Particle Multiplicities in Lead-Lead Collisions at the CERN Large Hadron Collider from Nonlinear Evolution with Running Coupling Corrections

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We present predictions for the pseudorapidity density of charged particles produced in central Pb-Pb collisions at the LHC. Particle production in such collisions is calculated in the framework of k_t factorization. The nuclear unintegrated gluon distributions at LHC energies are determined from numerical solutions of the Balitsky-Kovchegov equation including recently calculated running coupling corrections. The initial conditions for the evolution are fixed by fitting Relativistic Heavy Ion Collider data at collision energies $\sqrt{s_{\rm NN}} = 130$ and 200 GeV per nucleon. We obtain $dN_{\rm ch}^{\rm Pb\text{-}Pb}/d\eta(\sqrt{s_{\rm NN}} = 5.5\ {\rm TeV})|_{\eta=0} \approx$ 1290–1480.

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It has been suggested that the nucleus-nucleus collisions performed at the Relativistic Heavy Ion Collider (RHIC) at the highest collision energies of 130 and 200 GeV per nucleon probe the color glass condensate (CGC) [1] regime of QCD governed by nonlinear coherent phenomena and gluon saturation. This claim is supported by the success of saturation models [2–5] in the description of the energy, rapidity, and centrality dependence of the particle multiplicities experimentally measured in d-Au and Au-Au collisions [6,7]. With collision energies of up to 5.5 TeV, the upcoming program in lead-lead collisions at the CERN Large Hadron Collider (LHC) is expected to provide confirmation for the tentative conclusions reached at RHIC and to discriminate between the different physical mechanisms proposed to explain particle production in high energy nuclear reactions (for a review of alternative approaches see, e.g., [8]).

The phenomenological models in [2–5] rely on the assumption that the saturation scale Q_{sA} that governs the onset of nonlinear effects in the wave function of the colliding nuclei is perturbatively large $\sim$1 GeV at the highest RHIC energies. Next, gluon production is calculated via the convolution of the nuclear unintegrated gluon distributions (ugd's) according to k_t factorization [9]. Under the additional assumption of local parton-hadron duality, the multiplicity in A-A collisions at central rapidity rises proportional to the saturation scale, $dN^{AA}/d\eta|_{\eta=0} \propto Q_{sA}^2$ [10]. On the other hand, the growth of the saturation scale with increasing energy (equivalently, decreasing Bjorken-x) is determined by the perturbative Balitsky-Kovchegov and Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner (BK-JIMWLK) nonlinear evolution equations [11,12] (for a complete set of references, see [1]), thereby establishing a direct link between the gluon saturation dynamics and the experimentally measured hadron yields. The combination of CGC initial conditions with a hydrodynamic description of the subsequent system evolution yields an equally successful description of RHIC multiplicity data [13], which lends support to the idea of initial state dominance in the determination of bulk features of multiparticle production.

The energy dependence of the saturation scale yielded by the BK-JIMWLK equations at the degree of accuracy of their original derivation, leading-logarithmic (LL) in $\alpha_s \ln(1/x)$ with α_s fixed, is $Q_s^2 \approx Q_0^2(x_0/x)^\lambda$ with $\lambda \approx 4.8\frac{N_c}{\pi}\alpha_s$ [14,15]. This growth is too fast to be reconciled with the energy dependence observed in RHIC multiplicity data, which indicate $\lambda \sim 0.2$–0.3 [4–6,13]. Such deficiency of the theory has been circumvented so far by leaving λ as a free parameter, often adjusted to the empirical value $\lambda \approx 0.288$ obtained in fits to small-x Hadron Electron Ring Accelerator (HERA) data in deeply inelastic lepton-proton scattering in the framework of saturation models [15,16].

Higher order corrections to the BK-JIMWLK equations have been calculated recently via all orders resummation of $\alpha_s N_f$ contributions [17]. Such corrections bring substantial modifications to the LL kernel, including running coupling effects, and result in a significant slowdown in the speed of the evolution, among other quantitatively important dynamical effects [18].

