Mean Velocity Profile in Confined Turbulent Convection

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In this Letter, highly resolved measurements of the horizontal velocity inside the boundary layer of turbulent Rayleigh-Bénard convection are reported. They were performed in a cylindrical box with an aspect ratio $\Gamma = 1.13$ which was filled with air with a Prandtl number $\mathrm{Pr} = 0.7$. The horizontal velocity was measured along the central axis close to the cooling plate in a range of Rayleigh numbers between $\mathrm{Ra} = 10^{11}$ and $\mathrm{Ra} = 10^{12}$ using a two-dimensional laser Doppler velocimeter. We demonstrate that the profile of the mean velocity strongly differs from that of classical shear flows like the Blasius shape of a laminar flat plate boundary layer or a turbulent logarithmic velocity profile with standard coefficients.

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Rayleigh-Bénard (RB) convection in a closed box with approximately equal horizontal and vertical extent is characterized by the existence of a mean flow often called wind [1–3]. Whereas the temperature field in this type of flow is well known from previous measurements at Rayleigh numbers up to $\mathrm{Ra} = 10^{12}$ [4–6] the resolution of presently available velocity data [7–9] is insufficient to characterize the velocity boundary layer in the vicinity of the heating and cooling plates in sufficient detail. In the present Letter we report measurements of the mean horizontal velocity close to the cooling plate of a large-scale RB experiment, which for the first time are sufficiently well resolved to distinguish between competing theoretical predictions [10–12].

The heat transport in turbulent RB convection has received considerable attention in the past [10–17] due to its relevance to geophysical and astrophysical fluid dynamics as well as engineering applications like room ventilation [18]. Theoretical predictions and their comparison with experiments usually focus on the dependence of the nondimensional heat flux, embodied in the Nusselt number Nu, on the nondimensional temperature difference described by the Rayleigh number Ra. Phenomenological theories predicting the dependence Nu(Ra) usually rely, either explicitly or implicitly, upon assumptions about the shape of the profile $v(z)$ of the horizontal component of the wind in the vicinity of the heating and cooling plates. For instance, Shraiman and Siggia [13] have assumed a logarithmic velocity profile in deriving the relation $\mathrm{Nu} \sim \mathrm{Ra}^{2/7}$ that was supposed to explain the experiments by Castaing *et al.* [11]. Similarly, albeit for a vertical rather than a horizontal plate, Hoelling and Herwig [19] have relied on logarithmic velocity and temperature profiles to derive Nu(Ra). Grossmann and Lohse [12,15] in turn have employed the Blasius profile characteristic of the laminar flow around a flat plate as the basis for their predictions of the Nusselt number over a wide range of Ra. That assumption is also the starting point for the discussion of the heat transport at ultrahigh Rayleigh numbers in low temperature helium experiments [16]. In order to test the validity of these assumptions it is necessary to perform experiments that satisfy two conditions. Namely they have (i) to achieve high Rayleigh numbers and (ii) to guarantee a sufficiently high spatial resolution in order to be able to distinguish between competing theories. As a matter of fact, none of the previous measurements [7–9] could satisfy these requirements simultaneously.

In the present work we overcome the limitations of previous experiments by studying RB convection in a large-scale facility, the "Barrel of Ilmenau." Its size of 7.15 m in diameter and 6.30 m in height allows us to meet both the requirement of high-Rayleigh number and high spatial resolution of velocity measurement at the same time. The Barrel of Ilmenau consists of an adiabatic cylinder in which air with Prandtl number $\mathrm{Pr} = 0.7$ is heated from below and cooled from above. It can be described by totally five dimensionless parameters, the Rayleigh number $\mathrm{Ra} = \alpha g \Delta T H^3/\nu\kappa$, the Prandtl number $\mathrm{Pr} = \nu/\kappa$ and the aspect ratio $\Gamma = D/H$ characterizing the input parameter set as well as the Reynolds number $\mathrm{Re} = UH/\nu$ and the Nusselt number $\mathrm{Nu} = \dot{Q}_C/\dot{Q}_D$ which are the response of the system. In these definitions α is the isobaric thermal expansion coefficient, g the acceleration of gravity, ΔT the temperature difference between both horizontal plates, H the distance between the heating and the cooling plate, ν the kinematic viscosity, κ the thermal diffusivity, $\dot{Q}_C$ and $\dot{Q}_D$ are the convective and the diffusive heat flux and U is the global velocity of the mean flow. A detailed description of the experimental facility in which turbulent convection can be investigated in a parameter field of $10^5 < \mathrm{Ra} < 10^{12}$, $1 < \Gamma < 150$, and $\mathrm{Pr} = 0.7$ is given in [6].

