Gravitational Wave Production at the End of Inflation

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We consider gravitational wave production due to parametric resonance at the end of inflation, or "preheating." This leads to large inhomogeneities that source a stochastic background of gravitational waves at scales inside the comoving Hubble horizon at the end of inflation. We confirm that the present amplitude of these gravitational waves need not depend on the inflationary energy scale. We analyze an explicit model where the inflationary energy scale is $\sim 10^9$ GeV, yielding a signal close to the sensitivity of Advanced Laser Interferometer Gravitational Wave Observatory and Big Bang Observer. This signal highlights the possibility of a new observational "window" into inflationary physics and provides significant motivation for searches for stochastic backgrounds of gravitational waves in the Hz to GHz range, with an amplitude on the order of $\Omega_{\rm gw}(k)h^2 \sim 10^{-11}$.

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A successful model of inflation must have a "graceful exit" that describes the transition from the accelerated phase to a thermalized Universe [1]. A widely studied mechanism for achieving this is preheating (e.g., [2–16]). After inflation, the inflaton (or a related field) oscillates about the bottom of its potential, driving the resonant amplification of specific momentum modes of some coupled field(s). This renders the Universe inhomogeneous, and the resulting spatial gradients source gravitational waves. For grand unified theory (GUT) inflation, the present peak frequency is between 1 MHz and 1 GHz [5,15]. It was conjectured that the characteristic frequency is inversely proportional to the inflationary scale, while the amplitude can be *independent* of this scale [15], leading to a signal potentially detectable by future iterations of Laser Interferometer Gravitational Wave Observatory (LIGO) and Big Bang Observer (BBO). We confirm this conjecture by numerically computing the gravitational wave spectrum in a toy model of preheating following low-scale inflation. While simple inflationary models typically involve GUT scale physics, many stringy models have a much lower inflationary scale, so this signal may eventually lead to new constraints on these models. The tools developed for this analysis will allow us to explore fully realistic preheating models in an expanding background. Furthermore, our computational strategy applies to any inhomogeneous phase in the Universe and may have applications beyond the present problem.

Computational strategy & results.—During parametric resonance, momentum modes of a field χ are pumped by an oscillating field ϕ. In simple models ϕ is the inflaton, but in hybrid models ϕ is the direction orthogonal to the inflationary trajectory, which induces the "waterfall" transition [17,18]. In either case the Lagrangian can be expressed as

$$\mathcal{L} = -\tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\chi)^2 - V(\phi,\chi). \tag{1}$$

We numerically simulate the nonlinear field evolution in a conformally rigid spacetime background. We can then compute the spatial parts of $T_{\mu\nu}$ at any given time. The tensor contribution to the metric perturbation and Einstein equations read

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j, \tag{2}$$

$$\bar{G}_{\mu\nu}(t) + \delta G_{\mu\nu}(x,t) = 8\pi G[\bar{T}_{\mu\nu}(t) + \delta T_{\mu\nu}(x,t)], \tag{3}$$

where the overbar denotes the homogeneous background values. The perturbation is *transverse-traceless*, so

$$h^i_i = 0, \qquad h^i_{j,i} = 0. \tag{4}$$

The Fourier components of the h_{ij} obey

$$\ddot{\tilde{h}}_{ij} + 3\frac{\dot{a}}{a}\dot{\tilde{h}}_{ij} - 2\left(\frac{\dot{a}^2}{a^2} - 2\frac{\ddot{a}}{a} - \frac{k^2}{a^2}\right)\tilde{h}_{ij} = \frac{16\pi G}{a^2}\tilde{S}^{\rm TT}_{ij}, \tag{5}$$

using the convention

$$\tilde{f}(k,t) = \int d^3x f(x,t)e^{2\pi i k\cdot x} \tag{6}$$

and keeping only the transverse-traceless source.

