Quantum Computing with Collective Ensembles of Multilevel Systems

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We propose a new physical approach for encoding and processing of quantum information in ensembles of multilevel quantum systems, where the different bits are not carried by individual particles but associated with the collective population of different internal levels. One- and two-bit gates are implemented by collective internal state transitions taking place in the presence of an excitation blockade mechanism, which restricts the population of each internal state to the values zero and unity. Quantum computers with 10–20 bits can be built via this scheme in single trapped clouds of ground state atoms subject to the Rydberg excitation blockade mechanism, and the linear dependence between register size and the number of internal quantum states in atoms offers realistic means to reach larger registers.

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Quantum computers have the potential to solve a number of difficult mathematical problems such as factoring and unstructured search much more efficiently than their classical counterparts [1–3]. This power stems mainly from the quantum superposition principle, which allows a single system to simultaneously explore the entire (computational) state space. Although impressive quantum state control has been demonstrated in very high dimensional systems such as Rydberg state manifolds [4,5] and molecules [6,7], the number of qubits one can store in such individual systems scales only logarithmically with the Hilbert space dimension: for instance, a thousand states correspond to the Hilbert space dimension of only ten logical bits $2^{10} = 1024$. Genuine scalability aiming at several tens of logical bits can thus be reached only through composing individual quantum systems. Most quantum computing proposals use a large number of two-level quantum systems, each representing a physical qubit. The tensor-product space of N such particles accommodates N qubits of information [Fig. 1(a)], and a quantum algorithm is carried out with sequences of unitary operations on single qubits and pairs of qubits, the so-called universal quantum gates [8].

Controlling the joint state of many particles with suitable interactions to mediate two-bit gates between any pair of qubits is an outstanding challenge. In this Letter, we propose a new encoding and computing strategy that uses an ensemble of collectively addressed identical quantum systems each of which has $(N + 1)$ long-lived internal states $|i\rangle$, $i = 0, \ldots, N$. We shall regard the $|i = 0\rangle$ state as a reservoir state, initially populated by all members of the ensemble, and formally associate the computational register state $|b_1, b_2, \ldots, b_N\rangle$ ($b_i = 0, 1$) with the symmetric state of the ensemble with b_i ensemble members populating state $|i\rangle$ [Fig. 1(b)]. There is thus a direct correspondence between the binary representation of a register state and the Fock state representation, providing the zero and unity populations of different single-particle states, so that, in principle, only $K = N$ ensemble members suffice to represent all 2^N possible register states. In practice, however, ensemble sizes about an order of magnitude larger than the register size, or even more, improve a number of properties of our proposal, and do not set major experimental problems: ensembles of thousands of atoms are routinely produced and manipulated in quantum optics laboratories.

How can we prepare symmetric ensemble states and perform logical operations with at most one particle in

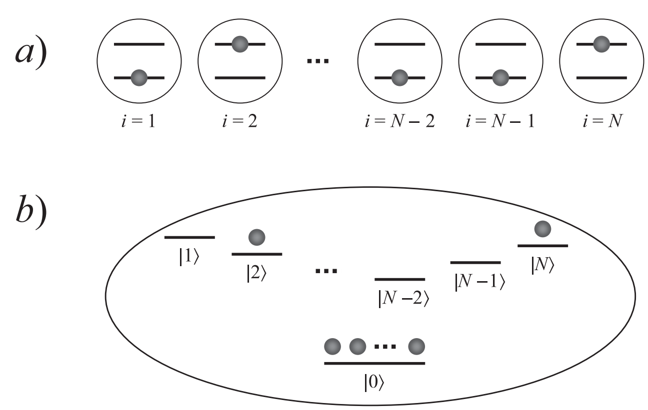

FIG. 1. Encoding of qubits in two-level and multilevel atoms. (a) Conventional qubit encoding of the N-bit state $|01\ldots001\rangle$ in N two-level systems. (b) Qubit encoding in the symmetric states of an ensemble of $(N + 1)$-level systems. The state $|0\rangle$ is a reservoir state, populated initially by all atoms. Bullets represent the number of atoms populating the different single-particle states to encode the N-bit state $|01\ldots001\rangle$.

