<sub>2</sub>. The numbers in the parentheses give the standard deviation in *d*. These values of *d* compare favorably with those determined from the Halder-Wagner plots [Figs. 1(c) and 1(d)].

#### **III. AC MAGNETIC RESPONSE**

The first five harmonics of the ac magnetic response comprising the real and imaginary components of *linear* ( $\chi_1$ ) as well as second to fifth-order *nonlinear* magnetic susceptibilities ( $\chi_2-\chi_5$ ) of the nanocrystalline Ni<sub>5</sub>Al<sub>3</sub>/NiO samples, S<sub>1</sub> and S<sub>2</sub>, were measured over the temperature range 1.8–320 K at the rms amplitudes  $h_{ac} = 0.1$ , 1 and 10 Oe and frequencies 11 Hz  $\leq \omega \leq$  9999 Hz of the ac driving field, using Quantum Design PPMS-ACMS option.  $\chi_1(T)$  was also measured at  $h_{ac} = 1$  Oe and  $\omega = 111$  Hz in fixed superposed dc fields,  $H \equiv H_{dc}$ , in the range 0–100 Oe. All the measurements were



FIG. 2. Transmission electron microscope (TEM) images of the nanoparticle agglomerates, high-resolution TEM images of single nanoparticles, and selected area electron diffraction (SAED) patterns of the samples  $S_1$  (left panels) and  $S_2$  (right panels).

carried out on the discs of 3 mm diameter spark-cut from the 8-mm diameter nanocrystalline  $Ni_5Al_3/NiO$  discs, prepared by inert gas condensation.

The nonlinear magnetic susceptibilities, defined by the expansion of the magnetization *m* in powers of a weak external magnetic field  $h = h_{ac} + h_{dc}$ , as

$$m = m_0 + \chi_1 h + \chi_2 h^2 + \chi_3 h^3 + \chi_4 h^4 + \chi_5 h^5 + \dots$$
 (1)

[where  $m_0$  is the spontaneous magnetization,  $\chi_1$  is the linear susceptibility and  $\chi_n$  (n = 2, 3,...) are the nonlinear susceptibilities] were measured because they enable an unambiguous distinction to be made between different types of magnetic order. To briefly illustrate this point, the above relation with  $m_0 \neq 0$  describes ferromagnetic order at  $T < T_C$ , as opposed to the paramagnetic state at  $T > T_C$  or a spin glass (SG) state at all temperatures (including the critical region near the SG transition temperature) when  $m_0 = \chi_2 = \chi_4 = 0$ .

#### **IV. LINEAR SUSCEPTIBILITY**

## A. Frequency dependence

Figure 3 displays the temperature variations of the linear ac magnetic susceptibility ( $\chi_1$ ) for the samples S<sub>1</sub> and S<sub>2</sub> at fixed frequencies ranging from 11 to 9999 Hz in the temperature range 2 K  $\leq T \leq 200$  K without superposed dc field. The main observations are as follows. In both the samples S<sub>1</sub> and S<sub>2</sub>,  $\chi_1(T)$  exhibits a peak at  $T_p \simeq 108$  K (where  $d\chi_1/dT = 0$ ) and an inflection point at  $T_i \simeq 138$  K [where  $d\chi_1(T)/dT$  goes through a *sharp minimum*]. That  $T_p$  and  $T_i$  mark the temperatures at which two different types of spin glass transitions occur and the exact nature of these transitions will become clear in the subsequent text. For a given frequency, these characteristic temperatures remain essentially unaltered when  $h_{ac}$  increases from 0.1 to 1.0 Oe, but change drastically at  $h_{ac} = 10$  Oe. At any given  $h_{ac}$  in both S<sub>1</sub> and S<sub>2</sub>,  $T_p$  and  $T_i$  get displaced to higher temperatures with increasing frequency and the values of  $\chi_1$  at these temperatures decrease, as is normally the case with spin glasses or superparamagnets.