In this Letter we report direct measurements of both horizontal velocity components close to the cooling plate in the central axis of the RB cell using a two-dimensional laser Doppler velocimeter system. The measurements were performed at fixed aspect ratio $\Gamma = 1.13$ at which the global flow usually develops a large single roll. This type of flow is referred to as confined convection. By varying the temperature difference between the heating and the cooling plate from $\Delta T = 4$ K up to $\Delta T = 60.5$ K eleven

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different Rayleigh numbers between $\mathrm{Ra} = 10^{11}$ and $\mathrm{Ra} = 10^{12}$ were covered (see Table I) [20].

For our measurements we have adopted Cartesian coordinates (x, y, z) which originate at the center of the cooling plate where x points roughly in the mean flow direction and z is normal to the wall and oriented downward. Since it is known from previous measurements [21,22] that the orientation of the horizontal velocity vector oscillates periodically over a wide angle a simple one-dimensional measurement of the velocity would not yield a correct estimate of the total mean velocity compared with a real two-dimensional measurement. Therefore we measured two components simultaneously by capturing one-hour time series of both $v_x(z, t)$ and $v_y(z, t)$ at 39 different z positions starting at $z = 90$ mm up to a minimum distance of $z = 0.47$ mm to the cooling plate. Typical sampling rates were between $n = 20\ \mathrm{s}^{-1}$ and $n = 1\ \mathrm{s}^{-1}$ depending on both the particle density inside the flow and its velocity. From the measured velocity components the magnitude $v(z, t) = [v_x^2(z, t) + v_y^2(z, t)]^{1/2}$ and the relative angle $\phi(z, t) = \arctan[v_y(z, t)/v_x(z, t)]$ against the x axis were computed as functions of time. The composition of the mean values $v(z)$ of every time series gives the mean velocity profile.

One important difference to classical turbulent shear flows becomes immediately obvious after inspecting the raw velocity data and the profiles in Fig. 1. Whereas the velocity of the incoming flow in a wind tunnel is virtually steady, the velocity of our wind shows large deviations over time scales of hours (see inset of Fig. 2). This results in a wiggly appearance of our profiles which should not be considered as a signature of insufficient measurement time. (The corresponding temperature data, published in du Puits *et al.* [6] are entirely smooth.) A particularly strong breakdown of the mean velocity over a period of several hours corresponding to five measurement points in the profile is highlighted with a circle in Fig. 2.

TABLE I. Set of parameters and summarized results of the velocity measurements at constant aspect ratio $\Gamma = 1.13$.

Ra $[10^{11}]$	ΔT [K]	$\lvert\bar{v}_{\max}\rvert$ [m/s]	Re_τ	v_τ [cm/s]	δ_v [mm]	δ_m [mm]	S
1.23	4.0	0.19	170	1.5	14.1	8.4	1.68
1.68	5.7	0.22	147	1.9	10.6	6.2	1.70
1.96	7.0	0.22	168	2.0	11.5	7.0	1.64
2.62	9.8	0.27	171	2.3	10.0	6.0	1.65
3.39	13.6	0.31	184	2.6	9.5	5.8	1.64
4.14	17.6	0.34	169	2.6	8.0	4.7	1.70
5.38	25.5	0.41	195	3.3	7.7	4.8	1.61
6.40	31.7	0.44	204	7.4	7.6	5.5	1.39
7.48	40.0	0.51	249	4.1	8.3	5.4	1.53
8.64	49.0	0.56	280	12.8	8.7	6.5	1.34
9.77	60.5	0.63	321	3.8	9.0	5.8	1.54

