Collective Atomic Recoil Lasing with a Partially Coherent Pump

G. R. M. Robb and W. J. Firth

Scottish Universities Physics Alliance (SUPA), Department of Physics, University of Strathclyde, 107 Rottenrow, Glasgow, G4 0NG, United Kingdom

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We investigate the effect of pump phase noise on the collective backscattering of light by a cold, collisionless atomic gas. We show that for a partially coherent pump field, the growth rate of the backscattered field is reduced relative to that for a coherent pump, but the backscattered intensity can be increased. Our results demonstrate that fluctuations and noise can play a counterintuitive role in nonlocally coupled many-body systems.

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The interplay between noise and nonlinearity has long been of interest to researchers from a broad range of disciplines, as it produces numerous fascinating and often counterintuitive phenomena. In nonlinear optics, collective nonlinear interactions between cold matter and light, such as collective light scattering [also termed collective atomic recoil lasing (CARL)] and collective cooling have been the subject of many theoretical and experimental studies over the past decade [1–7]. Studies of collective and cooperative phenomena using cold atoms in cavities are also of wider relevance. Together with experimental advances they allow fundamental studies of coherent and incoherent interactions in nonlocally coupled many-body systems which are relevant to plasma and beam physics, condensed matter, and even neuroscience. In many complex systems in nature, the environment is intrinsically noisy. In this Letter, we study the effect of pump phase noise on the CARL interaction.

A schematic diagram showing the situation under consideration is shown in Fig. 1. Two optical fields, a strong pump field and a weak probe field, interact with a cold, collisionless atomic gas and the probe field circulates in a high-finesse cavity. It is assumed that the probe field is coherent, but that the pump field is only partially coherent, its phase executing a random walk. It is also assumed that the atomic cloud is a classical collisionless gas and that the optical fields are classical and almost frequency degenerate ($|\omega_1 - \omega_2| \ll \omega_{1,2}$) ($\omega_2$ refers to the mean frequency of the pump field). The pump and probe fields, which are approximately counterpropagating where they interact with the atoms, give rise to a spatially periodic optical potential with approximate period $\lambda/2$, where $\lambda = \lambda_{1,2}$ is the pump or probe wavelength (similar to the case of a 1D optical lattice). The atoms move in this potential under the action of the dipole force to form a density grating with spatial period $\lambda/2$. The moving atoms in turn drive the evolution of the backscattered cavity (probe) field.

Collective backscattering of a partially coherent pump field which is far detuned from any atomic resonance and sufficiently strong to remain undepleted during the atom-light interaction is described by the classical CARL equations [2] modified to include the effect of a stochastic pump phase ϕ,

$$\frac{d\theta_j}{d\tau} = p_j, \tag{1}$$

$$\frac{dp_j}{d\tau} = -(Ae^{i(\theta_j - \phi)} + \mathrm{c.c.}), \tag{2}$$

$$\frac{dA}{d\tau} = \langle e^{-i(\theta-\phi)} \rangle + i\delta A, \tag{3}$$

$$\frac{d\phi}{d\tau} = \xi(\tau), \tag{4}$$

where $\theta_j = 4\pi z_j/\lambda$ is the scaled position of the jth atom on the scale of the optical potential, $p_j = (mv_j)/(\hbar k\rho)$ is the scaled momentum of the jth atom where m is the atomic mass and $k = 2\pi/\lambda$ is the pump or probe wave number. $\tau = \omega_r \rho t$ is an adimensional time variable and the collective atom-field coupling is determined by the CARL parameter [1], $\rho = (\frac{\Omega g\sqrt{n}}{2\Delta\omega_r})^{2/3}$ where $\Omega = \frac{dE_2}{\hbar}$ is the pump Rabi frequency, $\omega_r = 2\hbar k^2/m$ is the two-photon recoil frequency, $g = \sqrt{\frac{\omega d^2}{2\epsilon_0 \hbar}}$ is the atom-mode coupling constant,

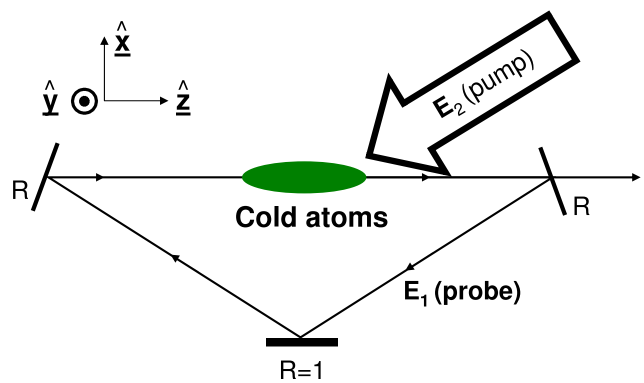

FIG. 1 (color online). Schematic diagram showing collective light scattering in a high-finesse unidirectional cavity. The misalignment of the pump field from the z axis is exaggerated.