each long-lived register state? In the case of atomic ensembles, our candidate solution to this problem is the so-called Rydberg blockade [9]. This mechanism is a consequence of the large dipole-dipole interaction [10], which strongly shifts the energies of Rydberg excited atoms with separations of several μm, so that the presence of one Rydberg atom is enough to prevent the excitation of all other atoms in the ensemble [11]. This blockade is able to entangle two atoms and can thus be used to achieve two-bit quantum gates in individual-atom systems [9]. It has also been suggested as a means to encode one qubit of information in a mesoscopic ensemble of two-level atoms [12] and perform arbitrary rotations of the qubit. Finally, it allows the implementation of a conditional quantum gate on the qubits stored in two different ensembles, either directly if the ensembles are within the range of the dipole-dipole interaction or by transferring the states of both ensembles into a single intermediate ensemble and carrying out the gate here, before transferring the (entangled) qubits back to their original ensembles [12]. The coupling of the mesoscopically populated reservoir state to the symmetric state with precisely a single atom transferred to another state is enhanced by the factor $\sqrt{K}$ compared to the single-atom coupling (K is the number of atoms in the ensemble), and may hence be strong enough to yield appreciable coupling even to field states with only a single photon: transfer of quantum states between samples may thus be achieved by exchange of photons. Although it also relies on Rydberg blockade, our scheme differs fundamentally by encoding the entire N-bit quantum register in a single collectively addressed mesoscopic ensemble.

Let us now show how to implement one- and two-bit gates in an ensemble of $(N+1)$-level atoms, making use of optical transitions and the Rydberg blockade mechanism. In mesoscopic ensembles, single-qubit rotations are a little more complex in their structure than in a single-atom implementation, as illustrated in Fig. 2. Rather than coupling qubit levels $|0\rangle$ and $|1\rangle$ by a site specific coherent, e.g., Raman, process [Fig. 2(a)], a coherent coupling of states with zero and one Rydberg excited atoms enables rotations in that space, and swapping the population between any atomic level $|i\rangle$ and the Rydberg state before and after this unitary rotation effectively implements the rotation on the ith qubit [Fig. 2(b)].

Here, we assume that any of the stable single-atom states can be coupled coherently to the Rydberg state $|r\rangle$, or to different Rydberg states $|r\rangle$, $|r'\rangle\ldots$ via suitable intermediate excited states. This is not a trivial task, since optical fields in general can resonantly excite all Zeeman substates in the ground state manifold, and if such states are used to encode the qubit register, we may need to apply external fields to lift their degeneracy or to apply techniques from control theory and act with shaped pulses that leave all register states untouched, and excite only the relevant one. A possible implementation using a homogeneous magnetic field to Zeeman select the desired states is described below.

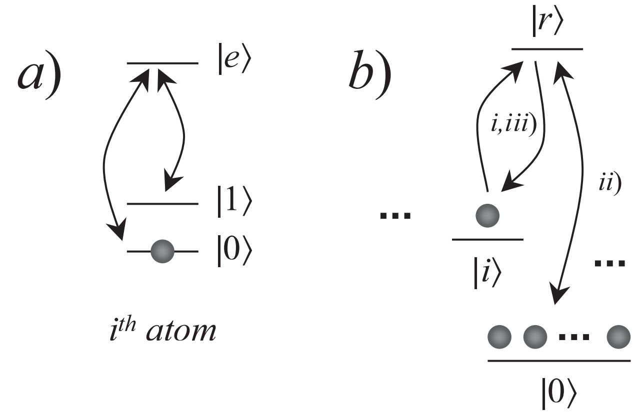

FIG. 2. In conventional encoding with one qubit per atom (a), rotation of the ith qubit can, e.g., be accomplished by a Raman process via an excited state of the ith atom. In our ensemble representation (b), the rotation of the ith qubit is accomplished by (i) a π-pulse transfer of the level population in $|i\rangle$ to $|r\rangle$, (ii) a coherent coupling in the two-state system with zero and one atom in $|r\rangle$, blocking further transfer of atoms to the Rydberg state and, finally, (iii) a π-pulse transfer of the level population in $|r\rangle$ to $|i\rangle$.

At this point it is important to recall that the coherent driving of the transition to the ith register level is enhanced by the symmetric coupling to all the single-particle components of the collective state, but unlike previous analyses [12], this enhancement is not the square root of the number of atoms $\sqrt{K}$, which is a fixed number, but only of the number of atoms available for the transition, $\sqrt{K-\sum_{j\neq i}b_j}$, because the population residing in the other register states is not coupled. During quantum computation, the ensemble will populate a superposition of states with accordingly different values of the coupling strengths and thereby introduce an inhomogeneity in the system. If $N \ll K$, variations in the coupling parameters may be significantly reduced and, if needed, simple composite pulses [13] can ensure robustness against these small variations.

As sketched in Fig. 3, the implementation of two-bit gates in our ensemble scheme is very different from the single-atom Rydberg blockade proposal, where the dipole-dipole interaction disturbs the resonance condition when two different atoms are exposed to resonant driving fields [Fig. 3(a)]. In our scheme, the excitation of the Rydberg state from a logical "1" of the ith qubit, i.e., from a single atom in the state $|i\rangle$, prevents the resonant driving of an atom in register state $|j\rangle$, and hence a conditional phase or NOT gate can be applied to the jth qubit [Fig. 3(b)].

