Impact of a Projectile on a Granular Medium Described by a Collision Model

Jérôme Crassous,* Djaoued Beladjine, and Alexandre Valance

Groupe Matière Condensée et Matériaux, UMR CNRS 6626, Université de Rennes 1, Campus de Beaulieu Bâtiment 11A, 35042 Rennes Cedex, France

(Received 28 August 2007; published 14 December 2007)

We propose a model for the propagation of energy due to the impact of a granular projectile on a dense granular medium. Energy is transferred from grain to grain during binary collision events. The transport of energy may then be viewed as a random walk with a split of energy during successive collisions. There is a qualitative and quantitative agreement between this simple description and experimental results.

DOI: 10.1103/PhysRevLett.99.248001 PACS numbers: 45.70.−n

The description of the collision of a projectile onto a granular assembly is a problem of major importance in various physical and geophysical situations, such as the formation of impact craters [1,2], the formation of planets [3], or the aeolian sand transport [4,5].

The interaction between the projectile and the granular medium is a complex process during which the kinetic energy of the projectile is transferred to the granular material. Only a part of this energy is restituted as kinetic energy of ejected particles [6] or formation of a rim of a crater [1], the rest of the energy being dissipated by the granular material. The different modes for the conversion of the initial kinetic energy are the consequence of the way the energy propagates and dissipates into the granular material. The identification of such mechanisms appears as a major task for the description of the collision between a projectile and a granular medium.

In this Letter, we propose that the energy transfer may be described by a succession of binary collisions between grains in the dense granular medium, the energy being shared between particles during the collision. In order to validate this description, we focused on the collision of one grain with a half space of granular medium composed of identical grains. For such collisions, experimental data have been reported [7], and they allow us to accurately test our model. This very simple model reproduces remarkably well the experimental observations. The propagation of the energy may be represented by a random walk from grain to grain with a split of the energy at each step. Many applications of this description are possible, from stress propagation in granular media to the description of dissipation in dense granular flow.

Our approach for treating the energy transfer is inspired by the description of the impact of one sphere on a linear chain of touching spheres. Hinch *et al.* [8] have studied analytically and numerically the dynamics of such a system, and they found that the propagation of compression pulses along the line of beads is essentially located on one or two contacts at a given time. They also pointed out that the nonlinearity of the contact laws between spheres are responsible for this localization, as shown by Nesterenko [9] in a continuous approach. As a consequence of such a localization, the transfer of mechanical energy through a line of beads can be accurately approximated by a succession of independent binary collisions. A striking illustration of such dynamics is the Newtown cradle, where the last bead of the chain is ejected with ≈ 97% of the input kinetic energy. As long as projectile velocity V is such that $mV^2 >> mgd$, where d is the bead diameter and g the gravitational constant, the underlying hypothesis of a strong nonlinearity of the contact law between spheres still holds. We then expect that a description of the system dynamics in terms of successive binary collisions should capture the essential physical feature of the splash function.

The description of the propagation of a mechanical wave in a 3D random packing of spheres is however a little bit trickier than for the simple case of a line of beads. First, in a grazing binary collision between a moving bead and a bead at rest, only a part of the impacting kinetic energy is transferred to the target. Second, since beads have in average 6 neighbors in a random packing, collisions involve generally more than two beads.

Figure 1 illustrates the procedure used to describe the propagation of a mechanical wave into a packing of beads in terms of successive collisions. When bead (0) impacts a bead of the surface layer of the packing, a collision takes place and is treated as binary. A part of the impulsion is transferred to the impacted bead (1), the incident one being

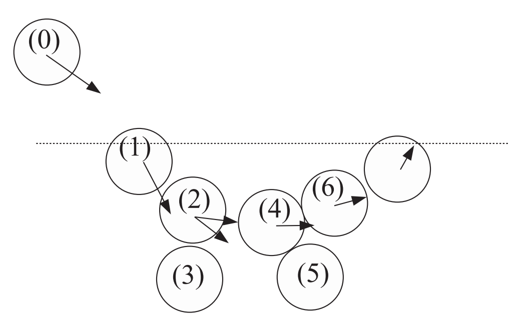

FIG. 1. A chain of binary collisions leading to the ejection of a bead.

