Relaxation Dynamics in Quantum Electron Glasses

Z. Ovadyahu

Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

It is experimentally shown that, depending on the carrier concentration of the system n, the dynamics of electron glasses either *slows down* with increasing temperature or it is *independent* of it. This also correlates with the dependence of a typical relaxation time (or "viscosity") on n. These linked features are argued to be consistent with a model for dissipative tunneling. The slow relaxation of the electron glass may emerge then as a manifestation of friction in a many-body quantum system. Our considerations may also explain why strongly localized granular metals are likely to show electron-glass effects while semiconductors are not.

Coupling to the environment is known to have nontrivial effects on the dynamics of quantum systems. This has been discussed in the context of a two-state-system coupled to an oscillator bath [1]. Much less is understood in real systems, most notably in systems that contain many, and possibly strongly interacting two-state-systems [2]. In this work we study experimentally such a quantum system, specifically the electron glass. It is shown that the carrier concentration of the system plays a unique role in determining the temperature dependence of its dynamics as well as its "viscosity." The results are compared with a model presented in Ref. [1] that seems to account for the dependence of the glassy dynamics on both temperature and carrier concentration. This, in turn, may indicate the relevance of the orthogonality catastrophe [3] to the slow relaxation in electron glasses.

Samples used in this study were thin films of either crystalline or amorphous indium-oxide (to be referred to as In_2O_{3-x} and In_xO respectively). The thickness for a particular batch (typically, $30–200$ Å) was chosen such that at the measurement temperatures all the samples had sheet resistance $R_\square$ within the range 10 MΩ–100 MΩ. The present study focused on the $4–6$ K range of temperature which was achieved either in a ^{3}He fridge with the sample attached to a copper cold-finger, or by employing non-Ohmic fields with the sample immersed in liquid helium [4]. No difference was found in the results using either technique in this range of temperatures.

The main method used in this work to characterize the dynamics of the electron glasses is the double-conductance-excitation (DCE) technique more fully described in [4], which also includes a comprehensive discussion of other techniques for measuring dynamics and their associated caveats. The procedure is illustrated in Fig. 1 along with the data components that are involved in the analysis. Starting with a voltage $V_g(1)$ held at the gate, and the sample equilibrated under the fixed external conditions (temperature T, or electric field F), one monitors the conductance as function of time $G(t)$ to obtain the equilibrium conductance. Next, the gate voltage is switched to $V_g(2)$ and is maintained there for a "waiting

time" t_w that for all the experiments reported here was 180 s. Finally, the gate voltage is switched back to $V_g(1)$, and $G(t)$ is measured for an additional period of time. Here we use the ratio $\eta \equiv \Delta G(0)/\Delta G(t_w)$ as a measure of dynamics (viscosity) where (cf. Fig. 1) $\Delta G(0) \equiv \Delta G' - \Delta G_{eq} = G(0) - 1 - \Delta G_{eq}$; $\Delta G_{eq} \equiv \frac{G[V_g(2)]}{G[V_g(1)]}$ (cf. inset to Fig. 1), and $\Delta G(t_w) \equiv G(t_w) - 1$. Note that $\Delta G(t_w)$ depends on how far the sample conductance has drifted in phase-space during t_w towards its equilibrium state (set by $V_g(2)$ and the external conditions, e.g., T). If, for example, a full equilibrium is reached during t_w, $\Delta G(t_w)$ will obviously equal $\Delta G(0)$ yielding $\eta = 1$. If, on the other hand, relaxation is infinitely slow, $\Delta G(t_w)$ will be zero ($\eta = \infty$). The origin of ΔG_{eq} is the thermodynamic field effect. This
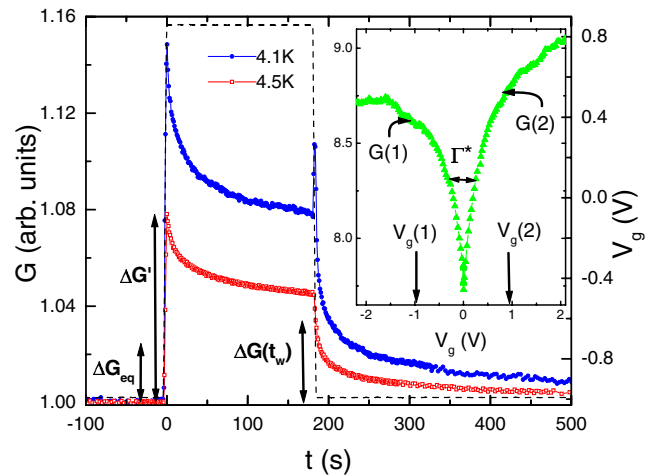

