Magnetic Double-Gradient Instability and Flapping Waves in a Current Sheet

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A new kind of magnetohydrodynamic instability and waves are analyzed for a current sheet in the presence of a small normal magnetic field component varying along the sheet. These waves and instability are related to the existence of two gradients of the tangential (B_τ) and normal (B_n) magnetic field components along the normal ($\nabla_n B_\tau$) and tangential ($\nabla_\tau B_n$) directions with respect to the current sheet. The current sheet can be stable or unstable if the multiplication of two magnetic gradients is positive or negative. In the stable region, the kinklike wave mode is interpreted as so-called flapping waves observed in Earth's magnetotail current sheet. The kink wave group velocity estimated for the Earth's current sheet is of the order of a few tens of kilometers per second. This is in good agreement with the observations of the flapping motions of the magnetotail current sheet.

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Introduction.—Thin current layers are typical structures in the heliosphere, including the solar corona, solar wind, and planetary magnetospheres. We address some of the magnetohydrodynamic aspects concerning the stability of current layers which are still poorly understood. In particular, CLUSTER observations in Earth's magnetotail current sheet indicated the appearance of strong wave perturbations propagating across the current sheet. Many event studies indicated very large current sheet variations and a predominant wave propagation in the transverse direction with respect to the magnetic field plane. The existence of such kinds of waves associated with flapping motions was confirmed in many statistical studies [1–7] which allowed one to identify them as the "kink"-like perturbations. The plasma sheet flapping observations are interpreted as crossings of a quasiperiodic dynamical structure produced by almost vertical slippage motion of the neighboring magnetic flux tubes. The frequency of the flapping motions, estimated from observations, is $w_f \sim 0.035\ \mathrm{s}^{-1}$ [1]. For a majority of the observed events [4], a group speed of the flapping waves was found to be in the range of a few tens (30–70) kilometers per second. The wavelengths and spatial amplitudes are estimated to be of the order of 2–5 R_E (R_E is the Earth's radius) [7].

A preferential appearance of one (kinklike) mode of the flapping motion was reported by [3]. CLUSTER observations give rise to the assumption that the flapping motions are notably more frequent in the central part of the tail than near the flanks. In the near-flank tail regions the motions of flapping waves are predominantly from the center to the flanks [2]. These experimental results confirm an internal origin of the flapping motions, due to some processes (like magnetic reconnection) localized deep inside the magnetotail. On the basis of CLUSTER observations of reconnection events, a relationship between the flapping motion and the reconnection process was investigated by [8]. During the reconnection events the current sheet exhibits strong flapping motions that propagate towards the flank of the tail.

With regard to a theoretical aspect of the problem, the ballooning-type mode in the curved current sheet magnetic field was claimed to be able to propagate azimuthally in flankward directions from the source [9]. This ballooning theory was applied in the WKB approximation implying the condition that the wavelength scale is much less than the curvature radius. This condition can hardly be fulfilled in the plasma sheet with a small normal component of the magnetic field. Another point is that according to the theory of [9], both kinklike and "sausage"-like deformations of the current sheet are equally possible, and the question arises about a reason, why the observed flapping perturbations of the current sheet are mainly associated with the kinklike wave modes.

In this Letter, we propose a new approach to explain the existence of the kinklike flapping wave oscillations propagating across the current sheet. In a framework of a rather simple magnetohydrodynamic consideration, we elucidate a physical reason of the flapping wave oscillations of the current sheet, which is related with gradients of the tangential and normal magnetic field components with respect to the normal and tangential directions, respectively.

Statement of problem.—A geometrical situation of the problem and coordinate system are illustrated in Fig. 1. We apply a system of incompressible ideal magnetohydrodynamics for nonstationary variations of plasma sheet parameters

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) + \nabla P = \frac{1}{4\pi}\mathbf{B}\cdot\nabla\mathbf{B}, \qquad (1)$$

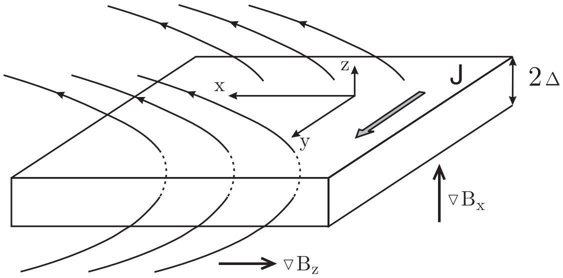

FIG. 1. Geometrical situation of the problem.