Since the Curie temperature  $T_c$ , at which a thermodynamic phase transition from a ferromagnetic (FM) state to a paramagnetic (PM) state occurs, does not depend on the choice of the experimental time-window, the dependence of  $T_p$  and  $T_i$  on frequency completely rules out the possibility of a FM-PM phase transition at these temperatures. A standard measure of the frequency-induced peak-shift that facilitates a direct comparison between different spin glass (SG) and/or superparamagnet (SP) systems is the fractional change in  $T^*$  (in the present case,  $T^*$  stands for  $T_p$  or  $T_i$ ) per decade of frequency,  $\Delta T^*/(T^* \Delta \log_{10} \omega); T^*(\omega) \longrightarrow T_g$  in the limit  $\omega \longrightarrow 0$  while  $T_g$  denotes the transition temperature for a SG or the blocking temperature for a SP. The values of the quantities  $\Delta T_p/(T_p \Delta \log_{10} \omega)$  and  $\Delta T_i/(T_i \Delta \log_{10} \omega)$ , determined in this work for  $S_1$  and  $S_2$  at  $h_{ac} = 0.1$ , 1, and 10 Oe, are listed in Table II. These values are closer to the peak-shift per decade of frequency, previously reported in the chiralitydriven spin glass system Ni<sub>3</sub>Al nanoparticle aggregates [25] and for the canonical spin glasses (AgMn, CuMn, AuMn, PdMn) [37-39]. This agreement strongly indicates that the nanocrystalline systems in question could be spin glasses.

In order to verify whether or not the observed magnetic ordering conforms to that of a spin glass, we follow the customary approach to ascertain if the frequency-induced shift in  $T_p$  and  $T_i$  can be explained in terms of the critical slowing down (CSD) model [40–42] for a spin glass. This model



FIG. 3. Linear susceptibility,  $\chi'_1(T)$ , for the nanocrystalline samples  $S_1$  and  $S_2$  in the temperature range 2 K  $\leq T \leq 200$  K at the rms amplitudes of the ac driving field  $h_{ac} = 0.1$ , 1, and 10 Oe and frequencies,  $\omega$ , ranging from 11 to 9999 Hz. Insets highlight the small but finite shift of  $T_p$  [the temperature at which  $\chi'_1(T)$  peaks] and  $T_i$  [where  $d\chi'_1(T)/dT$  goes through a minimum] to higher temperatures with increasing frequency. The vertical dashed line, marking the  $T_p$  or  $T_i$  for the lowest frequency, serves as a reference for the frequency-induced shift.

TABLE II. Relative shift in the transition temperatures  $T_p$  and  $T_i$  per decade of frequency.

Sample	$h_{\rm ac}$ (Oe)	$\Delta T_p / [T_p \ \Delta \log_{10} \omega]$	$\Delta T_i / [T_i \ \Delta \log_{10} \omega]$
$\overline{S_1^a}$	0.1	0.0050(2)	0.0053(3)
	1.0	0.0055(2)	0.0043(3)
	10.0	0.0124(5)	0.0182(5)
$S_2^{a}$	0.1	0.0053(3)	0.0046(3)
	1.0	0.0071(3)	0.0042(2)
	10.0	0.0113(2)	0.0038(3)
nc-Ni <sub>3</sub> Al <sup>b</sup>	1.0	0.0055(1)	0.0022(2)
CuMn <sup>c</sup>	0.1	0.0050	-
AuMn <sup>d</sup>	0.1	0.0045	-
AgMn <sup>e</sup>	0.1	0.0060	_

<sup>a</sup>Present work.

<sup>b</sup>Reference [25].

<sup>c</sup>Reference [37].

<sup>d</sup>Reference [38].

<sup>e</sup>Reference [39].

predicts the following functional dependence of  $T^*$  on  $\omega$ :

$$\omega = \omega_0 \left[ \frac{T^*(\omega) - T_g}{T_g} \right]^{z\nu},\tag{2}$$

where  $\omega_0$  is the attempt frequency,  $T_g = T^*(\omega = 0)$  is the true spin glass freezing temperature, z is the dynamic critical exponent that characterizes the relaxation rate of the correlated spin-cluster dynamics (the critical slowing down) and  $\nu$  is the critical exponent for the spin-spin correlation length. The *linear* log<sub>10</sub>( $\omega/\omega_0$ ) versus log<sub>10</sub>( $[T^*(\omega) - T_g]/T_g$ ) plots in Fig. 4 testify to the validity of the CSD model with the choice of the parameters (Table III) that are typical of canonical spin glasses CuMn, PtMn, etc. For conventional

TABLE III. Critical slowing down model parameters. SG(i) and SG(p) stand for the spin glass (SG) transitions occurring at frequency-dependent temperatures  $T_i$  and  $T_p$ , respectively.

