

Creating and Probing Multipartite Macroscopic Entanglement with Light

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We propose an immediately realizable scheme showing signatures of multipartite entanglement generated by radiation pressure in a cavity system with a movable mirror. We show how the entanglement involving the inaccessible massive object is unraveled by means of field-field quantum correlations and persists within a wide range of working conditions. Our proposal provides an operative way to infer the quantum behavior of a system that is only partially accessible.

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Entanglement is at the heart of physical investigation not just because of its critical role in marking the boundary between classical and quantum world but also because of its exploitability in tasks of quantum information processing. So far, theoretical and experimental endeavors have been directed towards the demonstration of entanglement between well-isolated microscopic systems. Nevertheless, the observation of nonclassical correlations in systems of macroscopic objects and in situations of ineludible exposition to an uncontrollable environment is crucial [1]. However, it is in general difficult to infer the quantum properties of a large object affected by the action of the external world.

Here, we propose a scheme for the inference of quantum entanglement in a system that, besides experiencing open dynamics, is only partially accessible to detection. Our scheme is designed in a setup involving a mechanical oscillator coupled to two light modes. The general setting we address is currently of great interest as it represents a candidate for quantum limited measurements, state engineering, and for decoherence tests [2]. More recently, optomechanical systems comprising large mechanical systems have been used to demonstrate control induced by dynamical backaction [3]. In these setups, the mechanical oscillator is not directly accessible and one has to design proper strategies to infer its dynamics. Similarly to Ref. [4], our scheme uses an ancillary cavity field interacting with the optomechanical device. However, while the scheme proposed there is devised to work in conditions of negligible backaction of the ancillary field over the mechanical system, our study explores the probing field-oscillator coupling conditions and sets a distinctive interaction structure. In essence, we treat the whole field-oscillator-field system as intrinsically *tripartite* and investigate the behavior of any entanglement being set. Differently from Marshall *et al.* [2], we do not require stringent (interferometric) stability and use only classical states of

the fields, whose large intensity improves the optomechanical coupling and makes the scheme more robust. Moreover, our analysis includes *ab initio* the most relevant sources of noise in this setup. These would make the revelation of any quantum behavior difficult. Remarkably, we show that genuine multipartite entanglement is set in the optomechanical system, although no direct interaction between the fields is considered. Signatures of oscillator-field entanglement are clear in the quantum correlations between the fields. Our study, together with weak assumptions concerning the underlying model, paves the way to probe information about a system by getting entangled with it.

The effectiveness of the protocol endures in a wide range of operating conditions set by the mass of the oscillator and its initial temperature. Although one expects that larger values of these parameters would push the system towards increasingly *classical* dynamics, nevertheless entanglement still persists [4] and can be inferred. Our work therefore provides an experimentally relevant situation where multipartite entanglement is set, in a counter-intuitive and unexpected way, under severe environmental influences. It serves a pragmatic way of inferring optomechanical entanglement while showing an interesting trade-off between multipartite quantum correlation, indirect interaction, and noise.

Model.—We consider two cavities labeled *a* and *b*, each in a Fabry-Perot configuration, sharing a movable mirror. Each cavity is coupled to an external field of frequency ω_{lj} , input power P_j with strength E_j ($j = a, b$). The system is sketched in Fig. 1. The field of cavity *j*, locked at frequency $\omega_j \simeq \omega_{lj}$, is described by the annihilation (creation) operator \hat{j} (\hat{j}^\dagger). In terms of field quadratures, $\hat{j} = (\hat{x}_j + i\hat{p}_j)/\sqrt{2}$. The mirror is modeled as a single bosonic mode with frequency ω_m and mass μ . It undergoes quantum Brownian motion due to its contact to a bath at

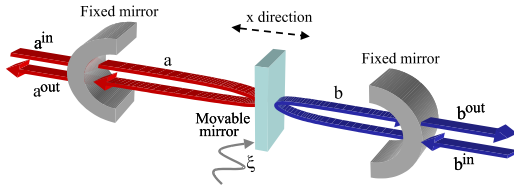


FIG. 1 (color online). The proposed experiment. The fields of two driven cavities $j = a, b$ interact with a movable mirror. Input (output) fields are indicated as \hat{j}^{in} (\hat{j}^{out}), and $\hat{\xi}$ is responsible for the Brownian motion of the mirror at temperature T .