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{V}, \tag{2}$$

$$\nabla\cdot\mathbf{V} = 0, \qquad \nabla\cdot\mathbf{B} = 0. \tag{3}$$

Here $\mathbf{V}$, $\mathbf{B}$, ρ, P are the velocity, magnetic field, density, and total pressure, respectively. The total pressure is defined as the sum of the magnetic and plasma pressures. We consider specific wave perturbations propagating across the magnetic field lines, which are much slower than the magnetosonic modes. In this case the incompressible approximation seems to be appropriate.

We focus our study on the very slow wave modes existing only in the presence of a gradient of the B_z component in the magnetotail current sheet along the x direction. The background conditions are considered to be rather simple with a slow dependence of the B_z component on the x coordinate

$$\begin{gathered} B_x = B^* b_x(\bar{z}), \qquad B_z = \varepsilon B^* b_z(\bar{x}), \qquad B_y = 0, \\ \mathbf{V} = 0, \qquad \bar{y} = y/\Delta, \qquad \bar{z} = z/\Delta, \qquad \bar{x} = x/L_x. \end{gathered} \tag{4}$$

Here Δ is a thickness of the current sheet, and L_x is a length scale of the B_z variation along the current sheet.

We introduce normalized small perturbations marked by sign "tilde" which are considered to be functions of time and two spatial coordinates (y, z)

$$\begin{gathered} B_x = B^*[\tilde{b}_x + b_x(\bar{z})], \qquad B_y = \varepsilon B^* \tilde{b}_y, \\ B_z = \varepsilon B^*[b_z(\bar{x}) + \tilde{b}_z], \qquad P = P_0 + \tilde{P} B^{*2}/(4\pi), \\ V_x = \tilde{v}_x V_A, \qquad V_y = \tilde{v}_y V_A, \qquad V_z = \tilde{v}_z V_A, \\ \bar{t} = t V_A/\Delta, \qquad V_A = B^*/\sqrt{4\pi\rho^*}, \qquad \nu = \Delta/L_x. \end{gathered} \tag{5}$$

Here P_0 is the background total pressure, the parameter ε means the ratio of the background normal and maximal tangential components of the magnetic field, and the parameter ν characterizes the gradient of the normal magnetic field component. For the background conditions considered in our model [$B_z(\bar{x})$, $B_x(\bar{z})$], equation $\nabla\cdot\mathbf{B} = 0$ is fulfilled for arbitrary independent parameters ε and ν.

Linearizing Eqs. (1)–(3) for the normalized perturbations, neglecting high order terms $\sim\nu^2\varepsilon$ and $\sim\varepsilon^2$, we assume $\nu \gg \varepsilon$ and retain the main term $\sim\nu\varepsilon$.

Substituting Fourier harmonics [$\propto \exp(i\bar{\omega}\,\bar{t} - i\bar{k}\,\bar{y})$], we obtain finally a system of equations for Fourier amplitudes

$$i\bar{\omega}\tilde{v}_x = \varepsilon\left(\tilde{b}_z \frac{db_x}{d\bar{z}} + b_z \frac{d\tilde{b}_x}{d\bar{z}}\right), \tag{6}$$

$$i\bar{\omega}\tilde{v}_y - i\bar{k}\tilde{P} = 0, \qquad i\bar{\omega}\tilde{v}_z + \frac{d\tilde{P}}{d\bar{z}} = \varepsilon\nu\tilde{b}_x\frac{db_z}{d\bar{x}}, \tag{7}$$

$$i\bar{\omega}\tilde{b}_z - b_z\frac{d\tilde{v}_z}{d\bar{z}} + \nu\tilde{v}_x\frac{db_z}{d\bar{x}} = 0, \qquad i\bar{\omega}\tilde{b}_y - b_z\frac{d\tilde{v}_y}{d\bar{z}} = 0, \tag{8}$$

$$i\bar{\omega}\tilde{b}_x + \frac{db_x}{d\bar{z}}\tilde{v}_z = 0, \qquad -i\bar{k}\tilde{v}_y + \frac{d\tilde{v}_z}{d\bar{z}} = 0. \tag{9}$$