$n = N/V_c$ is the atomic number density (with respect to the cavity mode volume V_c), E_2 is the amplitude of the electric field of the pump field, and d is the dipole matrix element for the atomic transition. The pump-atom detuning $\Delta = \omega - \omega_a$ is assumed to be much larger than the linewidth of the atomic transition and $\delta = \frac{\omega_2-\omega_1}{\omega_r\rho}$ is the scaled pump-probe detuning. $A = \sqrt{\frac{2\epsilon_0}{n\hbar\omega\rho}}E_1$ is the scaled (complex) probe field amplitude, and the average $\langle\cdots\rangle \equiv \frac{1}{N}\times\sum_{j=1}^{N}(\cdots)_j$. The partial coherence of the pump field is described using a phase diffusion model for the pump field phase, ϕ [8], which is assumed to evolve according to Eq. (4), where $\xi(\tau)$ is a Gaussian random variable with zero mean and variance Γ such that $\overline{\xi(\tau)} = 0$ and $\overline{\xi(\tau)\xi(\tau - T)} = 2\Gamma\delta(T)$. This corresponds to a pump field with a Lorentzian line shape and linewidth (phase diffusion rate) of $\Delta\omega_{\rm pump} = \omega_r\rho\Gamma$ [8]. The temperature T of the atomic gas is modeled by an initial Gaussian distribution of the atomic momenta such that the probability of finding an atom with a value of scaled initial momentum p_0 in the range dp is $f(p_0)dp_0$, where $f(p_0) = \frac{1}{\sqrt{2\pi}}\exp(\frac{-p_0^2}{2\sigma^2})$, $\sigma = \frac{\sqrt{3mk_BT}}{\hbar k\rho}$, and k_B is Boltzmann's constant. We have assumed that the cavity finesse is sufficiently high that the cavity decay time is much longer than the time scale of the evolution of the optical field due to the CARL interaction, and for convenience that the average pump frequency ω_2 coincides with the frequency of a cavity mode.

In the case of an initially cold gas, it is possible to obtain an analytical description of the average scattered field intensity $\overline{AA^*}$ by deriving equations for the second order moments of the dynamical variables and neglecting terms higher than second order in $|A|$. The large set of stochastic ordinary differential equations (ODEs) (1)–(4) is thereby approximated by the set of linear evolution equations

$$\frac{d\overline{A^*A}}{d\tau} = \overline{A^*b} + \text{c.c.}, \tag{5}$$

$$\frac{d\overline{A^*b}}{d\tau} = -i\overline{A^*P} + \overline{b^*b} - (i\delta + \Gamma)(\overline{A^*b}), \tag{6}$$

$$\frac{d\overline{A^*P}}{d\tau} = -\overline{A^*A} + \overline{b^*P} - (i\delta + \Gamma)(\overline{A^*P}), \tag{7}$$

$$\frac{d\overline{b^*b}}{d\tau} = -i\overline{b^*P} + \text{c.c.}, \tag{8}$$

$$\frac{d\overline{b^*P}}{d\tau} = -i\overline{Ab^*} + i\overline{P^*P}, \tag{9}$$

$$\frac{d\overline{P^*P}}{d\tau} = -\overline{A^*P} + \text{c.c.}, \tag{10}$$

where the macroscopic variables b and P are defined as

$$b = \frac{1}{N}\sum_{j=1}^{N} e^{-i(\theta_j-\phi)}, \qquad P = \frac{1}{N}\sum_{j=1}^{N}(p_j e^{-i(\theta_j-\phi)}).$$

Regions of instability (scattered field amplification) can now be found by finding solutions of the characteristic equation corresponding to Eqs. (5)–(10), i.e.,

$$\begin{aligned}\lambda[\lambda^8 &+ 4\Gamma\lambda^7 + (6\Gamma^2 + 2\delta^2)\lambda^6 + (4\Gamma^3 + 4\Gamma\delta^2)\lambda^5 \\ &+ (\Gamma^4 + 2\Gamma^2\delta^2 + \delta^4)\lambda^4 + 4\delta\lambda^3\Gamma \\ &+ (4\Gamma^2\delta + 4\delta^3 - 27)\lambda^2 - 36\Gamma\lambda - 12\Gamma^2] = 0,\end{aligned}$$

with $\text{Re}(\lambda) > 0$. Figure 2 shows the behavior of the linear growth rate of the scattered intensity $\overline{A^*A}$ as a function of pump-probe detuning δ for different values of the pump linewidth Γ. It can be seen that in the case of a coherent pump ($\Gamma = 0$), the maximum growth rate occurs at $\delta = 0$ and there is a sharp cutoff such that no instability occurs for $\delta > \sqrt[3]{\frac{27}{4}} \approx 1.9$. It can also be seen that pump phase diffusion ($\Gamma > 0$) broadens the region of instability or gain and removes the sharp instability cutoff at $\delta > 0$.