Figure 4 shows a specific implementation of the above ideas for Cs atoms, whose nuclear spin $I = 7/2$ provides 16 stable Zeeman states in the $f = 3, 4$ hyperfine levels. For effective Rydberg blockade the atoms are transferred

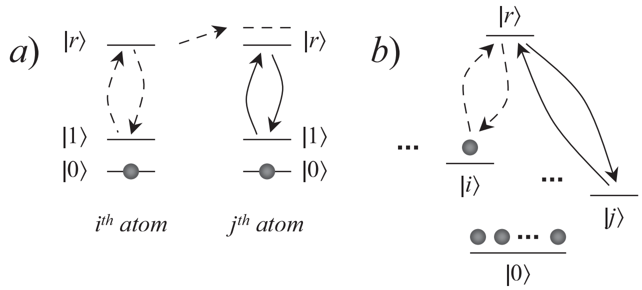

FIG. 3. Two-bit gates in two-level and multilevel atoms. (a) Two-bit gate via Rydberg blockade. If the ith qubit level $|1\rangle_i$ is excited to $|r\rangle_i$, the transition $|1\rangle_j \rightarrow |r\rangle_j$ is shifted out of resonance, and a 2π pulse on the jth atom yields a conditional phase. (b) Two-qubit gate in the ensemble representation. After a π-pulse on the transition $|i\rangle \rightarrow |r\rangle$, the transition $|j\rangle \rightarrow |r\rangle$ is blocked conditioned on the atomic population in $|i\rangle$, and a 2π-pulse yields a conditional phase.

by a two-photon excitation to a high lying ns state with $n \sim 70$. The hyperfine structure of the Rydberg level is unresolved so that the ground states are coupled to fine structure states $|ns_{1/2}, m_j = \pm 1/2\rangle$. Application of a magnetic field B to the atomic sample shifts all transition frequencies so that the only degenerate transitions are $|6s_{1/2}, f = 4, m = -4\rangle \leftrightarrow |ns_{1/2}, m_j = -1/2\rangle$ and $|6s_{1/2}, f = 4, m = 4\rangle \leftrightarrow |ns_{1/2}, m_j = 1/2\rangle$. We exclude one of these states, leaving 15 ground states available for the reservoir and encoding of up to 14 qubits. Each of the ground to Rydberg transitions are separated by at least $\frac{\mu_B B}{4\hbar}$, with μ_B the Bohr magneton, and as long as this quantity is large compared to the two-photon excitation frequency Ω any ground state can be selectively excited by appropriate choices of the laser frequencies. Typical numbers set by the need to respect the finite lifetime of the Rydberg level are [14] $\Omega/2\pi \sim 1$ MHz so that a modest field of $B \sim 15$ G will give suppression of undesired excitation at the 1% level.

In an ensemble with random atomic positions blockade is required between pairs of atoms with either the same or opposite signs of m, i.e., $m_j = \{\pm 1/2, \pm 1/2\}$ or $m_j = \{\pm 1/2, \mp 1/2\}$. In both cases an effective long range interaction occurs via the Förster mechanism [12] due to the near resonance of two-atom states $ns + ns \leftrightarrow np_{1/2(3/2)} + (n-1)p_{1/2(3/2)}$. Although the energy defects vary by about a factor of 3 at large n for different combinations of the $p_{1/2}$ and $p_{3/2}$ states, it can be shown that the interaction is always strong and deviates from isotropic by only about 15% for atomic separations greater than several μm with the approximate form [15] $U = \pm\sqrt{(\frac{\hbar\delta}{2})^2 + \frac{4}{3}\frac{C_3^2}{r^6}}$ with δ an effective average two-atom energy defect and $C_3 = \frac{e^2}{4\pi\epsilon_0} \times \langle np\|r\|ns\rangle\langle(n-1)p\|r\|ns\rangle$. We see from Fig. 4 that by excitation of $n = 70$ we obtain close to 1 GHz of interaction at 3 μm and about 80 MHz at separations as large as 5 μm. This is sufficient for high fidelity blockade of small samples of up to several hundred atoms. It may be advantageous to apply an optical lattice potential with only a single atom per site. The atoms would still not require individual addressing but the trapping may reduce the effect of collisions among the atoms, while many atoms will be within the range of the dipole-dipole interaction.

Having both one- and two-bit gates at our disposal, and the ability to initialize the system, we only need to specify an effective readout mechanism. This can, for example, be achieved by use of the Rydberg blockade [16], or by coupling the register levels in a controlled manner to excited states from which, e.g., ionization can be observed.