 © 2007 The American Physical Society

reflected. Impulsion is then transmitted to the next bead [as bead (1) to bead (2) on Fig. 1], or reflected [as bead (2) which experienced a reflection on bead (3)]. Multiple collisions, as the bead (4) with beads (5) and (6), are treated as two successive binary collisions, the order for the collisions being randomly chosen. On Fig. 1, we consider the collision of the bead (4) with the bead (6), the collision of (4) with (5) (not shown on this figure) occurring after reflection of bead (4) on bead (6). This is a useful way to mimic the division of the kinetic energy without dealing with the indeterminacy of three (or more) body collisions. Chains of collisions stop when there are no more neighbors in the velocity direction. Beads are then considered as ejected, and their ejection velocities are computed. However, if the ejection velocity is too small (i.e., unable to reach an altitude greater than one diameter above the surface level), the ejected bead is disregarded in the experiment.

Numerical computations are used for evaluating all the possible chains of collisions in a 3D random packing of spheres of volume fraction $\phi \approx 0.60$. For a given impact, all the choices for successive collisions, i.e., transmissions or reflections, are computed. The tree of all possibilities is calculated and averaged on 1000 collisions. The collision law between two beads is described in a very simple way with only one adjustable parameter. We first treat the collision as purely elastic; i.e., energy is conserved, and the impulse is oriented along the normal to the contact surface. The velocities of the impacting and impacted beads are then computed and multiplied by a numerical factor $\beta < 1$ which accounts for the energy dissipation processes occurring during the collision. More complex contact laws involving friction and tangential restitution coefficient [10] were also tested and led to similar results.

We test our binary collision model by comparing the results of the model with experiments on the impact of one bead on a granular medium (see Fig. 2). We briefly recall the main experimental results used for comparison, and more experimental details may be found elsewhere [7,11]. The collision experiments were carried out using PVC beads of diameter $d = 6$ mm and mass $m = 0.2$ g. The packing of volume fraction $\phi = 0.60 \pm 0.02$ is built by displaying randomly the beads in a rectangular box of dimension $42 \times 42 \times 23$ cm. The incident bead is propelled onto the packing by means of an air gun. The impact speeds are varied from $50\sqrt{gd}$ to $200\sqrt{gd}$ whereas the impact angles are changed as well from 10° to 90°. The collision process is recorded via two fast video cameras, one placed perpendicularly to the incident plane, the other parallel to the incident direction. The 3D trajectories of the incident bead and the splashed ones are then reconstructed, and for each set of impact parameters investigated, experimental data are averaged on about 50 impacts.

The friction and normal restitution coefficients of the beads, defined and measured as in [10], are respectively $\mu = 0.19$ and $\varepsilon = 0.90$ at a relative velocity $V \approx 10\sqrt{gd}$. In the simulation, we set β to the expected value for a head-on collision $\beta = (1 + \epsilon)/2 = 0.95$. This value β is also the value for which the agreement between experimental result and model is the best. This is a strong indication that the model of dissipation in terms of restitution coefficient should capture the essential of the dissipative processes involved in the mechanical wave propagation within the packing.

We first focus on the rebound of the impacting particle. It is experimentally found that the mean effective restitution coefficient $\bar{e}$, where $\bar{\cdot}$ designs the average on several realizations of the same impact, and $e = V_r/V_i$ with V_i the incident velocity and V_r the rebound one, does not depend on the impact velocity in the range of experimental velocity. Moreover, impacting bead looses much more energy for head-on impacts than for grazing collisions. In other words, $\bar{e}$ decreases with increasing impact angle from ≈ 0.77 at 10° to ≈ 0.15 at normal incidence (see Fig. 3). The same trend is also observed on the average $\bar{e}_z$ of the vertical restitution coefficient $e_z = V_{rz}/V_{iz}$. The computed values of $\bar{e}$ and $\bar{e}_z$ are found in very good both qualitative and quantitative agreement with the experimental data. It should be stressed that, because the number of collisions before the impacting bead be reflected is between 1 and $\sim$3, the value of β is of very minor influence on these results. In contrast, the effective restitution coefficients depend on the distribution of the angle between the impacting bead and the first impacted bead.