Before investigating the effect of a partially coherent, phase-diffusing pump field on the nonlinear regime of the CARL interaction, we first recover the limit of a completely coherent pump field. This is done by setting $\Gamma = 0$, so that the pump phase is a constant which can be set to zero. In this limit Eqs. (1)–(3) reduce to the usual CARL equations [2]. The evolution of the scaled cavity mode intensity due to scattering of a coherent pump as calculated from a numerical solution of Eqs. (1)–(3) is shown in Fig. 3(a) for the case of an initially cold atomic gas ($\sigma = 0$) and different values of pump-probe detuning δ. It can be seen that in the case of a cold gas with degenerate pump and probe fields ($\delta = 0$), which is usually considered to be the ideal for the CARL interaction, the cavity

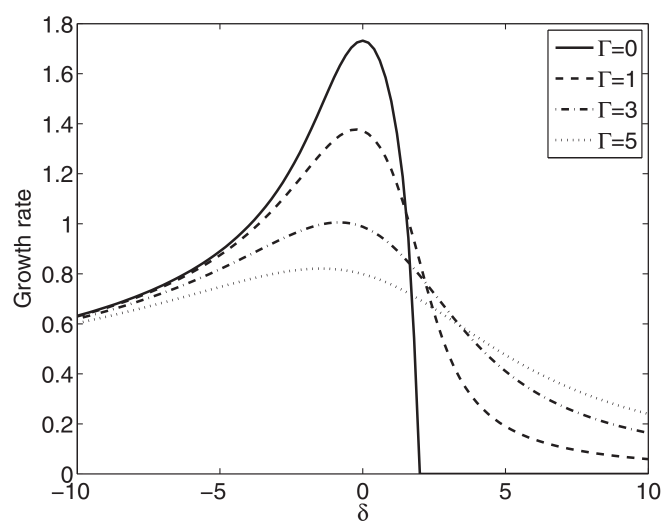

FIG. 2. Growth rate of the scaled cavity mode intensity $\overline{A^*A}$ as a function of pump-probe detuning δ for different values of the pump linewidth Γ.

mode or probe undergoes exponential amplification until saturation when $|A|^2 \approx 1.4$ followed by quasiperiodic oscillation. This implies that the real probe intensity at saturation scales as $I_{sat} \propto n^{4/3}$, indicating that the atoms are scattering collectively. This behavior was recently observed experimentally by Slama *et al.* [7] using a gas of ultracold ^{87}Rb atoms enclosed in a bidirectional ring cavity. In this coherent pump regime, amplification is due to strong bunching and trapping of the atoms in the optical potential, to form a strong density modulation with a spatial period of $\lambda/2$. When $\delta \neq 0$, the atomic velocity is not resonant with the phase velocity of the ponderomotive potential and the atoms become less strongly trapped in the optical potential. As predicted by the linear analysis described previously there exists a threshold value of δ, $\delta_c = \sqrt[3]{\frac{27}{4}}$, above which no amplification of the cavity mode occurs, as shown in Fig. 3(a).

The effect of pump phase diffusion on the nonlinear regime of the CARL interaction can be observed in Fig. 3(b), which shows the evolution of the scaled cavity mode intensity (averaged over 100 runs) for a partially coherent pump field with scaled phase diffusion rate (linewidth) $\Gamma = 5$, as calculated from the stochastic ODEs (1)–(3). It can be seen that the value of the cavity mode intensity at saturation $|A_{sat}|^2 \rightarrow \delta$ when $\delta > 0$. In contrast to the case of coherent pumping [Fig. 3(a)] for $\delta > 2$ there is strong amplification of the probe field. In fact, comparing with Fig. 3(a) it can be seen that for $\delta \geq 2$ with a partially coherent pump ($\Gamma = 5$), although the growth rate of the field is lower than in the case of the coherent pump, the cavity mode intensity exceeds that attained at saturation for the case of a coherent pump field.