FIG. 1 (color online).   Typical runs of the DCE protocol at two temperatures. The dependence of the conductance G on time is plotted as squares and the gate voltage as dashed line. Sample shown is In_2O_{3-x} deposited on a 0.5 μm SiO_2 spacer thermally grown on Boron-doped Si wafer. Inset shows the memory-dip of this sample measured in an independent set of experiments after allowing the sample to relax for 28 h under $V_g = 0$ V at $T = 4.1$ K. The difference between $G(2)$ and $G(1)$ is the basis for estimating ΔG_{eq} (see [4] for fuller details).

physical quantity is associated with the antisymmetric
contribution to the field effect as illustrated in the inset to
Fig. 1, which also depicts the characteristic width Γ^* of the
memory dip [5]. Note that $\Delta G(0)$ is a proper normalization; when the external conditions are changed, the degree
of sample excitation [when $V_g(2)$ is switched to $V_g(1)$ or
vice versa] will in general change too, and it will be
reflected in $\Delta G(0)$. Likewise, the value of ΔG_{eq} is in
general $\Delta G_{eq}(T)$ and should be measured for each temperature separately. The values of $\Delta G(0)$ and $\Delta G(t_w)$ are
extracted from the $G(t)$ data as in Fig. 1 as follows. First,
the times t_1 and t_2, where $V_g(2)$ reaches its final value, and
$V_g(1)$ is reinstated, respectively, are noted. These are used
as the origin ($t = 0$) for the two relaxations of $G(t)$, the first
after the $V_g(1)$ to $V_g(2)$ switch, the second after the switch
back. The respective values of $G(t_0)$ are found by extrapolation to t_0 using the logarithmic law [4] (t_0 is the resolution time of the measurement, typically $t_0 = 1$ s), and are
used to calculate $\Delta G(0)$ and $\Delta G(t_w)$ by subtracting the
appropriate baseline [i.e., the equilibrium G which for
$\Delta G(0)$ is different than for $\Delta G(t_w)$ due to the antisymmetric component shown in the inset to Fig. 1].

Results for η versus temperature based on DCE measurements are shown in Fig. 2 for six of the studied
samples. Three of these are for In_2O_{3-x} films, two of which
are doped with gold [Fig. 3(a)], and the other three are the
amorphous version [Fig. 3(b)]. For each sample, data were
taken starting at the lowest temperature, where the sample
was allowed to equilibrate for at least 24 h. A two hours
equilibration period was used for each higher temperature.