Sample	$h_{\rm ac}$ (Oe)	Transition	$\omega_0$ (Hz)	zv	$T_g$ (K)
<b>S</b> <sub>1</sub>	0.1	SG(p)	$1.12(15) \times 10^{12}$	5.4(4)	108.45(50)
	0.1	SG(i)	$3.42(38) \times 10^{12}$	5.5(5)	138.50(50)
	1.0	SG(p)	$8.26(6) \times 10^{11}$	5.1(1)	108.74(60)
	1.0	SG(i)	$2.50(7) \times 10^{12}$	5.1(3)	138.63(50)
	10.0	SG(p)	$3.30(60) \times 10^{10}$	5.3(4)	102.00(50)
	10.0	SG(i)	$5.20(20) \times 10^{10}$	5.5(5)	129.60(20)
S <sub>2</sub>	0.1	SG(p)	$7.52(90) \times 10^{11}$	5.0(1)	108.36(50)
	0.1	SG(i)	$2.39(40) \times 10^{12}$	5.0(2)	138.34(50)
	1.0	SG(p)	$1.44(60) \times 10^{12}$	5.6(2)	108.56(50)
	1.0	SG(i)	$2.18(70) \times 10^{12}$	4.99(4)	138.75(50)
	10.0	SG(p)	$9.60(20) \times 10^9$	5.0(2)	111.50(50)
	10.0	SG(i)	$5.00(10) \times 10^{11}$	4.5(5)	131.50(50)

spin glass systems,  $\omega_0$  is  $\sim 10^9 - 10^{13}$  and  $z\nu$  lies in the range 4–10. The presently determined values for the quantity  $z\nu$  are in excellent agreement with those ( $z\nu = 5.0(5)$  [43] and  $z\nu = 5.5(1)$  [44–48]), predicted by the three-dimensional Heisenberg chiral spin glass model which considers nearest-neighbor exchange interactions with Gaussian distribution.

The data presented in Fig. 3 assert that both samples  $S_1$  and  $S_2$  exhibit two SG-like transitions each: one at  $T_g \equiv T_i(\omega = 0)$  and the other at  $T_g \equiv T_p(\omega = 0)$ . The  $T_g$  values at different  $h_{ac}$  are given in Table III.

#### **B.** Direct-current field dependence

In order to find out whether or not the "zero-field" SG transitions are affected by finite static magnetic fields, the



FIG. 4. The linear  $\log_{10}(\omega/\omega_0)$  vs  $\log_{10}([T^*(\omega)-T_g]/T_g)$  plots with  $T^*(\omega)$  standing for either  $T_p(\omega)$  or  $T_i(\omega)$  and  $T_g = T^*(\omega = 0)$ , validate the critical slowing down model [40–42], proposed for a spin glass, in the samples S<sub>1</sub> (left) and S<sub>2</sub> (right).



FIG. 5. Temperature variations of the "in-phase" component of ac susceptibility,  $\chi'_1$ , at  $h_{ac} = 1$  Oe and frequency,  $\omega = 111$  Hz, measured in the superposed dc magnetic fields, on the samples  $S_1$  and  $S_2$ , over the temperature range 2 K  $\leq T \leq 200$  K.

"in-phase" ( $\chi'_1$ ) and "out-of-phase" ( $\chi''_1$ ) components of linear ac susceptibility were measured at  $h_{ac} = 1$  Oe and  $\omega = 111$  Hz in different superposed dc magnetic fields  $H \equiv H_{dc}$ .