We evolve the fields with LATTICEEASY [19], treating the spatial background as a rigid, expanding box. We assume the fields' initial inhomogeneity is derived from their quantum mechanical vacuum fluctuations. The $\tilde{h}_{ij}$ obey *ordinary* differential equations, which we solve with a fourth order Runge-Kutta integrator. Unlike [5,15], we use no flat-space results, and the gravitational wave amplitude can be followed throughout the simulations. Numerical noise is a significant problem, since the transverse-traceless source involves differences of terms involving derivatives, which are themselves computed via numerical differencing on the lattice—which can easily lead to a loss of numerical significance. In future work we plan to perform simulations at higher spatial resolution, with different algorithms for the field evolution, in order to produce highly accurate spectra for a variety of models over a large frequency range. Moreover, we have made simplifying assumptions about the subsequent evolution of

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the Universe, which can rescale the overall tensor spectrum. In models without resonance, the computed tensor spectrum is many orders of magnitude smaller than seen in resonant models, providing a "null check" of our algorithm, and we have recovered the key results of [5,15]. Compared to these results, the spectra in [5,15] are noticeably noisy, which is an artifact of the older algorithm. The amplitudes obtained with our new code are somewhat lower than the previous results, possibly due to better noise suppression.

We calculate the power spectrum using [20]

$$\rho_{\rm gw} = \frac{1}{32\pi G}\langle h_{ij,\mu}h^{ij,\nu}\rangle\bigg|_{\mu=\nu=0}. \qquad (7)$$

Evolving the h_{ij} and inserting (7) into Eq. (20) of [15] (and correcting a typographical error) we find:

$$\Omega_{\rm gw}(k)h^2 = \Omega_r h^2\left(\frac{g_0}{g_\star}\right)^{1/3}\frac{d}{d\ln k}\left[\frac{\rho_{\rm gw}(a_e)}{\rho_{\rm tot}(a_e)}\right]. \qquad (8)$$

Here a_e is the scale factor at the end of the simulation, Ω_r is the present-day density of radiation, g_0 and $g_\star$ denote the number of thermal degrees of freedom at present and at matter-radiation equality, and h is the dimensionless Hubble constant at the present epoch. We thus assume that the Universe is radiation dominated between the end of our simulation and matter-radiation equality, during which time $\Omega_{\rm gw}(k)$ is constant, and matter dominated thereafter. In the future we plan to consider more realistic transfer functions [21,22]. Moreover, we implicitly assume that the Universe promptly thermalizes after preheating, which requires a trilinear coupling we have not included here [14].

We work with

$$V(\phi,\chi) = \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{2}g^2\phi^2\chi^2. \qquad (9)$$

In standard quadratic chaotic inflation, μ is fixed by the power spectrum and is not a free parameter. Since we are interested in the end of inflation, Eq. (9) need not describe the potential in the epoch when cosmological perturbations were generated, and μ is a free parameter. To see this, consider

$$V = \frac{(M^2-\lambda\sigma^2)^2}{4\lambda} + \frac{m^2}{2}\phi^2 + \frac{h^2}{2}\phi^2\sigma^2 + \frac{g^2}{2}\phi^2\chi^2. \qquad (10)$$

During inflation, ϕ is large and $\sigma = 0$. When $\phi = M/h$, the field rapidly evolves towards the minimum at $\sigma = M/\sqrt{\lambda}$. This is the waterfall phase transition. Further, we assume that $\sigma = \langle\sigma\rangle$ and that there is no σ production via a $\sigma^2\chi^2$ term, giving

$$V(\phi) = \frac{1}{2}\left(m^2 + \frac{h^2M^2}{\lambda}\right)\phi^2. \qquad (11)$$

This is a mass term for ϕ. In practice, we need $m \ll M$ in order to get the correct perturbation spectrum, so the effective mass is simply $\mu^2 = h^2M^2/\lambda$. Here, we simply take μ to be a free parameter. The inflationary energy density is then approximately $\mu^2\phi_0^2$, where ϕ_0 is the field value at the beginning of our simulation, which we assume to be roughly coincident with the end of inflation. We take $\phi_0 \approx 0.2m_{\rm pl}$, so the inflationary energy scale is set by $\mu^{1/2}$. For hybrid inflation, $M/h \approx 0.2m_{\rm pl}$, and for $\mu = hM/\sqrt{\lambda} = 10^{-18}m_{\rm pl} \sim 10$ GeV, we need $h^4/\lambda \approx 3\times 10^{-16}$, which demands a small but not miniscule value of h, if we assume $\lambda \sim 0.1$.