As a further check, the dependence of the dynamics on
temperature was also assessed by measuring the excess
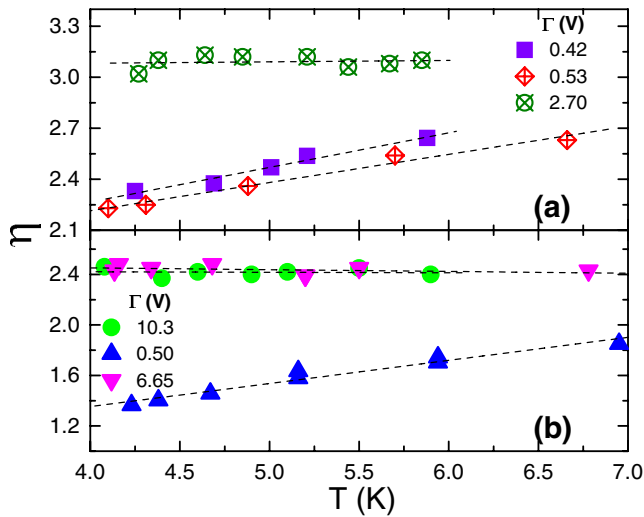conductance versus time $\Delta G(t)$. Such data are available
anyhow as part of the more sensitive DCE procedure [4],
and few examples for the properly normalized $\Delta G(t)$ are
given in Fig. 3. The results of DCE measurements on the 17
samples studied in this work are shown in Fig. 4. These
suggest that, at the studied range of temperatures, the
relaxation of the electron glass either *slows down* upon
increase of temperature, or it is *temperature independent*.
Note that data are shown for films of In_2O_{3-x}, In_xO (with a
broad range of x, i.e., stoichiometry), Au-doped In_2O_{3-x},
and the granular aluminum studied by Grenet *et al.* [6].
These are quite different systems in terms of microstructure, electron-phonon coupling, and carrier concentrations,
and they usually obey different $G(T)$ laws (though all are
activated). It is thus remarkable that their $\partial\eta/\partial T$ exhibits a
unified correlation with a single parameter—the width Γ^*
of the memory dip (cf. inset to Fig. 1).

Note that the temperature dependent dynamics is observed only for samples with $\Gamma^* \lesssim 2$ V. For small Γ^*
$\partial\eta/\partial T$ decreases fast with Γ^*, and saturates for $\Gamma^* \gtrsim$
2 V (Fig. 4). This functional dependence of $\frac{\partial\eta}{\partial T}[\Gamma^*]$ is
strikingly similar to the dependence of the typical relaxation time $\tau[\Gamma^*]$ measured by the two-dip experiment in a
series of In_xO films (all with similar values of $R_\square$) [7]. It
has been further shown by Vaknin *et al.* [5,7] that Γ^*
increases monotonically with the carrier concentration n
of the system that, for the range of Γ^* in Fig. 4, spans n
values from $\approx 3 \times 10^{19}$ cm^{-3} for undoped In_2O_{3-x} to
$\approx 10^{23}$ cm^{-3} for granular metals. The pattern that emerges
from our results is that, all other things being equal, the
wider is the memory-dip Γ^* of a sample (or equivalently,

FIG. 2 (color online). The dependence of the dynamics parameter η on temperature for typical samples with specified
values of Γ^*; (a) Undoped In_2O_{3-x} (full squares), In_2O_{3-x} doped
with 2% and 3.7% Au (diamonds and circles, respectively)
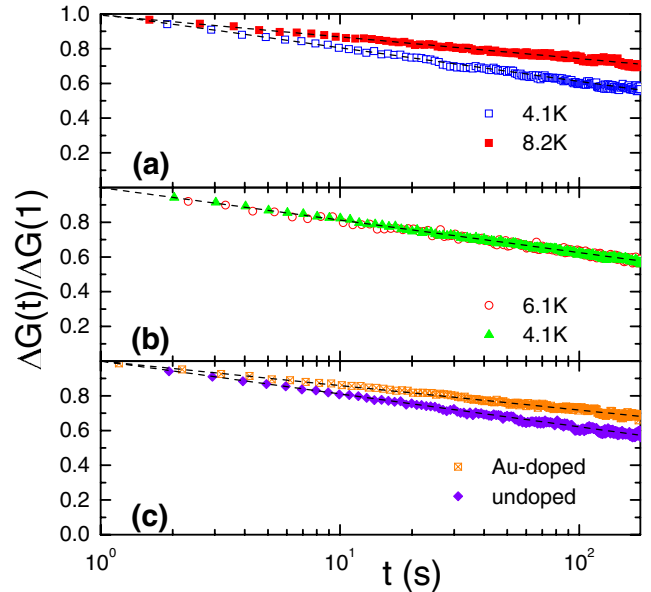(b) In_xO samples. Dashed lines are best fits.