TABLE IV.	Parameters	for the fits	based on	Eq. (3)	١.
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Parameters	$S_1$	$S_2$
Field Talige (Oe)	0 ≤ H ≤ 30	0 ≤ Π ≤ 30
$T_p(0)$ (K)	108.92(25)	108.94(26)
$C_p$ (Oe <sup><math>-n_p</math></sup> )	0.0188(3)	0.0195(3)
n <sub>p</sub>	0.67(1)	0.67(2)
$T_i(0)$ (K)	138.40(50)	138.00(30)
$C_i (\operatorname{Oe}^{-n_i})$	0.0097(2)	0.0072(1)
n <sub>i</sub>	0.67(1)	0.67(2)

As is evident from Fig. 5, in both samples, the peak in  $\chi'_1(T)$  (also in  $\chi''_1(T)$  but not shown) gets progressively suppressed, smeared out and displaced to lower temperatures with increasing  $H_{dc}$ .  $H_{dc}$  shifts the inflection point (and hence  $T_i$ ) also to lower temperatures but at a slower rate than in the case of  $T_p$ . Figure 6 shows that the temperatures  $T_p(H)$  [at which  $\chi'_1(T)$  peaks] and  $T_i(H)$  [which corresponds to the inflection point in  $\chi'_1(T)$  at fixed H] vary with the static field in accordance with the relation

$$t_x = \frac{T_x(0) - T_x(H)}{T_x(0)} = C_x \ H^{n_x},$$
(3)

where the subscript x in Eq. (3) stands for either p or i and the parameters  $T_x(0)$ ,  $C_x$  and  $n_x$  are given in Table IV. The  $T_x(H)$  data, shown in Fig. 6, are re-plotted in the form of the *H*-*T* diagram in Fig. 7(a) to facilitate a direct comparison of  $T_x(H)$  with theoretical predictions. There are basically two types of theoretical models [25,49]: (I) the mean-field



FIG. 6.  $H^{2/3}$  power-law variations of the reduced field-induced shifts ( $t_p$  and  $t_i$ ) in the critical temperatures  $T_p$  and  $T_i$ .



FIG. 7. (a) *H*-*T* phase diagrams for the samples  $S_1$  and  $S_2$ . (b) Schematic sketchs of the H - T phase diagrams predicted by the mean-field (MF) and chiral-spin (CS) models for the isotropic and anisotropic cases, as elucidated in the text. The notations SG, CG, and PM refer to spin glass, chiral glass, and paramagnetic phases. AT and GT refer to the literature (see text).

(MF) models [49–51] for a three-dimensional (3D) isotropic Heisenberg vector-spin glass with [49,52] or without [50] weak random anisotropy (RA), in which the spin components interact with one another through infinite-range exchange interactions, and (II) the chiral-spin (CS) models [25,44-48,53] for an isotropic classical 3D Heisenberg spin glass system in which the spin-frustration-induced, energetically degenerate 'left-handed' and 'right-handed' chiral states of local noncoplanar spin structures coexist with spins coupled by nearest-neighbor exchange interactions. Like the MF models, the chiral-spin models deal with both the isotropic and weak RA cases. The MF models for a 3D Heisenberg spin glass with or without weak RA predict only one "zero-field" SG transition at  $T_g(0)$  which, in the isotropic case, branches out into the Gabay-Toulouse (GT) [50] and Almeida-Thouless (AT) [51] irreversibility lines in finite fields. RA does not affect the AT line but converts the GT line into the AT transition line at weak fields which crosses over to the GT line at higher fields [52]. The GT (AT) irreversibility line is given by Eq. (3) with the subscript x replaced by GT (AT) and  $n_{\text{GT}} = 2 (n_{\text{AT}} = 2/3)$ . In the isotropic case, the chiral-spin models predict that, when a 3D isotropic Heisenberg spin glass system is cooled in zero field, the system undergoes a chiral-glass (CG) transition at a finite temperature T = $T_{CG}(0)$  followed by a spin-glass (SG) transition at a lower temperature  $T = T_{SG}(0) < T = T_{CG}(0)$ . In finite fields, the CG and SG transition lines,  $T_{CG}(H)$  and  $T_{SG}(H)$ , have the same functional form at low fields as that of the GT and AT lines. In the presence of weak RA, the chiral-spin models for a 3D Heisenberg spin glass system make the following predictions. When H = 0, the CG and SG transitions occur simultaneously at a finite temperature  $T_{CG+SG}(0)$ , followed by another SG transition at a lower temperature  $T_{SG}(0) < T_{CG+SG}(0)$ . At sufficiently low fields (where RA dominates over H), the  $T_{CG+SG}(H)$  and  $T_{SG}(H)$  phase transition lines have the AT form. By contrast, at sufficiently high fields (where *H* swamps RA), the  $T_{CG+SG}(H)$  transition line has the GT form.