temperature T given by background modes. For standard Ohmic noise characterized by a coupling strength γ_m , this leads to a non-Markovian correlation function of the associated noise operator $\hat{\xi}$ of the form $\langle \hat{\xi}(t)\hat{\xi}(t') \rangle = (\gamma_m/\omega_m) \int \omega e^{-i\omega(t-t')} [1 + \coth(\zeta\omega/2)] d\omega/2\pi$ (with $\zeta = \hbar/k_B T$ and k_B the Boltzmann constant) [5]. For weak mirror-environment coupling, a Markovian description can be gained with the mirror motion that is damped at a rate γ_m . In a frame rotating at the frequency of the lasers, the Hamiltonian of the system is written as

$$\hat{H} = \frac{\hbar\omega_m}{2}(\hat{p}^2 + \hat{q}^2) + \hbar \sum_{j=a,b} [(\Delta_{0j} - \tilde{G}_{0j}\hat{q})\hat{j}^\dagger\hat{j} + iE_j(\hat{j}^\dagger - \hat{j})]. \quad (1)$$

Here, \hat{p} , \hat{q} are the mirror quadrature operators, $\tilde{G}_{0j} = (-1)^{\delta_{jb}} G_{0j}$ with $G_{0j} = (\omega_j/\ell_j)(\hbar/\mu\omega_m)^{1/2}$ the coupling rate between the mirror and the j th cavity, and $\Delta_{0j} = \omega_j - \omega_{lj}$. Cavity j has length ℓ_j and decay rate κ_j so that $|E_j| = (2\kappa_j P_j/\hbar\omega_{lj})^{1/2}$. In order to study the explicitly open dynamics at hand we go to the Heisenberg picture where the evolution of the system is well described by a set of linear Langevin equations for the fluctuations (around the respective mean values) of the fields' quadratures $\delta\hat{x}_{a,b}$, $\delta\hat{y}_{a,b}$ and the fluctuations of the mirror's quadratures $\delta\hat{q}$ and $\delta\hat{p}$. These equations also involve the input noise terms [4,6]. This is a well-established method which allows for the reconstruction of the quantum statistical properties of the system, provided the fluctuations of the operators are small compared to the mean values [7]. The dynamical Langevin equations are easily expressed in terms of the equilibrium position of the mirror $q_s = \sum_j \tilde{G}_{0j} |\alpha_{s,j}|^2 / \omega_m$, the stationary amplitudes of the cavity fields $\alpha_{s,j} = |E_j|/(\kappa_j^2 + \Delta_j^2)^{1/2}$, and the detunings $\Delta_j = \Delta_{0j} - \tilde{G}_{0j}q_s$. As anticipated above, for $\gamma_m \ll \omega_m$ we have $\langle \{\hat{\xi}(t), \hat{\xi}(t')\} \rangle \propto \delta(t-t')$, which restores the Markovian nature of the dynamical process [4].

Under these conditions, a detailed calculation shows that the evolution of the quadratures' fluctuations is described by the equation $\hat{\mathbf{f}}(t) = e^{\mathbf{K}t}\hat{\mathbf{f}}(0) + \int_0^t d\tau e^{\mathbf{K}(t-\tau)}\hat{\mathbf{n}}(\tau)$, with $\hat{\mathbf{f}}^T = (\delta\hat{x}_a, \delta\hat{y}_a, \delta\hat{x}_b, \delta\hat{y}_b, \delta\hat{q}, \delta\hat{p})$ the vector of the fluctu-

ations and the matrix

$$\mathbf{K} = \begin{pmatrix} -\kappa_a & \Delta_a & 0 & 0 & 0 & 0 \\ -\Delta_a & -\kappa_a & 0 & 0 & G_a & 0 \\ 0 & 0 & -\kappa_b & \Delta_b & 0 & 0 \\ 0 & 0 & -\Delta_b & -\kappa_b & -G_b & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_m \\ G_a & 0 & -G_b & 0 & -\omega_m & -\gamma_m \end{pmatrix}. \quad (2)$$