FIG. 3 (color online). Normalized relaxation versus time
curves for several samples. Data are based on $\Delta G(0)$ produced
by $V_g(1) \rightarrow V_g(2)$ excitation (cf. Fig. 1). (a) In_2O_{3-x} with $\Gamma^* =$
0.47 V. (b) In_xO with $\Gamma^* = 6.65$ V. (c) Comparing Au-doped
and undoped In_2O_{3-x} samples, both with $R_\square = 30$ MΩ, and
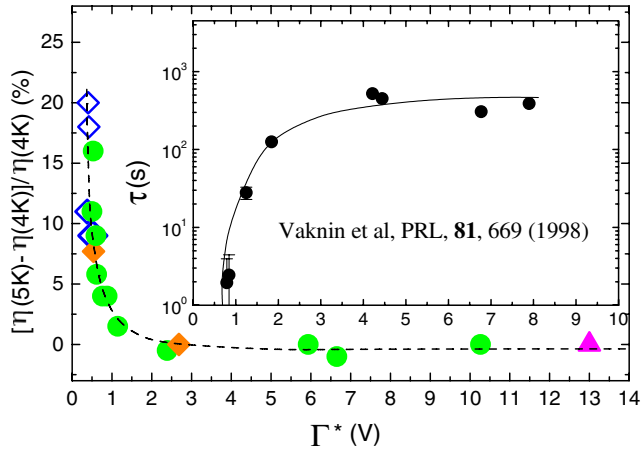both measured at 4.1 K. Dashed lines are best linear fits.

FIG. 4 (color online).   Temperature dependence of the relaxation dynamics vs Γ^* (evaluated on basis of a 0.5 μm thick SiO_2 spacer between the sample and the gate to allow comparison with the data in the inset). Empty diamonds are undoped In_2O_{3-x}, full diamonds are Au-doped In_2O_{3-x}, and circles are In_xO samples. Dashed line is a guide to the eye.

the larger is n), the *slower* is its relaxation (*higher* "viscosity"). In addition to the systematic correlation noted with the In_xO data, this can be seen, e.g., in Fig. 3(c) comparing the dynamics in two In_2O_{3-x} samples with identical $R_\square$ and the sample with the larger Γ^* exhibits slower dynamics. These results may be then summarized as follows: Dynamics in the electron-glass is more sluggish *and* less temperature dependent the higher is the carrier concentration n (or Γ^*) of the system. These correlations are the basis for the main conclusions of the Letter.

The linkage between temperature dependence and rate of relaxation lead us to consider the relevance of a quantum dissipation scenario where relaxation rate and T dependence are inherently connected. To make contact with the model, the following plausible assumptions are made: (i) The relaxation process in the electron-glass involves hopping transitions of electrons (or group of electrons) between localized states. Each of these may be modeled as two-state-system, with a "bare" Rabi frequency Δ. (ii) These tunneling events interact strongly with other (localized) band electrons, and thus coupling to the environment is Ohmic [8]. (iii) The two-state systems that are relevant for the sluggish relaxation involve configurations with $\varepsilon \ll k_B T$ where ε is the energy difference between the two states [9,10]. To have a concrete expression for the sake of the discussion, we use a specific form of the Leggett *et al.* model, which gives a renormalized tunneling rate Δ^* as [1]

$$\Delta^* \propto \left(\frac{\Delta^2}{\omega_c}\right)\frac{\Gamma(\alpha)}{\Gamma(\alpha + \frac{1}{2})}\left[\frac{\pi k_B T}{\hbar\omega_c}\right]^{2\alpha - 1}, \qquad (1)$$

where Γ is the gamma function, ω_c is the cutoff frequency of the oscillators bath that represents the environment, and α is the associated coupling constant.

Equation (1) should not be taken too literally in the context used here. The relaxation process of the electron-glass involves a wide spectrum of tunneling rates [11], and these events are presumably interdependent (hierarchical). To characterize this convoluted dynamics by a single "effective" Δ^* (and a single "effective" α) as we proceed to do next, cannot be expected to be more than a crude approximation. Other reservations will be mentioned below. Nevertheless it seems plausible that, if the underlying physics is relevant, the *qualitative* dependence on either T or α predicted by Eq. (1) would be reflected in the experimental results.