We now turn to the ejected particles. Normal impacts produce more splashed particles than grazing ones, and an increase of the impact speed results in an increase of the number of ejecta. Experimentally, is it found that the mean number of ejected beads varies as $\bar{n}_{ej} = (1 - \bar{e}^2) f(V_i/\sqrt{gd})$ where f is a increasing function of $V_i/\sqrt{gd}$. The dependence of $\bar{n}_{ej}$ on the impact angle is

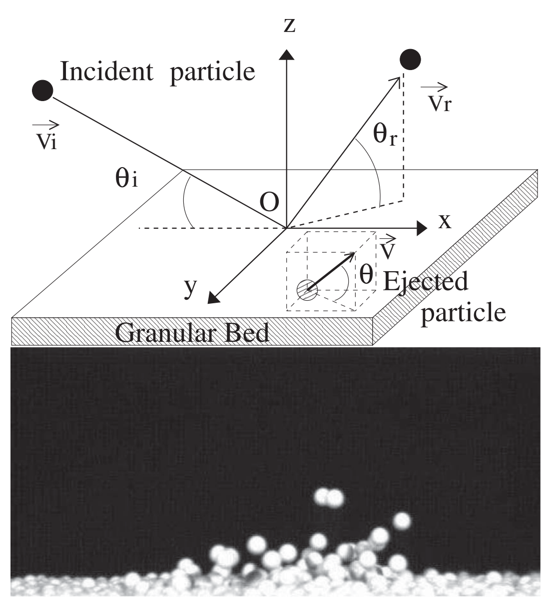

FIG. 2. Upper: Scheme of the collision process. Lower: Snapshot of a splash event 16 ms after the impact.

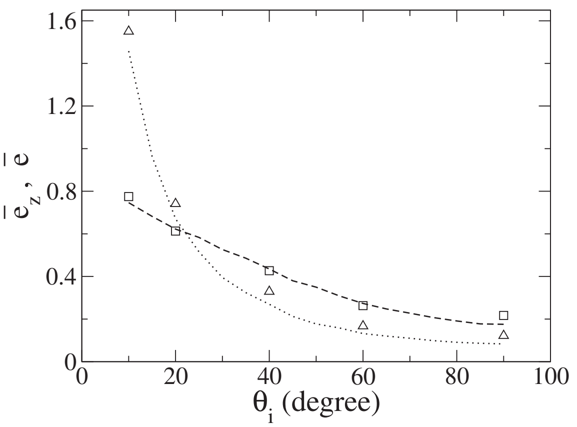

FIG. 3. (□) Mean restitution coefficients $\bar{e}$ and (△) mean vertical restitution coefficient $\bar{e}_z$ as a function of the impact angle for a velocity $V_i/(gd)^{1/2} = 107$. Dotted line and dashed line are the results of the binary collision model.

completely correlated to the variation of the fraction of energy communicated to the packing, $(1 - \bar{e}^2)$, with the impact angle (see Fig. 4). Accurate estimate of the number of ejecta at low impacting velocities is experimentally difficult, because only a few beads are ejected and hardly detach from the bed, leading to important experimental errors. Numerically, a bead is considered as ejected if its momentum is enough to raise the center of the bead at a height greater than one radius above the top of the surface layer of the packing. The latter is defined as the mean level of the envelop of the bottoms of virtual spheres deposited atop the packing. Dotted and dashed lines on Fig. 4 are the results of the model. They represent the mean number of ejecta $\bar{n}_{ej}$ as a function of the impact angle θ_i, the impact velocity being fixed to $107\sqrt{gd}$. The mean number of ejected beads increases with the impact angle, and is roughly doubled when the angle is increased from 10 to 90°, in agreement with the experiment. When normalized to kinetic energy transferred to the packing, the number of ejected beads $\bar{n}_{ej}/(1 - \bar{e}^2)$ appears constant as in the experiment. The variation of the normalized number of ejected beads with the impact velocity is shown in the inset of Fig. 4. It does not depend on the impact angle, and, as expected, it increases with increasing impact velocity. The number of ejected beads is found to agree very well with the experiments in the full range of impact velocities investigated experimentally. The model shows that there exists a critical impact velocity of order of $10\sqrt{gd}$ below which there are no splashed beads a fact that we are unfortunately unable to verify experimentally.