Here, $G_j = \sqrt{2}\alpha_{s,j}G_{0j}$ is an effective optomechanical coupling rate. The vector $\hat{\mathbf{n}}^T(t) = (\sqrt{2\kappa_a}\delta\hat{x}_a^{\text{in}}, \sqrt{2\kappa_a}\delta\hat{y}_a^{\text{in}}, -\sqrt{2\kappa_b}\delta\hat{x}_b^{\text{in}}, -\sqrt{2\kappa_b}\delta\hat{y}_b^{\text{in}}, 0, \hat{\xi})$ formally introduces the contribution from the input noise with $\delta\hat{a}_{\text{in},j} = (\delta\hat{x}_{\text{in},j} + i\delta\hat{y}_{\text{in},j})/\sqrt{2}$ satisfying $\langle \delta\hat{a}_{\text{in},j}(t)\delta\hat{a}_{\text{in},k}^\dagger(t') \rangle = \delta_{jk}\delta(t-t')$. In turn, this results in the noise correlation properties $\langle \{\hat{n}_\alpha(t), \hat{n}_\beta(t')\} \rangle / 2 = \mathbf{N}_{\alpha\beta}\delta(t-t')$ with $\mathbf{N} = (\oplus_{j=a,b} \kappa_j \mathbb{1}_2) \oplus \text{Diag}[0, \gamma_m(2\bar{n} + 1)]$. Here, \bar{n} is the mirror's mean phonon number and $\mathbb{1}_2$ the 2×2 identity matrix. We aim for studying the entanglement properties of the steady state, which is guaranteed to exist if the real parts of the eigenvalues of \mathbf{K} are negative. The explicit conditions for this to occur are obtained using a simple stability-analysis test [8]. In what follows, we take $(\omega_m, \omega_{lj}, \gamma_m, \kappa_j)/2\pi = (10^7, 3.7 \times 10^{14}, 100, 8.8 \times 10^7)$ Hz, $\ell_j = 1$ mm, $T = 0.4$ K, $P_a = 50$ mW, and $\mu = 5$ ng, which are achievable with present-day technology [3] (assuming $\kappa_b = \kappa_a$ does not affect the generality of our results). With these values, a stable steady state is secured for $\Delta_b < 0$ and for any P_b . The freedom of choosing P_b suits our purposes as we want to study the interaction of the fields with m for any value of the backaction induced by b .

Entanglement structure.—We consider the steady-state covariance matrix of the system

$$\mathbf{V} = \begin{pmatrix} \mathbf{L}_a & \mathbf{C}_{ab} & \mathbf{C}_{am} \\ \mathbf{C}_{ab}^T & \mathbf{L}_b & \mathbf{C}_{bm} \\ \mathbf{C}_{am}^T & \mathbf{C}_{bm}^T & \mathbf{L}_m \end{pmatrix}, \quad (3)$$

with $V_{\alpha\beta} = \langle \{\hat{f}_{ss,\alpha}, \hat{f}_{ss,\beta}\} \rangle / 2$ and $\hat{\mathbf{f}}_{ss} \equiv \hat{\mathbf{f}}(\infty)$. Here, \mathbf{L}_j is a 2×2 matrix which accounts for the local properties of mode j while \mathbf{C}_{jk} is a 2×2 matrix describing intermode correlations ($j, k = a, b, m$). The form of \mathbf{V} under the assumed stable conditions is given as $\mathbf{V} = \int_0^\infty d\tau (e^{\mathbf{K}\tau}) \mathbf{N} (e^{\mathbf{K}\tau})^T$. We can now study the behavior of the entanglement between the elements of the tripartite system. First, we characterize and quantify the bipartite entanglement in each intracavity field-mirror subsystem and in the field-field one. Physically, the radiation-pressure coupling displaces the state of the mirror, in the phase space, depending on the intensity of the field [2]. Therefore, we deal with a thermally weighted superposition of coherent states of the mirror correlated to photon-number states of the field. As the Langevin equations are linear, the initial Gaussian nature of the state of the system

is preserved. We use the logarithmic negativity $\mathcal{E}_{\mathcal{N}}^{jk}$ [9] and entanglement monotonicity that can be calculated using the symplectic spectrum of the matrix $\mathbf{V}_{jk}^P = \mathbf{P}\mathbf{V}_{jk}\mathbf{P}$ associated with the partially transposed density matrix of the $j - k$ subsystem. Here, $\mathbf{P} = \mathbb{1}_2 \oplus \boldsymbol{\sigma}_z$ (with $\boldsymbol{\sigma}_r$ the r -Pauli matrix and $r = x, y, z$) and \mathbf{V}_{jk} is the 4×4 matrix extracted from \mathbf{V} by taking the blocks in (3) relative to modes j and k [10,11].