Schematic sketches of the H-T diagrams for the 3D Heisenberg spin glass without (isotropic case) and with weak random anisotropy (anisotropic case), predicted by the meanfield (MF: type I) and the chiral-spin (CS: type II) models, are shown in the panels (a) and (b) for MF and (c) and (d) for CS in Fig. 7(b). The observed H-T diagrams [displayed in Fig. 7(a)], when compared with those yielded by the models of type I and type II [sketched in Fig. 7(b)], completely rule out the applicability of the MF models to the present case since they predict only one zero-field SG transition as against two H = 0 SG transitions observed in this work. Such a comparison also brings out clearly that the H-T diagrams for both the samples  $S_1$  and  $S_2$  conform well with only the weak random anisotropy case [i.e., with the shaded portion of the H-T diagram in the panel (d) of Fig. 7(b)] within the chirality-driven SG ordering scenario. Thus, the phase transition lines  $T_i(H)$  and  $T_p(H)$ , of the AT form at low fields, can be unambiguously identified with  $T_{CG+SG}(H)$  and  $T_{SG}(H)$ , respectively. Note that  $T_{CG+SG}(H)$  is the temperature at any given H (including H = 0) at which the paramagnetic (PM) to chiral-spin glass (CG) and the PM-SG phase transitions occur simultaneously, whereas  $T_{SG}(H)$  is the field-dependent temperature at which the CG + SG mixed phase transforms to the replica symmetry-breaking SG phase.

## V. NONLINEAR SUSCEPTIBILITIES

Figures 8 and 9 depict the temperature variations of the odd harmonics ( $\chi_1$ ,  $\chi_3$   $h_{ac}^2$ ,  $\chi_5$   $h_{ac}^4$ ) and even harmonics ( $\chi_2$   $h_{ac}$ ,  $\chi_4$   $h_{ac}^3$ ) of the ac magnetic response observed in the samples S<sub>1</sub> and S<sub>2</sub> over the temperature ranges 1.8–200 K at the ac driving



FIG. 8. Odd harmonics,  $\chi_1$ ,  $\chi_3 h_{ac}^2$ , and  $\chi_5 h_{ac}^4$  (top) and even harmonics,  $\chi_2 h_{ac}$  and  $\chi_4 h_{ac}^3$  (bottom) of the ac magnetic response as functions of temperature in the range 2 K  $\leq T \leq 200$  K at  $h_{ac} = 0.1$  Oe, 1 Oe and frequencies,  $\omega$ , ranging from 11 to 9999 Hz for S<sub>1</sub>. The inset depicts that, as a function of temperature, the temperature derivative of  $\chi_1(T)$ ,  $d\chi_1/dT$ , at  $\omega = 111$  Hz and  $h_{ac} = 1$  Oe goes through a *sharp minimum* at  $T_i$  and *changes sign* at  $T_p$ . The same features in  $d\chi_1/dT$  are observed at other values of  $\omega$  and  $h_{ac}$  as well.



FIG. 9. Odd harmonics,  $\chi_1$ ,  $\chi_3$   $h_{ac}^2$ , and  $\chi_5$   $h_{ac}^4$  (top) and even harmonics,  $\chi_2$   $h_{ac}$  and  $\chi_4$   $h_{ac}^3$ , (bottom) of the ac magnetic response as functions of temperature in the range 2 K  $\leq T \leq 200$  K at  $h_{ac} = 0.1$  and 1 Oe and frequencies,  $\omega$ , ranging from 11 to 9999 Hz for S<sub>2</sub>. The inset depicts that, as a function of temperature, the temperature derivative of  $\chi_1(T)$ ,  $d\chi_1/dT$ , at  $\omega = 111$  Hz and  $h_{ac} = 1$  Oe goes through a *sharp minimum* at  $T_i$  and *changes sign* at  $T_p$ . The same features in  $d\chi_1/dT$  are observed at other values of  $\omega$  and  $h_{ac}$  as well.