In this system of equations the derivative $db_z/d\bar{x}$ is assumed to be constant, and all other quantities are considered to be not dependent on the x coordinate. Therefore Eqs. (6)–(9) are treated as a system of ordinary differential equations with respect to the $\bar{z}$ coordinate. Excluding $\tilde{b}_x$ and $\tilde{b}_z$ in Eq. (6), we derive

$$\tilde{v}_x[-\bar{\omega}^2 + U(\bar{z})] = 0, \qquad U(\bar{z}) = \varepsilon\nu\frac{db_x}{d\bar{z}}\frac{db_z}{d\bar{x}}. \tag{10}$$

Generally, for a nonconstant $U(\bar{z})$, Eq. (10) yields $\tilde{v}_x = 0$.

From Eqs. (7)–(9), we finally obtain a second order ordinary differential equation for the $\tilde{v}_z$ velocity perturbation

$$\frac{d^2\tilde{v}_z}{d\bar{z}^2} + \bar{k}^2\tilde{v}_z\left(\frac{U(\bar{z})}{\bar{\omega}^2} - 1\right) = 0. \tag{11}$$

Further for simplicity we consider a piecewise constant function $U(\bar{z})$

$$U(\bar{z}) = \varepsilon\nu, \qquad -1 \le \bar{z} \le 1; \qquad U(\bar{z}) = 0, \qquad |\bar{z}| > 1, \tag{12}$$

which means that the current density is assumed to be constant within the current sheet.

Results.—A choice of the piecewise constant function $U(\bar{z})$ allows us to find analytical solutions which are of two kinds, kinklike and sausagelike modes. The kinklike mode is characterized by displacement of the current sheet center, and even function $\tilde{v}_z(\bar{z})$

$$\tilde{v}_z = C\exp[-\bar{k}(|\bar{z}| - 1)], \qquad |\bar{z}| > 1; \tag{13}$$

$$\tilde{v}_z = D\cos(\lambda\bar{z}), \qquad \lambda = \bar{k}\sqrt{\varepsilon\nu/\bar{\omega}^2 - 1}, \qquad |\bar{z}| \le 1. \tag{14}$$

An odd function $\tilde{v}_z(\bar{z})$ is relevant to the sausagelike mode characterized by variations of the thickness of the current layer without a displacement of its center

$$\tilde{v}_z = C\exp[-\bar{k}(\bar{z}-1)], \qquad \bar{z} > 1;$$
$$\tilde{v}_z = -C\exp[\bar{k}(\bar{z}+1)], \qquad \bar{z} < -1;$$
$$\tilde{v}_z = D\sin(\lambda\bar{z}), \qquad \lambda = \bar{k}\sqrt{\varepsilon\nu/\bar{\omega}^2 - 1}, \qquad |\bar{z}| \le 1. \tag{15}$$

Applying continuity conditions for $\tilde{v}_z$ and the first derivative $d\tilde{v}_z/d\bar{z}$ at the current layer boundaries, we obtain algebraic system corresponding to the kink mode

$$C = D\cos(\lambda), \qquad \bar{k}C = \lambda D\sin(\lambda), \tag{16}$$

and also we find a system for the sausage mode

$$C = D\sin(\lambda), \qquad -\bar{k}C = \lambda D\cos(\lambda). \tag{17}$$

Setting the determinants to vanish, we derive two equations corresponding to the kink and sausage modes, respectively

$$\tan(\lambda) = \frac{\bar{k}}{\lambda}(\text{“kink”}); \tan(\lambda) = -\frac{\lambda}{\bar{k}}(\text{“sausage”}). \tag{18}$$

These equations have discrete sequences of roots $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$. The main root is the minimal λ which corresponds to the maximal frequency.

By numerical solving these equations, we obtain two main roots $\lambda_{k,s}$ which determine the dimensional frequencies $\omega_{k,s}$ as functions of wave number for the kink and sausage modes

$$\omega_{k,s} = \omega_f \frac{k\Delta}{\sqrt{k^2\Delta^2 + \lambda_{k,s}^2}}, \qquad \omega_f = \sqrt{\frac{1}{4\pi\rho}\frac{\partial B_x}{\partial z}\frac{\partial B_z}{\partial x}}. \tag{19}$$

Here ω_f means a characteristic flapping frequency proportional to the square root of the multiplication of two gradients of the background magnetic field components, $\partial B_x/\partial z$ and $\partial B_z/\partial x$. The dimensionless functions $\omega_{k,s}/\omega_f$ are presented at the top panel in Fig. 2. Frequencies are monotonic functions of wave number, and they increase to the maximal asymptotic value ω_f for $k\Delta \to \infty$. The group wave velocity is shown in Fig. 2 as functions of wave number (the second panel).