We hold the resonance parameter $q = g^2m_{\rm pl}^2/\mu^2$ fixed [15]. This ensures that the same modes (in units of post-inflationary energy scale) are in resonance as μ is varied. We choose $q = 2\times 10^6$, which makes g unrealistically small as μ is decreased. However, our purpose is to give an "existence proof" that (p)reheating can generate a substantial gravitational wave background at low inflationary scales. This assumption ensures that the structure of resonance does not change with the inflationary energy scale, allowing us to isolate the aspects of gravitational wave production that depend only on the overall inflationary scale. Figure 1 shows the gravitational wave spectra for different inflationary energy scales. As predicted [15], the peak frequency scales inversely with energy, whereas the amplitude is scale independent.

The spectrum declines very rapidly at large k. This reflects the structure of the resonance bands, as modes with large k are never in resonance. Conversely, we see a relatively broad plateau, possibly attributable to modes moving through the resonance band as inflation continues. The amplitude of modes that are superhorizon throughout resonance is expected to rise as k^3 or faster [11]. The Universe grows significantly during resonance, so we cannot simultaneously resolve both resonant and superhorizon modes. In order to prevent contamination by numerical noise, we must choose our box to ensure that we resolve a significant number of high-frequency modes that are never resonantly amplified. A very large-scale simulation is thus needed to see the full shape of the peak.

The source term is potentially nonzero whenever the fields are inhomogeneous, but its amplitude rises rapidly during resonance, and then decreases as the Universe begins to thermalize. Most of the gravitational waves are thus radiated during a comparatively narrow time interval. In the models treated here, the Universe grows ~50% larger as 90% of the power is generated.

Discussion.—Bubble-wall collisions after a first order phase transition at the end of inflation [23–26] can also generate a present-day $\Omega_{\rm gw}(k)h^2 \sim 10^{-11}$. This spectrum has similar properties to the preheating background: the frequency scales inversely with the inflationary scale, while the amplitude need not depend on the inflationary scale. This process yields a maximum $\Omega_{\rm gw}(k)$ similar to that found here for parametric resonance, although the detailed spectra may differ considerably. This is to be

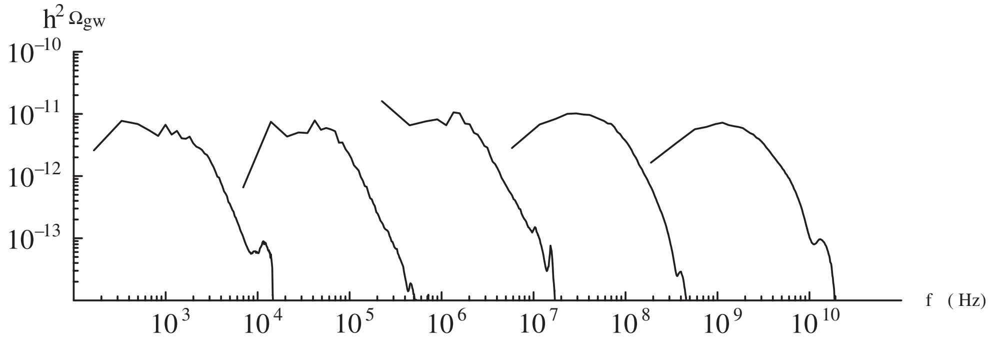

FIG. 1. We plot the spectrum of gravitational radiation produced during resonance with $\mu = 10^{-18}$ (left) through to 10^{-6} (right) in units where $m_{\rm Pl} \approx 10^{19}$ GeV $= 1$, where each spectrum has a value of $\mu 10^3$ times larger than the one immediately to the left. The corresponding initial energy densities run from $(4.5 \times 10^9\ \mathrm{GeV})^4$ to $(4.5 \times 10^{15}\ \mathrm{GeV})^4$ for our choice of ϕ_0. The plots are made on 128^3 grids, and the ''feature'' at high frequency is a numerical artifact.

expected since gradient terms source $\tilde{h}_{ij}$ and the amplitude of these is bounded by their contribution to the total energy density. If the gradient energy saturates this bound, $\Omega_{\rm gw}(k)h^2 \sim 10^{-11}$ or 10^{-10} is a generic result, in the absence of nonperturbative effects [27,28]. In our simulations the gradient energy peaks at around 25% of the total energy density. Since there are necessarily kinetic and potential contributions to the energy, this fraction cannot be significantly increased.