The reason for the observed behavior can be deduced from an analysis of the forces acting on the atoms. The counterpropagating pump and probe fields combine to form an optical potential with a stochastic phase velocity which fluctuates stochastically about a mean value $v_{ph} = \frac{\omega_1 - \omega_2}{\omega_1 + \omega_2} c$. Whether the atomic momentum increases or decreases depends on the atomic velocity relative to that of the optical potential. The stochastic phase velocity of the potential inhibits strong bunching and trapping of the atoms observed in the case of a coherent pump field [1,2]. Instead, with a noisy pump the interaction between the atoms and the field eventually leads to the mean atomic velocity synchronizing with the mean velocity of the optical potential, i.e., the atoms attain an average velocity equal to the average phase velocity v_{ph} of the optical potential formed by the counterpropagating pump and probe, i.e., $\langle v \rangle = v_{ph}$. In the scaled notation used here, this corresponds to the atoms eventually having scaled momenta distributed around a mean value $\langle p \rangle \approx -\delta$. Such synchronization between particle and wave (phase) velocities is also the physical origin of Landau damping or the Landau instability in a plasma [9]. Note that this simple picture neglects the dynamical phase evolution of the probe field, which modifies the phase velocity of the optical potential during the interaction which is the physical origin of the CARL instability in the case of a coherent pump [1,2]. Consequently, the relation $\langle p \rangle \rightarrow -\delta$ is best satisfied when $\delta > (\frac{27}{4})^{1/3}$, and the dynamical phase evolution of the probe field is negligible.

Because of conservation of momentum, which in the scaled variables used here can be derived from Eqs. (1)–(3) to be $\langle p \rangle + |A|^2 = \text{const}$, it can be seen that a decrease in $\langle p \rangle$ causes an increase in $|A|^2$, i.e., cavity mode amplification. Based on this simple argument, we therefore expect that saturation of the instability will occur when the scaled probe intensity $|A|^2 \approx \delta$. This implies that the real probe intensity $I_{sat} \propto n(\omega_2 - \omega_1)$, indicating that the atoms scatter effectively independently when a noisy pump is used. This simple mechanism for amplification is consistent with the results from numerical simulations shown in Fig. 3(b).

As amplification using a partially coherent pump field relies on synchronization between the atomic momentum or velocity distribution and the phase velocity of the optical potential, and not on a large degree of spatial bunching or trapping of the atoms, it should therefore be significantly less sensitive to thermal velocity spread in the atomic distribution, relative to the case of the coherent pump. Theoretical studies [2] supported by recent experimental evidence [7] show that collective backscattering with a coherent pump, i.e., usual CARL, requires a narrow velocity distribution ($\sigma \ll 1$) in order that thermal debunching or dephasing does not begin to wash out the strong periodicity in the spatial distribution of atoms necessary for strong probe amplification to occur. In contrast, Fig. 4(b) shows that for a partially coherent pump ($\Gamma = 5$), although a larger thermal velocity spread does reduce the growth rate of the probe amplification, the probe intensity at saturation is almost the same as in the case of the cold beam (i.e., $|A|^2 \rightarrow \delta$).

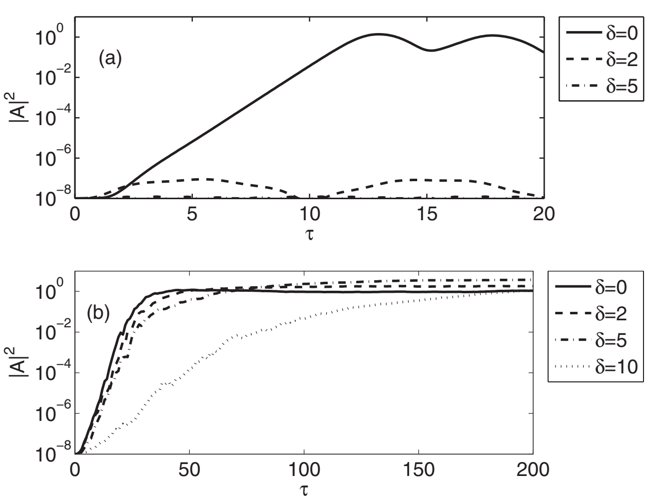

FIG. 3. Evolution of the scaled cavity mode intensity $|A|^2$ (averaged over 100 runs) due to scattering of a (a) coherent pump ($\Gamma = 0$) and (b) partially coherent pump ($\Gamma = 5$) by a cold gas ($\sigma = 0$) for different values of the pump-probe detuning δ.