The flapping wave perturbations become unstable when the multiplication of two magnetic gradients becomes negative. In particular, for the Earth's plasma sheet this condition corresponds to the case of decreasing B_z component towards Earth. The growth times of the instability for the kink and sausage modes are given by formulas

$$\tau_{k,s} = \tau_f \frac{\sqrt{\lambda_{k,s}^2 + k^2\Delta^2}}{k\Delta}, \qquad \tau_f = 1/\sqrt{\frac{-1}{4\pi\rho}\frac{\partial B_x}{\partial z}\frac{\partial B_z}{\partial x}}. \tag{20}$$

The instability growth times ($\tau_{k,s}/\tau_f$) are shown in Fig. 2 (bottom panel) as functions of wave number for the two wave modes. One can see from the figure that the unstable kink mode develops much faster than the sausage mode. In particular, for $k\Delta = 0.7$ the ratio of growth times is $\tau_s/\tau_k = 2$. Figure 3 illustrates a perturbation of the current sheet and the directions of plasma motion corresponding to the kink mode flapping.

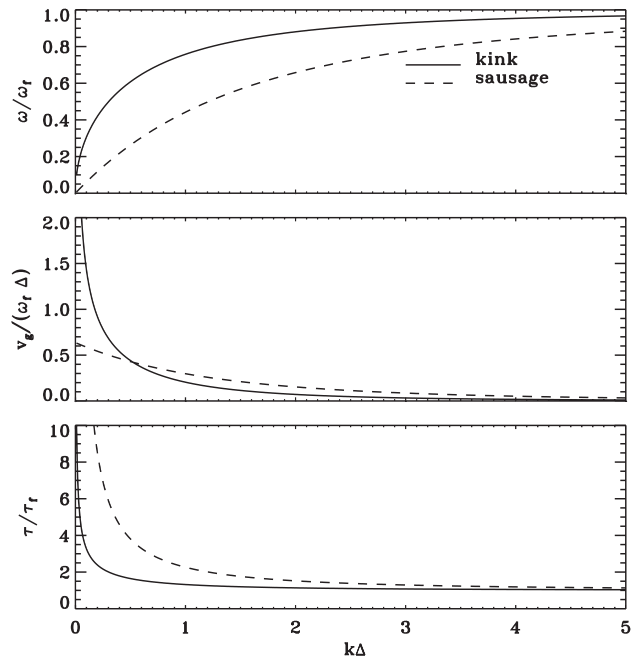

FIG. 2. Frequency, group velocity, and instability growth time as functions of wave number for two wave modes.

A qualitative explanation of the flapping instability and waves corresponding to the obtained solution is the following. Let us consider a plasma element of a unit volume at the center of the current layer as shown in Fig. 4. Initially, the equilibrium condition is assumed to be fulfilled

$$\frac{\partial P}{\partial z} = \frac{1}{4\pi}B_x\frac{\partial B_z}{\partial x}. \tag{21}$$

If we shift the plasma element along the z direction, the restoring force appears which is determined by a difference of two terms, $(\nabla P)_z$ and $[1/(4\pi)](\mathbf{B}\cdot\nabla\mathbf{B})_z$. In the new position of the magnetic tube element, the resulting force will be

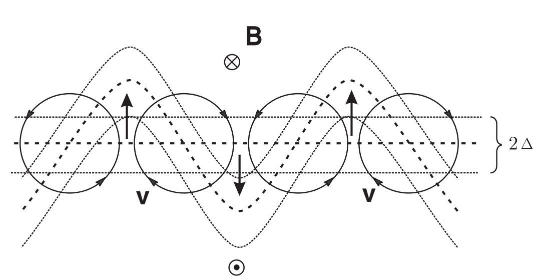

FIG. 3. Illustration to the kink mode. Perturbation of the current sheet and the corresponding directions of plasma motion.