In this Letter we demonstrate that this partial improvement is sufficient to describe the energy and rapidity dependence of the multiplicities in Au-Au collisions at the highest RHIC energies. Then we extrapolate to LHC energies and present predictions for Pb-Pb collisions at $\sqrt{s_{\rm NN}} = 5.5$ TeV.

We start by solving the nonlinear small-x evolution equation for the dipole-nucleus scattering matrix, $S(Y, r)$, including running coupling corrections [17,18]:

$$\frac{\partial S(Y, r)}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S], \quad (1)$$

where r is the dipole size and $Y = \ln(x_0/x)$. The first, *running coupling*, term of the evolution kernel, $\mathcal{R}[S]$, recasts the higher order corrections that amount to a modification of the LL small-x gluon emission kernel, leaving the interaction structure of the LL equation untouched, whereas the second, *subtraction*, term, $\mathcal{S}[S]$, accounts for the new interaction channels opened up by the higher order corrections. Explicit expressions for both terms as well as a detailed explanation of the numerical method used to solve Eq. (1) are given in [18]. The initial conditions for the evolution are taken from the semiclassical McLerran-Venugopalan (MV) model [19], aimed at describing the gluon distributions of large nuclei at moderate values of Bjorken-x, prior to the onset of quantum corrections. Thus, the initial dipole scattering amplitude, $\mathcal{N} = 1 - S$, reads

$$\mathcal{N}(Y = 0, r) = 1 - \exp\left\{-\frac{r^2 Q_0^2}{4}\ln\left(\frac{1}{r\Lambda} + e\right)\right\}, \quad (2)$$

where Q_0 is the initial saturation scale. The constant e under the logarithm acts as an infrared regulator and $\Lambda = 0.2$ GeV.

The *speed* of evolution, $\lambda = d\ln Q_s^2(Y)/dY$, extracted from numerical solutions of Eq. (1) corresponding to different initial conditions ($Q_0 = 0.5$, 0.75, 1, and 1.25 GeV) is plotted in Fig. 1. For $Y > 0$ the saturation scale is determined by the condition $\mathcal{N}(Y, r = 1/Q_s(Y)) = \kappa$, with $\kappa = 0.5$ (left plot) and $\kappa = e^{-1}$ (right plot). These results show two remarkable features of the solutions.

First, the running coupling corrections render the energy dependence of the saturation scale compatible with the one indicated by the analysis of experimental data. Thus, the λ values in Fig. 1 are slightly smaller than the one extracted from fits to HERA data, $\lambda = 0.288$ (except, perhaps, for $Q_0 \lesssim 0.5$ GeV at small rapidities). On average, they are compatible with $\lambda = 0.2$ reported in [13] as the optimal value to reproduce the energy and rapidity dependence of the multiplicities in Au-Au collisions at the highest RHIC energies. Second, they reveal the existence of two very distinct kinematical regimes: At small *preasymptotic* rapidities the evolution is strongly dependent on the initial conditions. In particular, denser systems, i.e., those associated to larger values of Q_0, evolve more slowly due to the relative enhancement of nonlinear effects with respect to

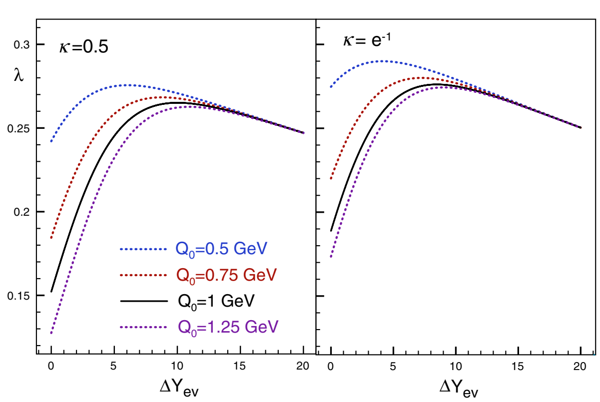

FIG. 1 (color online). $\lambda = \frac{d\ln Q_s^2(Y)}{dY}$, for $Q_0 = 0.5$, 0.75, 1, and 1.25 GeV (from top to bottom), and for $\kappa = 0.5$ (left plot) and $\kappa = e^{-1}$ (right plot).