In a first step we address the question as to whether the profile of the mean velocity is of Blasius type. In Fig. 1 two measured mean velocity profiles at $\mathrm{Ra} = 1.23 \times 10^{11}$ and $\mathrm{Ra} = 7.48 \times 10^{11}$ are plotted against two Blasius-solutions of the two-dimensional boundary layer equations [23]. In order to perform such a comparison we need to specify the path length x in the similarity parameter $\eta = z\sqrt{v_{\max}/2x\nu}$ of the Blasius solution. If we follow the implicit assumption of [12] that the development of the boundary layer starts at the outer edge of the plate, we have to take x as the radius $D/2 = 3.5$ m of the barrel. In this case our profile drastically differs from the Blasius profile, indicating that the picture of a laminar boundary layer developing from the rim of the cooling plate is unlikely to be correct. Next we specify x in such a way that the velocity gradient $\partial v/\partial z$ of the Blasius solution corresponds to the gradient of the measured profile extrapolated to $z = 0$. While at the lowest Ra the near-wall part of the profile up to approximately $v(z) = 0.5v_{\max}$ grows almost linearly, the high-Rayleigh-number profile deviates from linearity over a significant part of the measurement region. In the outer boundary layer region $0.5v_{\max} < v(z) < v_{\max}$ where the plume advection occurs, all measured profiles are generally more flat than the corresponding Blasius profile. It can be concluded that the Blasius profile does not provide a good approximation to the profile of mean velocity in confined turbulent RB convection.

Our next step consists in comparing the mean velocity with the logarithmic velocity profile in isothermal turbulent shear flows; see Fig. 2. For this purpose z and $v(z)$ have been recomputed into cross coordinates $z^+ = zv_\tau/\nu$ and $v^+(z^+) = v(z)/v_\tau$ with $v_\tau = \sqrt{\nu\partial v/\partial z|_{z=0}}$ (see Table I). Although direct measurements of v_τ are not possible in our experimental facility, the extrapolation of the measured

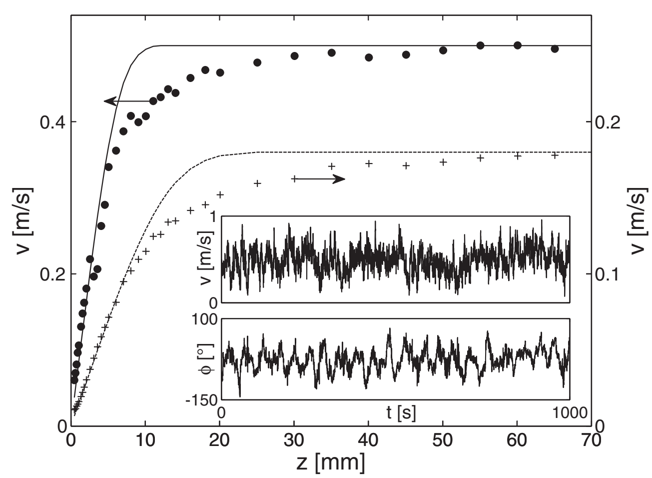

FIG. 1. Profiles of the mean velocity $v(z)$ for $\mathrm{Ra} = 7.48 \times 10^{11}$ (●, left axis) and $\mathrm{Ra} = 1.23 \times 10^{11}$ (+, right axis) together with the theoretical solution of the laminar shear layer by Blasius. The insets show typical time series of the velocity magnitude $v(t)$ and the relative angle $\phi(t)$.