Note first and foremost that Eq. (1) allows for the uncommon $\partial\eta/\partial T > 0$ dependence observed in the limit of small Γ^* (i.e., by assigning $\alpha \leq 1/2$), a nontrivial feature that is hard to explain in a classical scenario. Also, the consistency of (ii) with the data can be readily seen by interpreting the temperature dependence of the dynamics (Fig. 4) through Eq. (1). This obviously yields α increasing with Γ^*. As mentioned above, Γ^* increases monotonically with the carrier concentration n of the system [7]. Therefore it is natural to accept that the dissipative environment is the electronic sea. Secondly, over a considerable range of Γ^* (and thus n), $\partial\eta/\partial T \approx 0$. This means that α, while increasing with n, saturates at $\approx 1/2$ for large n. Intriguingly, this behavior is theoretically anticipated [8] for a *metallic* bath where it was shown that $1/2$ is the maximum value of α attainable in the high n limit [12]. That a similar situation may occur in an Anderson insulator, while not impossible (at least in the sense of α saturating for large n), remains an open question.

Another nontrivial aspect of the results is the dependence of dynamics on Γ^* at a given temperature. A pronounced decrease of Δ^* with α is in fact what one gets from Eq. (1) for the range of parameters relevant for these experiments. Associating ω_c with the Fermi energy [13] E_F gives $\omega_c \approx 10^{13}–5 \times 10^{14}$ sec for the In_xO series. Then using Eq. (1) for $T = 4$ K results in a sharp increase of Δ^* (by orders of magnitude) when α changes from 0.1 to $1/2$ (which occurs over the very narrow range of Γ^* cf. Fig. 4). The correlation between the data in Fig. 4 and the data in the inset in terms of the sharp change at the same value of Γ^* is then quite consistent with this picture [14].

The dramatic slowdown of the tunneling rate with n naturally explains why sluggish relaxation is peculiar to Anderson insulators with relatively high carrier concentrations. This feature is implicit in Eq. (1) as both ω_c and α increase with the carrier concentration. The underlying mechanism beyond these quantum "friction" effects is the Anderson orthogonality catastrophe (AOC) [3,8], which may naturally account for the relevance of the memory-dip Γ^*: As conjectured by several authors, the memory-dip is a reflection of an underlying density-of-states modulation brought about by disorder and interactions [15], and the AOC is the generic mechanism responsible for these effects [16].

The AOC is usually discussed only in the metallic regime. However, as pointed out by Ng, the long range Coulomb interaction makes it a viable mechanism for the strongly localized regime as well, although a real divergence may occur only in 3D [17]. At finite temperatures no divergence is expected anyhow as the frequencies that contribute to the AOC are cutoff at $\omega_l \approx k_B T/\hbar$. A state localized over a length ξ introduces $\omega_\xi \approx \hbar/(m\xi^2)$ as a cutoff (m is a the electron mass). Therefore, at $T \approx 4$ K, regions of the system where the localization length ξ is larger than $\approx 10^2$ Å are not more restrictive to the AOC than a metal would be at this temperature. Note that these $\xi's$ contain $\gtrsim 30$ electrons even for samples with the smallest n used in our studies. This presumably is the reason for the similar electron-glass properties shared by granular metals and the In_2O_{3-x} and In_xO samples and the lack of these effects in semiconductors. Semiconductors in the localized regime typically have $n \ll 10^{20}$ cm^{-3} we then expect that they exhibit very fast relaxation rates, and their (correspondingly narrow) memory-dip will be anyhow masked by the huge sensitivity of G to changes in V_g as already remarked elsewhere [18]. Sluggish relaxations observed in systems with small n are probably due to coupling to a slowly varying extrinsic potential, e.g., structural defects.