field amplitudes  $h_{ac} = 0.1$  Oe and 1 Oe, and frequencies in the range 11 Hz  $\leq \omega \leq$  9999 Hz. The insets of Figs. 8 and 9 serve to illustrate how the temperatures corresponding to the inflection point  $(T_i \equiv T_{CG+SG})$  and the peak  $(T_p \equiv T_{SG})$  in  $\chi_1(T)$  are accurately determined by the temperatures where the temperature derivative of  $\chi_1(T)$ ,  $d\chi_1/dT$ , goes through a sharp minimum and changes sign. It is evident from Figs. 8 and 9 that, contrasted with the very noisy signals for all the harmonics (odd and even) at  $h_{ac} = 0.1$  Oe, particularly at low frequencies, the linear and nonlinear susceptibility data taken at  $h_{ac} = 1$  Oe exhibit well-resolved features at most frequencies. This is so because, barring the first harmonics, the higher harmonics signals at  $h_{\rm ac} = 0.1$  Oe for all  $\omega$  are weaker by an order of magnitude than those at  $h_{ac} = 1$  Oe. The higher the order of harmonics, the weaker the signal, with the result that it approaches the instrumental resolution limit at  $\omega$  < 3333 Hz. Thus, the data taken at  $h_{ac} = 1$  Oe are better suited for a meaningful comparison with theoretical predictions. To facilitate such a comparison, rough sketches of the characteristic experimental signatures in  $\chi_1(T)$ ,  $\chi_3(T)$ ,  $\chi_5(T)$  and  $\chi_2(T)$ ,  $\chi_4(T)$  in the critical region near FM-PM and SG-PM transitions, predicted by the theory [54-57] for an ideal ferromagnet (FM) and spin glass (SG), are shown in Fig. 10. A direct one-to-one comparison between the experimentally observed (Figs. 8 and 9) and theoretically predicted (Fig. 10) temperature variations and the signs of the odd and even harmonics reveals the following. The welldefined *negative cusps* at  $T_p$  and  $T_i$  at all frequencies in  $\chi_3(T)$ and for  $\omega \gtrsim 1111$  Hz in  $\chi_5(T)$ , when  $h_{ac} = 1$  Oe, and the shift to higher temperatures of  $T_p$  and  $T_i$  with increasing  $\omega$  (clearly visible in Figs. 8 and 9) provide strong evidence for the two SG transitions in both the samples  $S_1$  and  $S_2$ . Note that in S<sub>2</sub>, an additional minimum at the temperature,  $T_k \simeq 125$  K, at which a 'knee' appears in  $\chi_1(T)$ , is observed at all  $\omega$  in  $\chi_3(T)$ . However, in sharp contradiction with the theoretical prediction that, for an *ideal* spin glass,  $\chi_2 = \chi_4 = 0$  at all temperatures, including those embracing the critical region near  $T_g$ , two negative-minima at the frequency-dependent  $T_p$ and  $T_i$  (positive peaks at  $T_p$  and  $T_k$ , and a negative minimum at  $T_i$ ) are observed in  $\chi_2(T)$  at all frequencies and in  $\chi_4(T)$  for  $\omega \gtrsim 1111$  Hz in the sample S<sub>1</sub> (S<sub>2</sub>). From the theoretical standpoint, a negative divergence at  $T = T_C$  in  $\chi_2(T)$  and  $\chi_4(T)$  is symptomatic of a conventional FM-PM phase transition (Fig. 10). The frequency dependence of the temperatures at which such minima/peaks occur completely rule out thermodynamic FM-PM phase transitions. Moreover, the occurrence of two successive FM-PM transitions in S<sub>1</sub> is incomprehensible: if the transition at  $T_p$  is a FM-PM transition, the transition at a higher temperature  $T_i$  cannot be a yet another FM-PM transition. The reverse is also true. We believe that the minima and/or peaks in  $\chi_2(T)$  and  $\chi_4(T)$ mark strong departures from ideal SG behavior and their origin lies in the frustration-driven chiral-spin structures (rather than the frustrated individual spins) prevalent in the Ni<sub>5</sub>Al<sub>3</sub> nanoparticle cores. Such spin structures interact with one another through the competing exchange interactions generated in the intervening structurally-disordered NiO shells having noncollinear antiferromagnetic order.



FIG. 10. Schematic depiction of the temperature variations of the odd harmonics ( $\chi_1$ ,  $\chi_3$  and  $\chi_5$ ) and even harmonics ( $\chi_2$  and  $\chi_4$ ) of the ac magnetic response at the reduced field h = 1, predicted by theory [54–58] for a ferromagnet (FM) and a spin glass (SG) in the critical region.