Since our algorithm works for an arbitrary extended source of gravitational radiation, we could apply it to other mechanisms that generate stochastic backgrounds. These include networks of cosmic superstrings [29], the TeV scale phase transition present in Randall-Sundrum models [30], a first order electroweak phase transition [31], cosmological bubble collisions [25], or large-scale turbulence in the presence of magnetic fields [27,28,32]. Moreover, [33] presents an alternative (and physically distinct) mechanism for generating gravitational waves at the end of inflation, and we expect that it will be possible to analyze this mechanism with our algorithm. Finally, [34] discusses the generation of gravitation waves by density perturbations during the radiation dominated era, and it would be interesting to explore the overlap between this second order calculation and our techniques.

The detectability of high-frequency background of stochastic gravitational radiation is an open question. We plot a schematic version of our results in Fig. 2, and we see that the proposed space-based detectors BBO and LISA are sensitive to much longer wavelengths than any signal likely to be generated during preheating from GUT scale inflation. It will be a stretch for Advanced LIGO to see the signals computed here. However, a further iteration of LIGO is likely to put significant constraints on any signal that would be generated during resonance after inflation occurring at scales around 10^9 GeV.

Detecting high-frequency gravitational waves is particularly challenging, since the required strain sensitivity at fixed $\Omega_{\rm gw}(k)$ scales as the cube of the frequency.

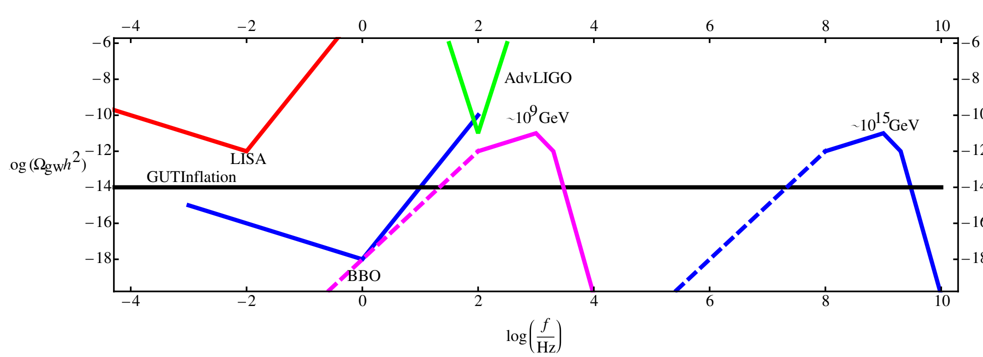

FIG. 2 (color online). We sketch the gravitational wave spectra obtained for the lowest and highest energy models computed here, relative to that of the Advanced LIGO goal, and the proposed LISA and BBO experiments. We see that inflationary models with lower energy scale may lead to a signal that is visible at LIGO scales if the sensitivity of LIGO is further improved, and with BBO. The tensor background generated by quantum fluctuations during GUT scale inflation is shown by the solid horizontal line. The dashed lines denote the inferred k^3 tails. The spectra generated by the inflationary scenarios considered in [5,15] roughly overlap with the 10^{15} GeV spectrum depicted above.

Consequently, it is easier to detect a background arising from preheating after low-scale inflation. With few exceptions, stringy models of inflation have $V^{1/4} \lesssim 10^{13}$ GeV, 2 orders of magnitude below GUT scales, and can be much lower [35]. Moreover, reheating after thermal inflation could lead to a signal in the LIGO or BBO bands [36]. In these models the primordial tensor spectrum is unobservable, but any preheating signal is easier to detect. Very low-scale inflation (at or near the TeV scale, for example [37,38]) could produce a signal visible to a BBO-style mission. Finally, future detector technologies might be sensitive to high-frequency gravitational radiation.

Our key result is an existence proof that parametric resonance following inflation can lead to a significant gravitational wave background, independently of the inflationary energy scale. Many questions remain. First, we have considered only models that are well described by (9), and our setup is a toy model, requiring a very small coupling parameter at low inflationary scales. This ensures the *structure* of resonance is independent of the energy scale, which allows us to isolate the impact on the inflationary energy scale on the height and location of the peak. However, this small coupling is not a prerequisite for preheating or resonance. Preheating is generically associated with enhanced inhomogeneities that will source gravitational radiation, so we can hope that similar results also apply to more realistic resonance scenarios, but these need to be explored in detail. Second, we have not yet considered overlapping backgrounds from other astrophysical processes. Third, we need to incorporate the finer details of the tensor mode transfer function and the postresonance thermalization of the Universe.

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