An implementation of our proposal with, say, 50–100 cold trapped cesium atoms in a far-off resonant optical trap may allow quantum computing with 14 qubits. This would exceed the performance of any other proposal imple-

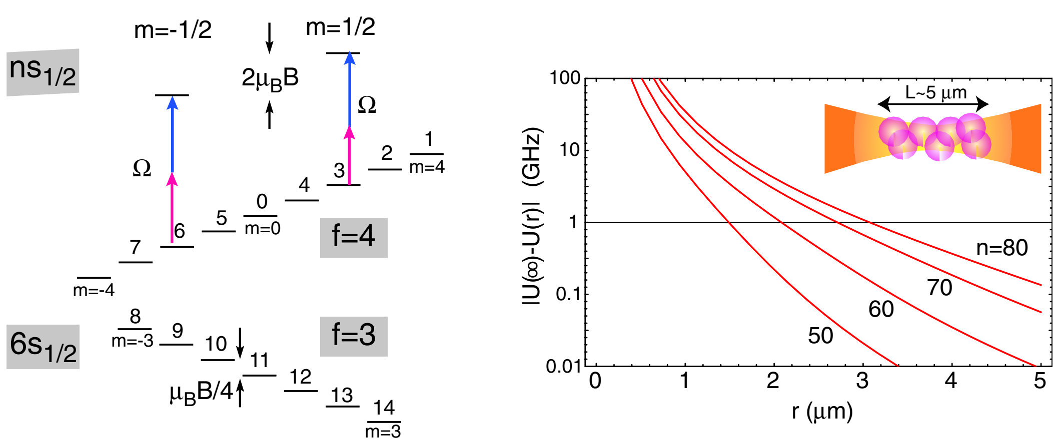

FIG. 4 (color online). Cesium level scheme and identification of qubit register. Encoding of reservoir state 0 and 14 register states in the Zeeman ground states of Cs. Coupling of $|3\rangle = |f = 4, m = 2\rangle$ and $|6\rangle = |f = 4, m = -2\rangle$ to Rydberg states is shown. The plot shows the dipole-dipole interaction strength versus atom separation r for Rydberg states with $n = 50$–80. As indicated in the inset, an atomic ensemble can be confined in an optical trap with a length scale of 5 μm.

mented to date. It is necessary, however, to develop strategies to scale the model to even larger numbers of qubits. To this end, we emphasize the linearity reported between single-particle internal Hilbert space dimension and number of qubits encoded. It is conceivable that atoms, or molecules, with more stable states can be used and approach the 100-bit regime, but perhaps the simplest approach may be to place a small number, say 10–20, of such samples so that they can be addressed by different laser fields, but still affect each other by the Rydberg blockade mechanism. Also, superconducting transmission lines work well with the large dipole moments of Rydberg excited states [17], and the use of mesoscopic ensembles of atoms also provides an efficient coupling to single photons [18,19]: both types of coupling may be used to communicate between different ensembles constituting a larger quantum computer. The number of qubits encoded simply add, and the set of one- and two-bit gates is equally straightforward to implement. Another, perhaps intriguing, approach towards larger registers is to consider mixtures of different species, which may be contained within the same volume, and where the Rydberg blockade may also apply between species. In this case, the total number of qubits is also found by adding the contributions from each species.

Though we discussed here an implementation of our encoding scheme making use of the Rydberg blockade mechanism to prevent transitions of an atomic ensemble out of the register Hilbert space, other physical systems can be considered that exhibit excitation blockade: for instance, hybrid schemes for quantum computing where an ensemble of atoms or molecules interacts with a single saturable two-level system (e.g., a Cooper pair box [20]) are promising candidates for implementation of our encoding scheme. The challenge is now to find ways to reliably control the internal state transitions and to identify the most efficient one- and two-bit gates. Further natural developments involve the analysis of the most relevant errors and the achievements of appropriate error-correcting codes.

In conclusion, we have proposed an approach to quantum computing that makes effective use of the single-particle Hilbert space dimension by encoding an N-bit quantum register in particles with only $(N + 1)$ internal states. The principles are new by the effective concentration of quantum information in single-particle multilevel systems. We emphasize, however, that, in our approach, we retain the basic qubit structure, which is essential for easy implementation of quantum gates. Conventional tensor-product state encoding requires individual access to the atoms and highly controlled mutual interactions between them, and a high degree of entanglement is present during computation. Instead, we use the collective population of levels, and all operations are carried out by only collective and symmetric access to the atoms. Interactions also play an important role and entanglement is present in our proposal, but in a very different manner. Thus, for example, the "classical" register states, which are simple product states [cf. Figure 1(a)], are now entangled states [cf. Figure 1(b)]. These physical and mathematical differences with conventional encoding will have important consequences for the further development of optimal algorithms, and, e.g., for error-correcting codes [1,21,22], where the typical errors on different qubits will definitely not be independent—the atoms will not lose a level, but the ensemble can lose a single atom, which calls for different correction measures.

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