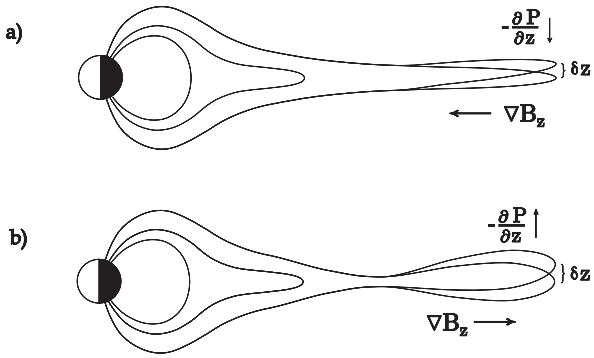

FIG. 4. Illustration to the kink flapping waves (a) and instability (b) in cases of positive and negative gradient of B_z.

$$F_z = -\frac{1}{4\pi} B_x(\delta z) \frac{\partial B_z}{\partial x} = -\frac{1}{4\pi} \delta z \left(\frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x} \right)_{z=0}. \quad (22)$$

This force accelerates plasma in the z direction

$$\rho \frac{\partial^2 \delta z}{\partial t^2} = -\delta z \frac{1}{4\pi} \frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x}. \quad (23)$$

This equation yields the characteristic flapping frequency ω_f, which is proportional to the square root of the gradients of the magnetic field components. This qualitative explanation of the instability is illustrated in Fig. 4 where panels (a) and (b) correspond to the stable and unstable situations, respectively. The current sheet is stable when the background total pressure has a minimum at the center of the sheet. Instability condition is reached in the opposite case when the total pressure has a maximum at the center of the sheet.

For example, we estimate this frequency for the parameters which seem to be reasonable for the conditions of the current sheet in the Earth's magnetotail,

$$B_x = 20 \text{ nT}, \quad B_z = 2 \text{ nT}, \quad \Delta \sim R_E, \quad n_p = 0.1 \text{ cm}^{-3},$$
$$k\Delta = 0.7, \quad \partial B_z/\partial x \sim B_z/L_x, \quad L_x \sim 5R_E. \quad (24)$$

For these parameters we find the characteristic flapping frequency $\omega_f \sim 0.03 \text{ s}^{-1}$, and also the group velocity $V_g = 60$ km/s.

Summary.—The flapping instability and waves are analyzed for a current sheet in a presence of two gradients of the B_x and B_z magnetic field components along the z and x directions, respectively. Both of these gradients play a crucial role for the stability of the current sheet. The instability occurs in the regions of the current layer where the multiplication of two gradients is negative. In particular, the instability can arise in a vicinity of a localized thinning of the current sheet [Fig. 4(b)]. In stable regions, the flapping waves are associated with the so-called "bursty bulk flows" or BBF's [3], which are the magnetic tubes rapidly moving through the center of the current sheet towards the Earth. These BBF's are considered to be the sources of the flapping wave oscillations propagating from the center of the current sheet towards the flanks in the $\pm y$ directions.

The analytical solution is obtained for the simplified model of the current layer with a constant current density. The frequency and the growth rate for the kink mode are found to be much larger than those for the sausage mode. For both modes, the frequencies are monotonic increasing functions of the wave number. The corresponding wave group velocities are decreasing functions of the wave number, and they vanish asymptotically for high wave numbers. For the typical parameters of the Earth's current sheet, the group velocity of the kinklike mode is estimated as a few tens of kilometers per second that is in good agreement with the CLUSTER observations. A strong decrease of the group velocity for high wave numbers means that the small scale oscillations propagate much slower than the large scale oscillations. Because of that, the propagating flapping pulse is expected to have a smooth gradual front side part and a small scale oscillating backside part.

The double-gradient flapping waves studied in our model propagate in the direction perpendicular to the planes of the background magnetic field lines, and thus they cannot be stabilized by the magnetic tension. For the kink mode, the magnetic field planes are just shifting with respect to each other.

The effects of compressibility can be neglected when the phase speed of wave perturbations is much less than the sonic speed. This condition is fulfilled for our solution. The assumption $\nu \gg \varepsilon$ used in our model seems to be appropriate for steady magnetotail conditions. For example, it is well fulfilled in the current sheet equilibrium solution of Kan [10]. The neglected second order terms $O(\varepsilon^2)$ are responsible for the small effects related to the Alfvén waves propagating in the z direction. These second order effects are subjects for future study.

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