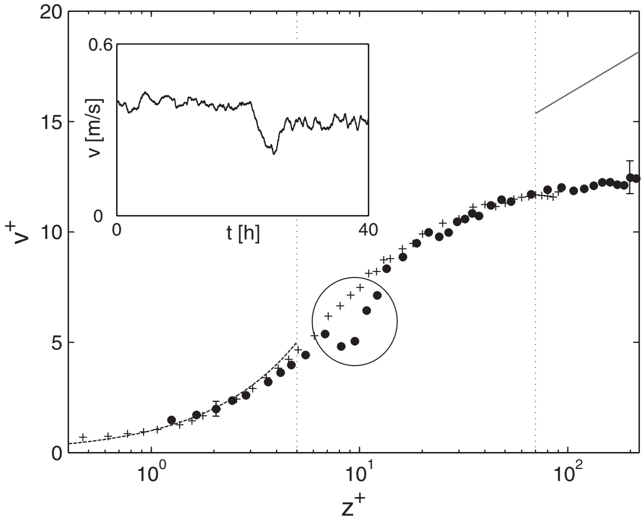

FIG. 2. Profiles of the mean velocity $v^+(z^+)$ for two different Rayleigh numbers Ra $= 7.48 \times 10^{11}$ (●) and Ra $= 1.23 \times 10^{11}$ (+). The error bars indicate the typical 95% confidence interval. The dashed line below $z^+ = 5$ presents the linear scaling $v^+ = z^+$ close to the wall, the full line above $z^+ > 70$ is the logarithmic scaling $v^+ = 1/\kappa \ln(z^+) + 5$. Occasional breakdown of the wind (see circle) is a particular feature of highly turbulent convection which we have also observed in an independent hot-wire velocity measurement, as shown in the inset.

velocity profile to the point of origin was found to provide reliable estimates for the friction velocity. The linear scaling of v^+ over z^+ in the viscous sublayer below $z^+ < 5$ is reflected quite well by the measured profiles (see the dashed line in Fig. 2) confirming the correctness of the order of magnitude of v_τ. Further away from the surface of the cooling plate and beyond the so-called buffer layer ($5 \leq z^+ < 70$) a zone follows where the mean velocity profile of the fully developed turbulent boundary layer is characterized by its logarithmic growth ($z^+ \geq 70$). Since the access into the cell was limited by our measurement technique to positions up to $z = 90$ mm corresponding to $z^+_{\max} \approx 100$ at Ra $= 10^{11}$ and $z^+_{\max} \approx 200$ at Ra $= 10^{12}$ we can not finally judge whether or not the shapes of the measured profiles are of a logarithmic form. However, it is obvious that they deviate from the mean velocity profile of a canonical turbulent shear layer which can be described by $v^+ = [\ln(z^+) + C]/\kappa$ with the constants $\kappa = 0.41$ and $C = 5.0$ [23]. The fact that the mean velocity in this part of the boundary layer grows not as fast as it does in a classical turbulent shear layer, suggests an enhanced momentum transport normal to the wall. One of the possible reasons could be the permanent plume advection from the boundary layer into the outer flow which stimulates an additional wall-normal flow component. However, it could be caused as well by vertical components in the very complex and strongly three-dimensional outer flow from which vortices penetrate into the near-wall region.

At this point we wish to focus our attention onto a special feature of highly turbulent convection which is particularly marked in Fig. 2. For $8 < z^+ < 12$ corresponding to a time interval of four hours during the measurement of the Ra $= 7.48 \times 10^{11}$ profile, these four values of the mean velocity are significantly reduced against the natural shape of the curve. Since we observed those so-called "break downs" also in additional measurements using hot film sensors (see inset of Fig. 2) and we checked our measurement technique very carefully we can definitely exclude any measurement errors. In contrast to simple shear flows the wind in turbulent RB convection is more complicated and contains apart the well-known reversals, rotations and cessations [24,25] some further unknown events which should be the object of ongoing research.

A more quantitative approach to the characterization of the shape of the mean velocity profiles consists in the investigation of the shape factor $S = \delta_v/\delta_m$ defined as the ratio between the displacement thickness δ_v and the momentum thickness δ_m with

$$\delta_v = \int_0^\infty \left\{1 - \frac{v(z)}{v_{\max}}\right\} dz, \quad (1)$$

$$\delta_m = \int_0^\infty \left\{1 - \frac{v(z)}{v_{\max}}\right\}\left\{\frac{v(z)}{v_{\max}}\right\} dz. \quad (2)$$

While the velocities v^+ are exclusively related to the momentum transport at the wall, the shape factor reflects the behavior of the velocity over the complete boundary layer, particularly in the outer region. For our profiles we obtain shape factors in the range between $S = 1.35$ to $S = 1.70$ where the lower values are related to the higher Ra, as shown in Fig. 3. In each case the shape factors are clearly smaller than $S = 2.6$—the value of a laminar shear layer—confirming the enhanced wall-normal momentum transport as already seen in the profiles of the dimensional velocity.