more dilute systems. Such dependence on the nature of the evolved system is completely washed out by the evolution and, at high enough rapidities, all the solutions reach a common speed of evolution. The onset of this universal *scaling* regime is reflected in Fig. 1 by the convergence of all the individual trajectories into a single curve for $Y \gtrsim 15$. The studies of more exclusive properties of the solutions carried out in [14,18] suggest that the full scaling regime is reached at even larger rapidities, $Y \gtrsim 80$. Moreover, sizable scaling violations have been detected in HERA data [20] and in particle spectra in d-Au collisions at RHIC [21]. These observations raise the question of whether the scaling ansatz that connects HERA and RHIC phenomenology through the universality property of the solutions is an adequate one at presently available energies.

In analogy to [4,5], we calculate the pseudorapidity density of charged particles produced in nucleus-nucleus collisions within the k_t-factorization framework via

$$\frac{dN_{\rm ch}}{dyd^2b} = C\frac{4\pi N_c}{N_c^2 - 1}\int\frac{d^2p_t}{p_t^2}\int^{p_t} d^2k_t\alpha_s(Q)\varphi\left(x_1, \frac{|\underline{k}_t + \underline{p}_t|}{2}\right) \times \varphi\left(x_2, \frac{|\underline{k}_t - \underline{p}_t|}{2}\right), \quad (3)$$

where p_t and y are the transverse momentum and rapidity of the produced particle, $x_{1,2} = (p_t/\sqrt{s})e^{\pm y}$, $Q = 0.5\max\{|p_t \pm k_t|\}$, and b is the impact parameter of the collision. The lack of impact parameter integration in this calculation and the gluon to charged hadron ratio are accounted for by the constant C, which sets the normalization. The nuclear unintegrated gluon distribution entering Eq. (3) is related to the inclusive gluon distribution, $\varphi(x, k) \propto \frac{d[xG(x,k^2)]}{d^2kd^2b}$ and is given in terms of the dipole scattering amplitude evolved according to Eq. (1):

$$\varphi(Y, k) = \int\frac{d^2r}{2\pi r^2}\exp\{i\underline{r}\cdot\underline{k}\}\mathcal{N}(Y, r). \quad (4)$$

The relation between the evolution variable in Eq. (1) and Feynman-x of the produced particle is taken to be $Y = \ln(0.05/x_{1,2}) + \Delta Y_{\rm ev}$. Since the relevant values of Bjorken-x probed at midrapidities and $\sqrt{s_{\rm NN}} = 130$ GeV at RHIC are estimated to be ~0.1–0.01, the free parameter $\Delta Y_{\rm ev}$ controls the extent of evolution undergone by the nuclear gluon densities resulting from Eq. (1) prior to comparison with RHIC data. Similar to [4], large-x effects have been modeled by replacing $\varphi(x, k) \to \varphi(x, k) \times (1 - x)^4$. The running of the strong coupling, evaluated according to the one loop QCD expression, is regularized in the infrared by freezing it to a constant value $\alpha_{\rm fr} = 1$ at small momenta. Finally, in order to compare Eq. (3) with experimental data it is necessary to correct the difference between rapidity, y, and the experimentally measured pseudorapidity, η. This is achieved by introducing an average hadron mass, m. The variable transformation, $y(\eta, p_t, m)$, and its corresponding Jacobian are given by Eqs. (25)–(26) in [3]. Corrections to the kinematics due to the hadron mass are also considered by replacing $p_t^2 \to m_t^2 = p_t^2 + m^2$ in the evaluation of $x_{1,2}$. Remarkably, the optimal value found in comparison with data, $m \sim 0.25$ GeV is in good quantitative agreement with the hadrochemical composition of particle production at RHIC.