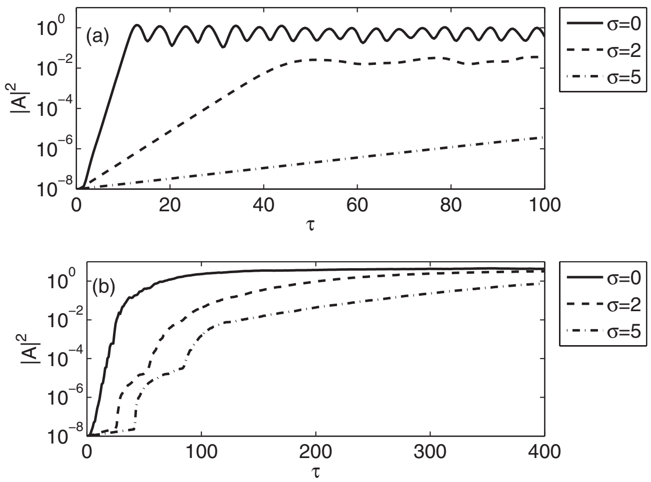

FIG. 4. Evolution of scaled cavity mode intensity (averaged over 100 runs) due to scattering of (a) a coherent pump ($\Gamma = 0$) and (b) a partially coherent pump ($\Gamma = 5$) showing the effect of increasing temperature (velocity spread, σ). Note that in (a) the value of δ used was $\delta = \sigma$, as this is the condition for maximum growth rate [11], whereas in (b) a value of $\delta = 5$ was used.

One possibility for experimental observation of the partially coherent pump regime ($\Gamma > 1$) described in this Letter can be deduced from an inspection of the recent experimental observation of CARL or superradiant Rayleigh scattering carried out by Slama *et al.* [7]. In these experiments an ultracold gas of ^{87}Rb atoms was enclosed in a high-finesse bidirectional ring cavity, similar to the schematic layout shown in Fig. 1, with the significant difference that the pump field is also a cavity mode which counterpropagates with respect to the probe field. The high finesse of the cavity means that the pump field has a very narrow linewidth ($\Delta\omega_{\rm pump} \approx 2\pi \times 20$ kHz). The characteristic growth time of the CARL instability reported in [7] was reported as being $t_g \approx 1\ \mu$s, which implies that $\omega_r \rho \approx t_g^{-1} = 10^6\ {\rm s}^{-1}$ and therefore that $\rho \approx 10$ in this experiment. Consequently, for the experiments reported in [7] the scaled pump linewidth $\Gamma = \Delta\omega_{\rm pump}/(\omega_r\rho) \approx 0.04$. As $\Gamma \ll 1$, then the experimental results are well described by a coherent pump model [2,7]. In order to reach the regime of partially coherent pumping described in this Letter, it would be necessary to increase the pump linewidth until it became at least comparable with the gain bandwidth of the CARL instability ($\Gamma > 1$). In principle this could be done by introducing some asymmetric losses into the cavity such that the effective finesse of the cavity for the pump field alone is reduced by a factor of at least $\sim$25. In order to do this while maintaining the same degree of coupling between the pump light and the atoms, i.e., the same value of the ρ parameter, it would therefore also be necessary to increase the pump power, decrease the pump-atom detuning (typically the atoms are very far detuned by $\sim$THz in [7]), or increase the atomic density n. From Fig. 3(b) the characteristic growth time of the probe field in the partially coherent pumping regime is expected to be about a factor of 3 longer than in [7], i.e., $\sim 3\ \mu$s. It was also shown in [7] that probe amplification disappeared when the temperature of the atoms exceeded $\approx 40\ \mu$K. The results shown in Fig. 4(b) suggest that in the partially coherent pumping regime, substantial probe amplification may be observable at significantly higher temperatures.

In conclusion, a theoretical study of the CARL interaction was performed in order to investigate the effect of pump phase noise. It was found that the saturation intensity of the backscattered field could be increased and was less sensitive to thermal velocity spread compared to the case of a coherent pump. Use of a partially coherent pump may allow the production of significant backscattering at temperatures significantly higher than in previous experiments [5,7]. The phenomenon of optical amplification with a partially coherent pump field described in this Letter has features of both recoil-induced resonances (RIR) [10] and CARL. Like RIR, maximum gain occurs off resonance ($\delta > 0$), only a small degree of spatial bunching is present, and the atoms scatter independently. Unlike RIR, but in common with CARL, high gain exponential amplification of the probe field can occur. These results demonstrate that the effect of fluctuations and noise in a nonlocally coupled many-body system can counterintuitively enhance the response of the system (in this case the probe field intensity).

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