Finally, we address the question about the self-similarity of the profiles in the investigated range between Ra $= 10^{11}$

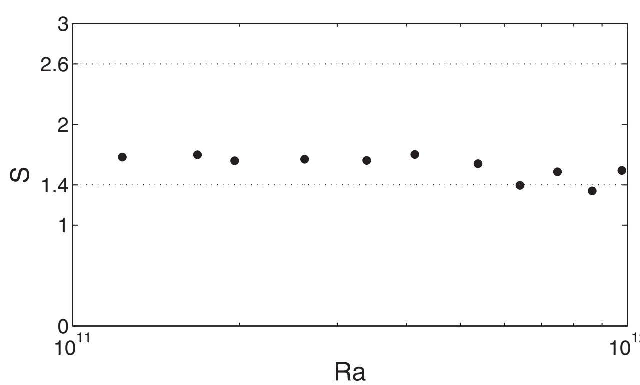

FIG. 3. Shape factor $S = \delta_v/\delta_m$ over Ra, the dotted lines $S = 2.6$ and $S = 1.4$ characterize the typical values for the laminar and the turbulent shear layer, respectively.

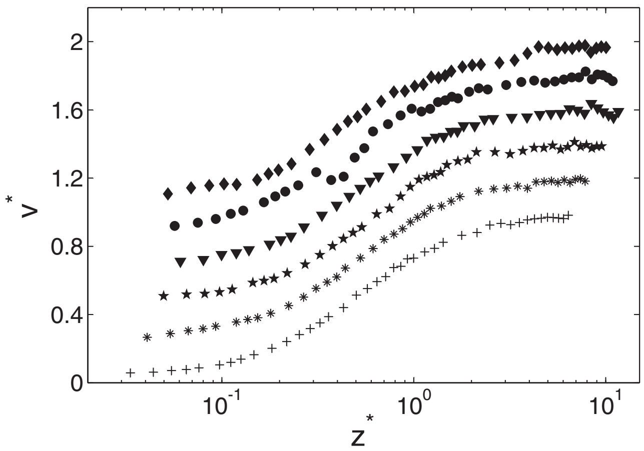

FIG. 4. Profiles of the mean velocity scaled by the maximum velocity and the displacement thickness for $Ra = 9.77 \times 10^{11}$ (◆), $Ra = 7.48 \times 10^{11}$ (●), $Ra = 5.38 \times 10^{11}$ (▼), $Ra = 3.39 \times 10^{11}$ (★), $Ra = 1.96 \times 10^{11}$ (*) and $Ra = 1.23 \times 10^{11}$ (+). The profiles are shifted against each other by +0.2.

and $Ra = 10^{12}$. In several RB experiments with liquid Helium a transition in the scaling law of the heat transport $Nu = f(Ra)$ has been observed [26]. Therefore, we also focused our interest to a possible transition in the shape of the mean velocity profile. In order to compare them, the profiles were normalized by the displacement thickness $z^* = z/\delta_v$ and the maximum velocity $v^* = v/v_{max}$ and they are plotted in Fig. 4. Despite small deviations, mainly caused by statistical errors due to the limited measurement time, all profiles up to the highest Ra at $Ra = 9.77 \times 10^{11}$ are similar and do not indicate any transition in the boundary layer structure. This finding confirms the results of previously published temperature measurements [6] in which the mean temperature profile showed little changes in this parameters range.

In summary, we have for the first time performed highly resolved 2-*d* measurements of the horizontal velocity inside the boundary layer of confined turbulent convection at aspect ratio $\Gamma \approx 1$ and Rayleigh numbers between $Ra = 10^{11}$ and $Ra = 10^{12}$. It could be demonstrated that the gradient of the mean velocity profile does not remain constant over the boundary layer region in this regime of Ra as predicted by various theoretical models [10,12,14]. Particularly in its outer region the momentum transport is enhanced against the classical boundary layer models in shear flows, obviously due to an additional wall-normal velocity component which is not reflected in the classical Blasius equations. Its driving force could be the permanent plume advection from the boundary layer into the wind or wall-normal components of the outer flow. Because of missing wall-normal velocity measurements this question remains open and requires further experiments.

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