With this setup we find a remarkably good agreement with the pseudorapidity densities of charged particles measured in 0%–6% central Au + Au collisions at collision energies $\sqrt{s_{\rm NN}} = 130$ and 200 GeV. The comparison with data [6] constrains the free parameters of the calculation to the following ranges: $Q_0 \sim 0.75$–1.25 GeV, $m \sim 0.25$ GeV and $3 \gtrsim \Delta Y_{\rm ev} \gtrsim 0.5$. These ranges determine the uncertainty bands of the LHC extrapolation in Fig. 2. The best fits (solid lines in Fig. 2) are obtained with $Q_0 = 1$ GeV, $m = 0.25$ GeV, and $\Delta Y_{\rm ev} = 1$. The normalization constant, C, fixed at $\sqrt{s_{\rm NN}} = 130$ GeV and $\eta = 0$, is of order one in all cases. The line of argument that leads to these values is the following: First, the energy extrapolation from 130 to 200 GeV at central rapidities demands a moderate evolution speed $\lambda \sim 0.2$ [13]. From Fig. 1, that condition is met by either initial saturation scales $Q_0 \sim 1$ GeV and small evolution rapidities $\Delta Y_{\rm ev} \lesssim 3$ or at asymptotically large rapidities, $\Delta Y_{\rm ev} \sim 50$, which are kinematically excluded. In the physically accessible range, the solutions close to the scaling region, i.e., for $\Delta Y_{\rm ev} \sim 10$, result in too narrow pseudorapidity distributions independently of the value of Q_0. In the preasymptotic regime at fixed $\Delta Y_{\rm ev} \lesssim 3$, those solutions corresponding to $Q_0 \lesssim 0.75$ GeV yield exceedingly broad distributions. Thus, the energy and the pseudorapidity dependence independently constrain the parameters of the gluon distributions probed at RHIC to the same ranges. This provides the baseline for further evolution to LHC energies. In summary, these results indicate that the nuclear gluon densities probed at RHIC are in the preasymptotic stage of the evolution. This, together with the large values of the initial saturation scale required by data, suggests that the saturation of gold nuclei at RHIC energies is not dynamically generated by the evolution but, most likely, it is attributable to the nuclear enhancement factor that lies at the basis of the MV model, i.e., to the fact that the number of gluons in the nuclear wave function is large even at moderate energies due to the spatial superposition of a large number of nucleon's gluon fields.

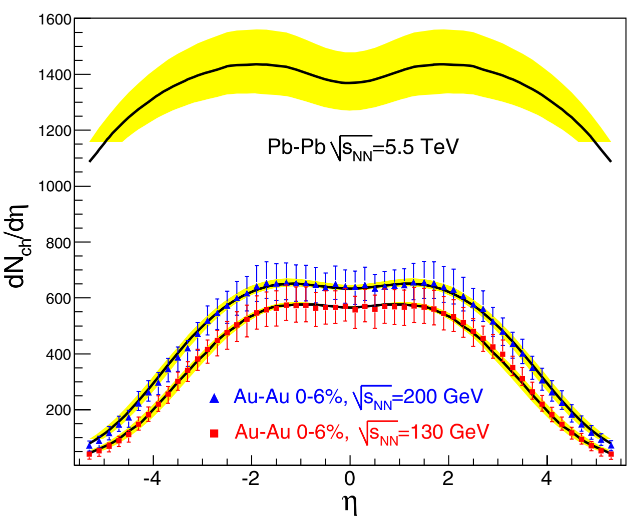

FIG. 2 (color online). Pseudorapidity density of charged particles produced in Au-Au 0%–6% central collisions at $\sqrt{s_{\rm NN}} = 130$ and 200 GeV and for Pb-Pb central collisions at $\sqrt{s_{\rm NN}} = 5.5$ TeV. Data taken from [6]. The upper, central (solid lines), and lower limits of the theoretical uncertainty band correspond to ($Q_0 = 1$ GeV, $\Delta Y = 1$), ($Q_0 = 0.75$ GeV, $\Delta Y = 3$), and ($Q_0 = 1.25$ GeV, $\Delta Y = 0.5$), respectively, with $m = 0.25$ GeV in all cases.