In summary, we have shown that at liquid helium temperatures, the dynamics of several electron glasses exhibit some surprising similarities with the behavior expected of a single two-state-system coupled to an electronic bath. These include an unusual temperature dependence of dynamics, and a strong suppression of relaxation rates caused by coupling to a dissipative bath. It would be interesting to further test some of the conjectures raised in this paper by carrying out similar studies at lower temperatures (where η should become T independent). On the theoretical side, the intriguing question is how the orthogonality catastrophe and, in particular, the behavior of the coupling constant α as function of n are affected when the metallic system crosses over to the localized regime. These issues clearly deserve a serious theoretical elucidation.

[1] A. J. Leggett *et al.*, Rev. Mod. Phys. **59**, 1 (1987) and references therein.

[2] L. F. Cugliandolo *et al.*, Phys. Rev. B **66**, 014444 (2002).

[3] P. W. Anderson, Phys. Rev. Lett. **18**, 1049 (1967).

[4] Z. Ovadyahu, Phys. Rev. B **73**, 214208 (2006).

[5] A. Vaknin, Z. Ovadyahu, and M. Pollak, Phys. Rev. B **65**, 134208 (2002).

[6] Grenet *et al.*, Eur. Phys. J. B **56**, 183 (2007).

[7] A. Vaknin, Z. Ovadyahu, and M. Pollak, Phys. Rev. Lett. **81**, 669 (1998).

[8] J. Kondo, Physica (Amsterdam) **125B+C**, 279 (1984); **126B+C**, 377 (1984).

[9] The excess conductance relaxation that we focus on here has relative amplitude of typically few percents which suggests that the excess energy in small relative to T. The dc conductivity on the other hand, is composed of hops that involve $\varepsilon \gg k_B T$. In this case the "biased" expression should be used (Eq. 7.18 in [1]), which would merely modify the values of the Miller-Abrahams resistors in the system.

[10] This expression is valid for $T \gg \frac{\hbar\Delta}{k_B\alpha}[\frac{\Delta}{\omega_c}]^{\alpha/(1-\alpha)}$, which is easily satisfied over the entire range of parameters used here for all transitions with $\Delta \lesssim 10^{10}$ sec.

[11] A. Vaknin, Z. Ovadyahu, and M. Pollak, Phys. Rev. B **61**, 6692 (2000).

[12] Y. Tanabe and K. Ohtaka, Phys. Rev. B **34**, 3763 (1986).

[13] In general, the upper cutoff should be of the order of zero-point energy of the tunneling particle rather than E_F, as pointed out by Yu. Kagan and N. V. Prokof'ev, Sov. Phys. JETP **63**, 1276 (1986) [Zh. Eksp. Teor. Fiz. **90**, 2176 (1986)]. For an Anderson insulator, these values are of the same order as long as the tunneling particle has the electronic mass.

[14] Note that τ defined by the two-dip-experiment is exponentially more sensitive than either the DCE or the single relaxation technique [4]; If, (e.g., Fig. 3), the dynamics is assessed by the logarithmic slope a in $\Delta G = 1 - a\log(t)$, then the two-dip-experiment would give $\tau \propto 10^{(-1/2a)}$ (the factor 2 results from the excitation-relaxation symmetry inherent to the two-dip-experiment, [5]).

[15] C. C. Yu, Phys. Rev. Lett. **82**, 4074 (1999); Phys. Status Solidi B **230**, 47 (2002); M. Müller and L. B. Ioffe, Phys. Rev. Lett. **93**, 256403 (2004); A. B. Kolton, D. R. Grempel, and D. Dominguez, Phys. Rev. B **71**, 024206 (2005); Eran Lebanon and Markus Mueller, Phys. Rev. B **72**, 174202 (2005).

[16] Yuval Gefen *et al.*, Phys. Rev. B **65**, 081106 (2002).

[17] Tai Kai Ng, J. Phys. Condens. Matter **2**, 10 205 (1990).

[18] M. Ben Chorin, Z. Ovadyahu, and M. Pollak, Phys. Rev. B **48**, 15 025 (1993).