We start with the entanglement between field a and mirror m , which is maximum when field b is absent [4]. However, the backaction induced by b changes this picture. A way to see this is to fix the working point for the a cavity to those values corresponding to the maximum of entanglement with m . Then, we tune the effects of b (by changing Δ_b and P_b) and we study the behavior of the $a - m$ entanglement from negligible to strong backaction induced by b . The results are shown in Figs. 2(a) and 2(b). Evidently, for large Δ_b , $\mathcal{E}_{\mathcal{N}}^{am}$ achieves values close to its maximum. Indeed, in a far-detuned cavity the input power entering a cavity is reduced, which brings the system to a situation of small backaction. We note that a reduction in $\mathcal{E}_{\mathcal{N}}^{am}$ is accompanied by a raise of $\mathcal{E}_{\mathcal{N}}^{bm}$, as shown in Fig. 2(b): our calculations reveal that entanglement in the $b - m$ subsystems is pronounced in the region of moderate Δ_b where $\mathcal{E}_{\mathcal{N}}^{am}$ suffers the effects of b 's backaction. Large detunings lower $\mathcal{E}_{\mathcal{N}}^{bm}$ which, eventually, goes to zero for $\Delta_b \gg \omega_m$. By comparing the results obtained for increasing P_b (with Δ_b set at the value corresponding to the maximum of $\mathcal{E}_{\mathcal{N}}^{ab}$), we see that $\mathcal{E}_{\mathcal{N}}^{ab}$ disappears, slowly with respect to $\mathcal{E}_{\mathcal{N}}^{am}$ (at $P_b \approx P_a$, $\mathcal{E}_{\mathcal{N}}^{am} = 0$, and $\mathcal{E}_{\mathcal{N}}^{ab} \neq 0$).

We may use the entanglement between a and b as a tool to see signatures of entanglement between a and m . As a and b never directly interact, all $\mathcal{E}_{\mathcal{N}}^{ab} \neq 0$ is necessarily due to a mediation by the mirror. That is, in this system the mirror acts as a bus for the cross talk of the fields. Any entanglement between a and b is thus an indication of a coherent field-field interaction through the mirror. As the input fields are prepared in pure coherent states, $\mathcal{E}_{\mathcal{N}}^{ab}$ must be the result of an effective entangling field-field interaction [12]. Under the assumptions of our model, $\mathcal{E}_{\mathcal{N}}^{ab}$ can be taken as a signature of optomechanical entanglement. If initially a and b are in pure states, $\mathcal{E}_{\mathcal{N}}^{ab} > 0$ strictly indicates mirror-field entanglement. While, in general, one

could construct models where two systems become entangled via the coupling to a system that remains separable with respect to the rest [13], the interaction studied in our scheme provides strong evidence for mirror-cavity entanglement. The case of $|\Delta_b|/\omega_m \sim 0.5$ and $P_b \sim 10$ mW is interesting as it corresponds to $\mathcal{E}_{\mathcal{N}}^{am} \simeq \mathcal{E}_{\mathcal{N}}^{bm}$ with $\mathcal{E}_{\mathcal{N}}^{ab}$ achieving its maximum, thus optimizing the overall entanglement distribution. The corresponding tripartite state violates the positivity of partial transposition criterion [11] with respect to all partitions, indicating that the state is not biseparable with respect to any split and reflecting multipartite entanglement in this sense [14,15]. Although this can exclude biseparability with respect to any split, there still is the possibility of having a convex combination (i.e., a mixture) of biseparable states with respect to different subsystems. An extreme case is the situation of having an entangled state between two parties only but no knowledge about which specific modes share this entanglement. Nevertheless, the optomechanical state at hand turns out to be also genuinely tripartite entangled in the stronger sense that it is not a convex combination or a mixture of such biseparable states. This has been verified by means of the application of optimal entanglement witnesses for Gaussian states built out of solutions of semidefinite optimization programs [16,17]. Hence, genuine tripartite entanglement is shared between the subsystems, which is a very important result. It provides a motivated counter example to the study in Ref. [13] and shows unexpected resilience of multipartite entanglement to severe environmental effects. The study of quantitative tight lower bounds to the optomechanical entanglement is the focus of further investigations. Clearly, the entanglement-inference protocol suggested here works under the assumptions of the model at hand. However, a more general framework can be formulated where some of the assumptions on the mirror's phonon background, for instance, are relaxed.

