Comment on "Competing Interactions, the Renormalization Group, and the Isotropic-Nematic Phase Transition"

In a recent Letter, Barci and Stariolo (BS) [1] generalized the well-known Brazovskii model [2] to include an additional rotationally invariant quartic interaction and study this model in two dimensions ($d = 2$). Authors do not give any specific example of a microscopic system to which this generalized Landau-Ginzburg-Wilson (LGW) action corresponds. After a brief discussion of a possible renormalization group (RG) treatment, they proceed to study their model using a mean-field Hartree approximation. They then find that when $u_2 > 0$ the model exhibits striped (lamellar) phase, but when $u_2 < 0$ it shows a nematic order.

Brazovskii model is notoriously difficult to renormalize. There exist, however, well-known lattice models which exhibit exactly the same phenomenology and are much more tractable to the RG treatment. One example is Widom's isotropically spatially frustrated lattice model of microemulsion [3]. The model exhibits isotropic-to-lamellar phase transitions, which can be studied by mapping it directly onto anisotropic $O(6)$ field theory [4]. This field theory has the lower critical dimension $d_l = 2$ and the upper critical dimension $d_u = 4$ (away from the isotropic Lifshitz point for which $d_l = 4$ and $d_u = 8$). Since existence of a lattice diminishes the role of fluctuations, we expect that the lower critical dimension for the field theory considered in Ref. [1] should be $d_l > 2$. Below the lower critical dimension no mean-field theory can be trusted *even qualitatively*. Since the Hartree approximation used in Ref. [1] is nothing more than a mean-field theory, it is bound to fail in $2d$. In this Comment, I will argue that the situation of the theory of Ref. [1] is even more difficult, since the lower critical dimension for this model is actually $d_l = 3$.

Let us first consider the lamellar phase found when $u_2 > 0$. Suppose that the symmetry is broken in such a way that the lamellae are parallel to the x axis (parallel to the x-y plane in 3D). We want to study the fluctuations of the interfaces separating the high and the low order parameter states. Suppose that one of the interfaces lies along the x axis (is in the x-y plane in 3D). At finite temperature, this interface will fluctuate. We want to find an effective Hamiltonian which controls these fluctuations. Clearly, this Hamiltonian must be invariant under the transformation $h \rightarrow -h$, where h is the height of the interface over the projection plane. Furthermore, since the original LGW action of Ref. [1] is invariant under arbitrary translations and rotations, the effective interfacial Hamiltonian must be invariant under $h \rightarrow h + a + \mathbf{b} \cdot \mathbf{x}$, where $\mathbf{x}$ is an arbitrary vector in the projection plane of the lamella, and a and $\mathbf{b}$ are arbitrary constants. To leading order in h, the interfacial Hamiltonian must, therefore, be $\mathcal{H} = \frac{\kappa}{2} \int d^{d-1}\mathbf{x}(\nabla^2 h)^2$, where κ is an effective bending modulus. We now study the fluctuations of these interfaces. Define a local width of an interface w as $w^2 = \langle [h(\lambda_0/2) - h(0)]^2 \rangle$, where λ_0 is the wavelength, $\lambda_0 = 2\pi/k_0$, of the order parameter in the symmetry broken phase. It is then easily found that for a two-dimensional system (1D interfaces) the interfacial width w diverges as $w^2 \sim T\lambda_0^2 L/\kappa$, where L is the system size. Thus, lamellar order is impossible in $2d$. In 3D ($2d$ interfaces), w diverges as $w^2 \sim (T\lambda_0^2/\kappa)\ln(L/\lambda_0)$, allowing for a pseudo-long-range lamellar order. This fact was already known to Landau in the 1940s and is noted in the original Brazovskii paper [2], who also finds logarithmic divergences beyond the Hartree approximation. It is, then, clear that no mean-field theory (Hartree included) can be used to study Brazovskii type models in $2d$.

For $u_2 < 0$, BS find an isotropic-nematic transition with *mean-field exponents*. They then speculate that fluctuations will turn this transition into the Kosterlitz-Thouless (KT) one. Nowhere is this statement proven explicitly and, indeed, no such proof is possible. In $2d$ the fluctuations can completely destroy a mean-field phase transition, as happened for the lamellar phase discussed above. The fluctuations can also change the nature of the phase transition to something very different and nonuniversal. Simply because the coarse-grained LGW action has a nematic symmetry does not imply that a microscopic (fine-grained) model will have a KT transition. There is no such strong universality in $2d$. For example, there is a class of generalized XY models all with the same LGW action, but their critical behavior depends on the precise form of the microscopic interaction potential [5]. To conclude, the isotropic-to-lamellar phase transition found by BS cannot exist in $2d$. As far as the isotropic-nematic transition, nothing about its order or its universality class can be said based on the coarse-grained LGW action presented in Ref. [1].

This work was supported in part by the CNPq.

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Received 1 June 2007; published 30 November 2007
DOI: 10.1103/PhysRevLett.99.228903
PACS numbers: 05.70.Fh, 05.10.Cc, 64.70.−p, 68.35.Rh

[1] D. G. Barci and D. A. Stariolo, Phys. Rev. Lett. **98**, 200604 (2007).
[2] S. A. Brazovskii, Zh. Eksp. Teor. Fiz. **68**, 175 (1975) [Sov. Phys. JETP **41**, 85 (1975)].
[3] B. Widom, J. Chem. Phys. **84**, 6943 (1986).
[4] Y. Levin and K. A. Dawson, Phys. Rev. A **42**, 1976 (1990); M. C. Barbosa, Phys. Rev. B **42**, 6363 (1990).
[5] E. Domany, M. Schick, and R. H. Swendsen, Phys. Rev. Lett. **52**, 1535 (1984); R. L. C. Vink, Phys. Rev. Lett. **98**, 217801 (2007).