In terms of energy balance, the sum of the kinetic energy E of the splashed beads is experimentally proportional to the energy communicated to the packing: the ratio $E/(1 - \bar{e}^2)E_i$, where E_i is the kinetic energy of the incident bead, is found to be independent of the impact speed and incident angle, and to be equal to 4.5% (see Fig. 5). Numerically, the sum of the kinetic energy of the splashed beads in the collision process were calculated for different impact speeds and angles. The ratio $E/(1 - \bar{e}^2)E_i$ of the restored kinetic energy to the kinetic energy transferred to the packing by the impacting bead is found, within numerical uncertainties, independent on the impact angle. This ratio increases with the impact velocity as shown on Fig. 5. This is a consequence of the threshold used in the detection of the splashed beads. Indeed, since the dissipation rate in a collision is taken independent of the impact velocity, the kinetic energy of the ejected beads should scale linearly with the impacting bead velocity, the only nonlinearity effect in the restitution of kinetic energy being the detection threshold. The ratio $E/(1 - \bar{e}^2)E_i$ is found to depend on dissipative process and varies from 2% for $\beta = 0.88$ to 7% for $\beta = 0.97$ at an impact velocity of $V_i = 100\sqrt{gd}$.

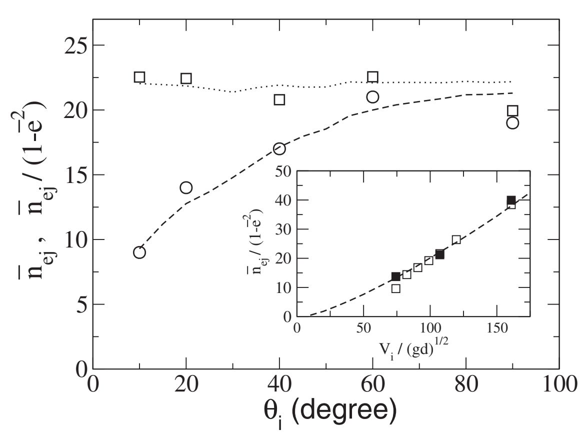

FIG. 4. (○) Mean number of ejected beads and (□) ratio of mean number of ejected beads to the fraction of kinetic energy transferred to the packing as a function of the impact angle θ_i. The incident speed is $V_i/(gd)^{1/2} = 107$. Inset : ratio of the mean number of ejected beads to the fraction of kinetic energy transferred to the layer as a function of the impact velocity $V_i/(gd)^{1/2}$ for two different impact angle, 10° (□) and 40° (■). Dotted and dashed lines are the results of the binary collision model.

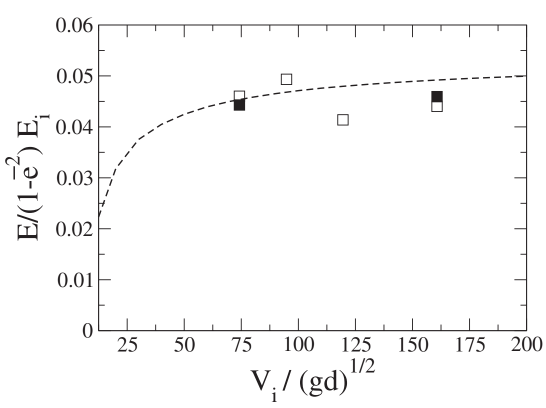

FIG. 5. Sum of the kinetic energy of the splashed grains as a function of the impact velocity for two different impact angles of 10° (□) and 40° (■). Dashed line is the prediction of the binary collision model.

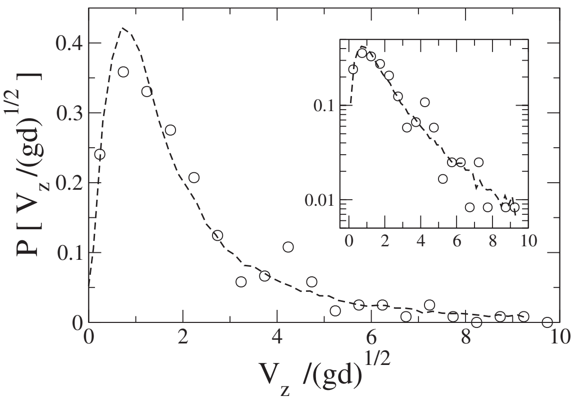

FIG. 6. Distribution of the dimensionless vertical ejection velocity for different impact velocities at a given impact angle of 10°. (○) are experimental data and dashed line the result of the binary collision model. Inset is the same data on a logarithmic scale.