Although the entanglement between the intracavity fields and the mirror is the focus of our study, the accessible quantities are the fields leaking out of the cavities. Here, we sketch an operative strategy to infer the correlations between the intracavity fields which requires the output fields' covariance matrix. We define $\hat{f}_{\alpha}^{\text{in}}(t) = \hat{n}_{\alpha}/\sqrt{2\kappa}$ ($\alpha = 1, \dots, 4$) so that the extracavity-field fluctuations are related to the intracavity ones by

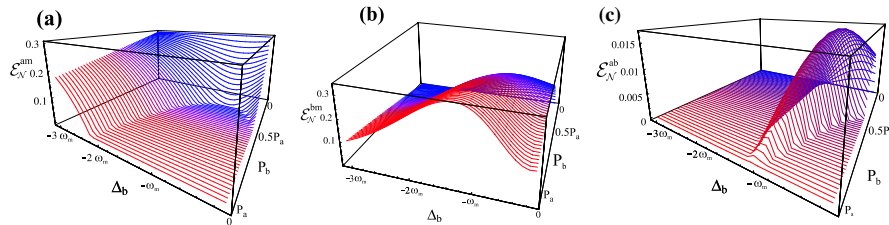


FIG. 2 (color online). Logarithmic negativity $\mathcal{E}_{\mathcal{N}}^{am}$, $\mathcal{E}_{\mathcal{N}}^{bm}$, and $\mathcal{E}_{\mathcal{N}}^{ab}$ [(a)—(c), respectively] against $\Delta_b \in [-3\omega_m, 0]$ and $P_b \in [0, P_a]$ for the parameters assumed in the text.

$\hat{f}_\alpha^{\text{out}}(t) = \sqrt{2\kappa_a}\hat{f}_\alpha(t) - \hat{f}_\alpha^{\text{in}}(t)$. It is convenient to introduce dimensionless extracavity quantities, which we define as $\hat{f}_{d,\alpha}^{\text{in,out}} = \lim_{t \rightarrow \infty} \int_t^{t+t_m} \hat{f}_\alpha^{\text{in,out}}(t') dt' / \sqrt{t_m}$, where t_m is the acquisition time chosen for a measurement of the output quadratures at the steady state. In this way, the output covariance matrix reads $\mathbf{V}^{\text{out}} = 2\kappa_a t_m \mathbf{V}_{ab} + \mathbf{V}^{\text{in}}$, where $V_{\alpha\beta}^{\text{in}} = \delta_{\alpha\beta}/2$. This expression suggests an operative way to infer the extracavity entanglement between a and b : for a set working point (with $t_m \sim \kappa^{-1}$, typically), \mathbf{V}^{out} is built up by homodyne measurements [4,18] and \mathbf{V}_{ab} deduced as $(\mathbf{V}^{\text{out}} - \mathbf{V}^{\text{in}})/2\kappa_a t_m$.

Working point.—We now address the performance of the scheme at different working points and discuss its noise resilience. Although the working point used up until now is optimal for entanglement inference, it is important to test the resilience of the protocol to variations of the critical physical parameters. These are the effective mass μ [19], the temperature, and the quality factors of the mirror. By keeping the other parameters unchanged, we have checked that an increase by 1 order of magnitude in μ (brought up to 50 ng) keeps the entanglement-inference scheme effective, although the maximum entanglement in each subsystem shrinks ($\mathcal{E}_{\mathcal{N}}^{ab}$ decays almost exponentially with μ with a e^{-1} descent at $\mu \sim 40$ ng). Moreover, the mirror needs to be only mildly protected from the environment. With the optimal conditions of $|\Delta_b|/\omega_m \sim 0.5$ and $P_b = 0.15P_a$, we have to reduce ω_m/γ_m by 2 orders of magnitude (from 10^5 to 3×10^3) or increase the temperature to 12 K (corresponding to $\bar{n} \sim 25\,000$) in order to lose tripartite entanglement. This is a very important result witnessing a striking robustness of the quantum features of the system. These adverse working conditions represent a rather pessimistic case as the (very conservative) value we chose for γ_m is 3 times larger, for instance, than the characteristic one for the structure reported in Ref. [20], and low initial temperatures can be used by means of passive cooling.

Conclusions.—We have discussed a state-of-the-art scheme to infer entanglement between a cavity field and a mirror by inducing quantum correlations in a tripartite system including an ancillary mode. We found practical working points at which the optomechanical entanglement is bounded by the correlations between the fields. This suggests the use of field-field correlation to reveal the mirror-field entanglement. Strikingly, genuine tripartite entanglement appears, robust against noise, despite the fact that no direct interaction between the fields is set. This paves the way to the inference of entanglement involving macroscopic objects and stimulates the usage of such entanglement as a building block in protocols for the indirect manipulation of the mirror state via optical control.

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