The extrapolation to LHC energies, done neglecting the differences between lead and gold nuclei and presented in Fig. 2, is now straightforward and completely driven by the nonlinear dynamics of gluon densities. For central Pb-Pb collisions we get

$$\frac{dN_{\rm ch}^{\rm Pb\text{-}Pb}}{d\eta}(\sqrt{s_{\rm NN}} = 5.5\ {\rm TeV}, \eta = 0) \sim 1290\text{–}1480, \quad (5)$$

with a central value corresponding to the best fits to RHIC data ~1390. These values are significantly smaller than those of other saturation based calculations [4,5,22], ~1700–2500, and compatible with the ones based on studies of the fragmentation region [23]. Such reduction is due to the lower speed of evolution yielded by Eq. (1) and to the proper treatment of preasymptotic effects, thereby going beyond the scaling ansatz. Importantly, the prediction for the midrapidity multiplicity in Eq. (5) is very robust against changes in the description of particle production and the implementation of large-x effects. This is illustrated in Fig. 3(a), where the following modifications to our setup have been considered [the itemization here follows the labeling in Fig. 3(a)]: (a) replacement of the ugd's in Eq. (3) by the modified gluon distributions $h(Y, k) = k^2 \nabla_k^2 \varphi(Y, k)$, as advocated in [24], (b) regularization of the strong coupling at the value $\alpha_{\rm fr} = 0.5$, (c) removal of the $(1 - x)^4$ corrections to the ugd's, and (d) putting $m = 0$. The results obtained with these alternative configurations do not deviate from the uncertainty band given in Eq. (5), confirming that our predictions are mostly driven by the properties of small-x dynamics. Oppositely, our predictive power at large pseudorapidities, $|\eta| \gtrsim 6$, is lessened by the sensitivity of the evolution to the initial conditions and by our relatively crude implementation of large-x effects, which are dominant in that region.

A common feature of the different CGC-based calculations for particle production in Pb-Pb collisions at the LHC is the prediction of a mild increase of $(dN_{\rm ch}/d\eta)/(N_{\rm part}/2)$

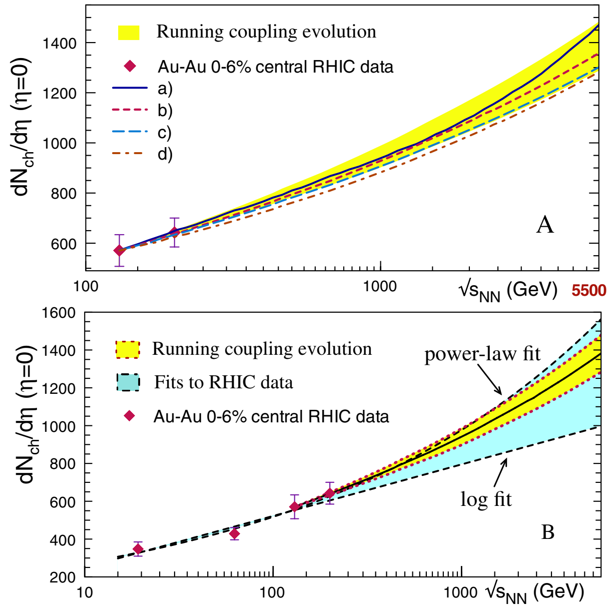

FIG. 3 (color online). Charged particle multiplicity in central Au-Au collisions at $\eta = 0$ versus collision energy. *Upper plot:* Results obtained with the setup leading to Eq. (5) (band) and several modifications of it (see text). *Lower plot:* Power-law, $a\sqrt{s}b$, and logarithmic, $a + b\ln s$, fits to RHIC data at $\sqrt{s_{\rm NN}} =$ 19.2, 64.2, 130, and 200 GeV.

with increasing number of participants, $N_{\rm part}$, [4,5]. Although a proper treatment of the collision geometry dependence of the multiplicities is beyond the scope of this work, the results in Eq. (5) could be used to renormalize the mentioned expectations at the most central collisions.

Purely empirical parametrizations of multiplicity data of a large variety of colliding systems allow a logarithmic dependence on collision energy (see, e.g., [25]). As shown in Fig. 3(b), RHIC data by themselves do not differentiate between this and other functional forms like power laws, negating any possibility to usefully constrain the expectations for LHC energies without further theoretical guidance. Our results, similar to other calculations based on perturbative QCD, exhibit a power-law growth of the midrapidity multiplicity with increasing collision energy. The higher order corrections utilized here for the first time provide a richer physics input and result in a noticeably smaller power than previously estimated. This fact is crucial to obtain a good description of both the energy and pseudorapidity dependence of existing data and is the key ingredient in the extrapolation to higher energies.

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