At this stage, it should be emphasized that the global nature of spin glass ordering emerges only in the limit  $\omega \rightarrow 0$ and  $h_{ac} \rightarrow 0$ . That this is indeed the case in the samples  $S_1$  and  $S_2$ , is borne out by the observation that  $\chi_2 = \chi_4 \rightarrow 0$  (indicated by very weak resolution-limited signals and the statistically scattered data around "zero") at  $h_{ac} = 0.1$  Oe as the ac driving-field frequency falls below 1111 Hz.

Thus a thorough discussion of the zero-field/in-field linear ac-magnetic susceptibility,  $\chi_1(T, H)$  and nonlinear susceptibilities  $\chi_2(T)$ ,  $\chi_3(T)$ ,  $\chi_4(T)$  and  $\chi_5(T)$  enables us to conclude that the Ni<sub>5</sub>Al<sub>3</sub>/NiO core/shell nanoparticle samples, S<sub>1</sub> and

S<sub>2</sub>, behave as a 3D nearest-neighbor Heisenberg spin glass wherein weak random anisotropy couples the chiral and spin degrees of freedom. As a result of this mixing, the CG and SG transitions occur simultaneously at  $T_i \equiv T_{CG+SG}$  where the chiral  $Z_2$  and the continuous spin-rotation SO(3) symmetries are simultaneously broken. The replica symmetry, not spontaneously broken at  $T_{CG+SG}$ , breaks at a lower temperature  $T_p \equiv T_{SG}$ .

## VI. SUMMARY AND CONCLUSION

From the results of several attempts to synthesize the intermetallic compound Ni<sub>5</sub>Al<sub>3</sub> in the nanocrystalline form by inert gas condensation technique, we conclude that, at the nanometer length scale, a stable Ni<sub>5</sub>Al<sub>3</sub> compound (core) cannot be formed without a NiO shell. Detailed Rietveld refinement of the room temperature x-ray diffraction (XRD) patterns revealed the presence of a minor NiO phase concomitant with a major Ni<sub>5</sub>Al<sub>3</sub> phase while the compositional analysis by the wavelength dispersion x-ray spectroscopy (WDS) yielded the composition of the samples as,  $S_1$ : 84.98 % Ni<sub>5+x</sub>Al<sub>3-x</sub> (x = 0.0144) + 15.02 % NiO and S<sub>2</sub> : 79.80 % Ni<sub>5-x</sub>Al<sub>3+x</sub> (x = 0.0384) + 20.20 % NiO. The phase fractions estimated from the detailed WDS analysis are in excellent agreement with those obtained from the Rietveld refinement of the XRD patterns. High-resolution transmission electron microscopy indicated the core/shell nature of the Ni<sub>5</sub>Al<sub>3</sub>/NiO nanoparticles and a log-normal crystallite size distribution with the mean size  $d \simeq 6 \,\mathrm{nm}$ . Zero-field

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(dc magnetic field, H = 0) linear ( $\chi_1$ ) and nonlinear ( $\chi_n$ with n = 2, 3, 4, 5) ac-magnetic susceptibilities have been measured as functions of temperature at various ac driving field amplitudes over three decades of frequency on the nanocrystalline samples S<sub>1</sub> and S<sub>2</sub>.  $\chi_1(T)$  and  $\chi_n(T)$  provide a conclusive evidence for the existence of two spin glass (SG) thermodynamic phase transitions: one at  $T_i(H = 0) \simeq 138$  K and the other at a lower temperature  $T_n(H=0) \simeq 108$  K in both S1 and S2. Linear ac-susceptibility in superposed dc magnetic fields demonstrates that the thermodynamic nature of these transitions is *preserved* in finite fields and the H-Tphase diagrams for the samples  $S_1$  and  $S_2$  are in very good agreement only with the H-T phase diagram predicted by the chirality-driven spin glass (SG) ordering model for a three-dimensional nearest-neighbor Heisenberg SG system with weak random anisotropy. A detailed comparison between theory and experiment permits us to unambiguously identify various zero-field and in-field SG phase transitions as (i) the *simultaneous* paramagnetic (PM): chiral-spin glass (CG) and PM-SG phase transitions at  $T_i(H)$ , and (ii) the replica symmetry-breaking SG transition at  $T_p(H)$ .

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