The distribution of the ejection speed of the splashed beads were also measured for different impact speeds and angles. Figure 6 shows a typical measurement of the probability density of the vertical component of the ejection velocity V_z measured at the top of the surface layer. These distributions are found weakly dependent of both the impact velocity and angle. The distributions exhibit a maximum at $V_z \approx 0.8\sqrt{gd}$ and decreases systematically with the ejection speed. The distribution of the vertical component of the ejection velocity, resulting from the collision model, is found independent of the impact velocity as experimentally observed. The calculated distribution is plotted in Fig. 6 and appears very close to the experimentally observed one. However, one should stress that the presence of a peak in the distribution of V_z is fully conditioned by the threshold used in the detection of splashed beads. Indeed, without such a threshold, the density number of ejected beads should increase for decreasing ejection velocity. In contrast, the behavior of the distribution at large ejection velocity do not depend on the detection threshold. At large values of ejection speed, the computed distribution is very close to the experimental one both qualitatively and quantitatively. It should be noticed that in this collision model, the largest values of ejection speed are associated to short sequences of binary collisions. Indeed, fragmentation and dissipation of the energy increase with the number of successive collisions in a given sequence, and long sequences produce a relatively large number of weakly energetic ejected particles compared to shorter sequences. The distribution (not reported in this Letter) of the number of collisions before ejection shows that shorter paths involve 3 collisions, and the more likely number of collisions is ~12. Similar trends are found for the norm of the ejection speed, and good agreement between the experiment and the model is also observed. Finally, the distributions of the ejection angle θ were also extracted experimentally and numerically and were found to be both independent of the impact velocity and of the incident angle. The mean ejection angle is invariant and found to be equal to 60° with respect to the horizontal plane.

In summary, we lightened the way a granular material reacts to the impact of a grain. A part of the kinetic energy of the impacting grain is transferred to the packing, which in turn ejects numerous grains. The transmission and the division of the kinetic energy was described by considering a transfer of kinetic energy based on successive binary collisions. The chains of collisions giving rise to ejected particles can be of various lengths resulting in a broad distribution for the ejection speed of the splashed particles. Such a description accurately describes the full set of experimental data. In such a description, the input momentum is transferred from one bead to another one by contact, and a chain of collisions from input to output can be described by a broken line through the random packing of spheres. Such broken lines may then be viewed as the trajectories of a random walker, as, for example, a photon propagating in a diffuse media. The ratio of the kinetic energy between the splashed particles and the impact one is then analogous to the albedo of a scattering surface. Such analogy should allow one to borrow concepts and results from radiative transfer diffusing transport theories for more insight into the description of impact processes with granular media.

The authors thank Madami Ammi, Isabelle Cantat, Renaud Delannay, Jim Jenkins, and Luc Oger for fruitful discussions. This work has been supported by ANR Grant No. NT05-4_42012 "MICMAC."

*jerome.crassous@univ-rennes1.fr

[1] A. M. Walsh, K. E. Holloway, P. Habdas, and J. R. Bruyn, Phys. Rev. Lett. **91**, 104301 (2003).
[2] J. S Uehara, R. P. Ojha, M. A. Ambroso, and D. J. Durian, Phys. Rev. Lett. **90**, 194301 (2003).
[3] G. Wurm, G. Paraskov, and O. Krauss, Phys. Rev. E **71**, 021304 (2005).
[4] R. A. Bagnold, *The Physics of Blown Sand and Desert Dunes* (Chapmann and Hall, Methuen, London, 1941).
[5] R. S. Anderson and P. K. Haff, Science **241**, 820 (1988).
[6] B. T. Werner, J. Geol. **98**, 1 (1990).
[7] D. Beladjine, M. Ammi, L. Oger, and A. Valance, Phys. Rev. E **75**, 061305 (2007).
[8] E. J. Hinch and S. Saint-Jean, Proc. R. Soc. A **455**, 3201 (1999).
[9] V. F Nesterenko, J. Appl. Mech. Tech. Phys. **5**, 733 (1983).
[10] S. M. Foerster, M. Y. Louge, H. Chang, and K. Allia, Phys. Fluids **6**, 1108 (1994).
[11] D. Beladjine, Ph.D. thesis, University of Rennes 1, 2007.