Bell's Inequality and Universal Quantum Gates in a Cold-Atom Chiral Fermionic p-Wave Superfluid

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We propose and analyze a probabilistic scheme to entangle two spatially separated topological qubits in a $p_x + ip_y$ superfluid using controlled collisions between atoms in movable dipole traps and unpaired atoms inside vortex cores in the superfluid. We discuss how to test the violation of Bell's inequality with the generated entanglement. A set of universal quantum gates is shown to be implementable *deterministically* using the entanglement despite the fact that the entangled states can be created only probabilistically.

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Introduction.—Topological quantum computation affords the amazing possibility that qubits and quantum gates may be realized using only the topological degrees of freedom of a system [1]. Since these degrees of freedom, by definition, are insensitive to weak local perturbations, the resulting computational architecture should be free of environmental decoherence. In a class of topological systems, the requisite (non-Abelian) statistical properties [2,3] are provided by the presence of Majorana fermion excitations described by the self-Hermitian operators $\gamma^\dagger = \gamma$. These excitations have been shown to occur naturally at the cores of vortices in a 2D spinless $p_x + ip_y$ superfluid or superconductor [4–6], where the interacting fermions are described by the many body Pfaffian wave function [2]. (It seems likely, but remains to be verified, that this wave function also describes the essential physics of the filling fraction $\nu = 5/2$ fractional quantum Hall (FQH) system [2,4]). It is encouraging that the spinless $p_x + ip_y$ superfluid of fermionic atoms is potentially realizable in an optical trap tuned close to a p-wave Feshbach resonance [7–9]. Our current work establishes the possibility of testing Bell's inequality in a p-wave fermionic superfluid on the way to eventual universal topological quantum computation using vortices in such a system.

In a $p_x + ip_y$ superfluid, one can define a topological qubit using a group of four vortices. Since the states of the qubit are associated with the composite states of the four *spatially separated* Majorana fermion excitations, they are immune to local environmental errors. One can implement some single-qubit gates by adiabatically moving (braiding) one vortex around another within the same vortex complex defining the qubit. Since the associated unitary transformations are purely statistical, there is, in principle, no error incurred in these gating operations. However, such a braid operation of one vortex from one qubit around another from a different qubit fails to provide a two-qubit gate [10]: the topological braiding operations allowed in a $p_x + ip_y$ superfluid, as in its FQH Pfaffian counterpart, are not computationally sufficient. Any composite state of the two qubits, accessible by braiding one excitation around another, can always be written as a product of the states of the individual qubits [10]. Therefore, in light of its experimental relevance, it is important to examine the problem of creating quantum entanglement in a $p_x + ip_y$ superfluid via some other, possibly nontopological, means (without incurring too much error) which, coupled with the available braiding transformations, may lead to universal quantum computation. This is all the more important because the other, more exotic, non-Abelian topological states, e.g., the SU(2) Read-Rezayi state [11], which can support universal computation via only the topologically protected operations [12,13], are presently much beyond experimental reach. In the 5/2 FQH state, nontopological interference of charge-carrying quasiparticle currents along different trajectories [10,14,15] was proposed to entangle qubits. Such an approach is not suitable for the superfluid, because the non-Abelian excitations here are vortices, which do not carry electric charge.

In this Letter, we show how to entangle two spatially separated topological vortex qubits in a $p_x + ip_y$ superfluid by using "flying qubits" formed by atoms in movable, external dipole traps. Controlled cold collisions between an atom in the dipole trap and an atom at the vortex qubit yield entanglement between the flying qubit and the vortex qubit. Subsequently, a measurement on a system comprising two flying qubits, entangled with two different vortex qubits, collapses the two vortex qubits on an entangled state. We show how to test the violation of Bell's inequality with the obtained entanglement. Finally, we show how to *deterministically* implement a set of universal quantum gates using the entangled state, although the entanglement among the vortex qubits itself can be generated only with a 50% success probability.

Topological qubit and flying qubit.—Consider a quasi-two-dimensional (xy plane) $p_x + ip_y$ superfluid of spin-polarized atoms [7–9], where vortices can be generated through rotation or external laser fields. For each vortex, there exists a zero-energy state that supports a Majorana fermion mode γ [2,5,6]. Two Majorana fermion states in two vortices can be combined to create an ordinary fermionic state $c = (\gamma_1 + i\gamma_2)/2$. Therefore, a natural definition of a vortex qubit may be given in terms of the unoccupied, $|0\rangle$, or occupied, $|1\rangle = c^\dagger|0\rangle$, states of two Majorana vortices. Here the state $|1\rangle$ contains an unpaired Fermi atom inside the cores of the vortex pair ("unpaired" as opposed to "paired" in a Cooper pair in the superfluid). However, such a definition does not allow the superposition states such as $(|0\rangle \pm |1\rangle)/\sqrt{2}$ because they violate the conservation of the total topological charge (the superfluid condensate conserves the fermion number modulo 2). To overcome this difficulty, a topological vortex qubit is defined through two pairs of vortices, i.e., with the states $|0\rangle_V \equiv |00\rangle$ and $|1\rangle_V \equiv |11\rangle$. The superposition states, $(|0\rangle_V \pm |1\rangle_V)/\sqrt{2}$, are now allowed. Various intrapair and interpair vortex braiding operations within a single qubit give rise to various single-qubit gates [14,16] (e.g., qubit-flip gate R, phase gate $\Lambda(\pi/2)$, and the Hadamard gate H) as depicted schematically in Fig. 1. Finally, the state of the vortex qubit can be read out in the $\{|0\rangle_V, |1\rangle_V\}$ basis by fusing the vortices pairwise and detecting the number of unpaired atoms in the core [6].

The flying qubit is constructed using an atom trapped in the ground state of a movable optical dipole trap which is itself formed by overlapping two identical laser beam traps. One laser beam trap can then be adiabatically moved out to split the composite trap into two traps, L, R; see Fig. 2(a). This yields a superposition state for the atom, $(|01\rangle_{LR} + |10\rangle_{LR})/\sqrt{2}$. Here L and R denote the left and the right traps, respectively. Now, concentrating on the left trap only, one can define the two states of the flying qubit, $|0\rangle_F = |01\rangle_{LR}$, $|1\rangle_F = |10\rangle_{LR}$.

Entanglement between two topological qubits.—Using flying qubits as auxiliary degrees of freedom, one can generate entangled states between two vortex qubits. The basic idea of the entanglement generation (EG) is illustrated in Figs. 2(b) and 2(c). Initially, a vortex qubit, V, is prepared in the state $|0\rangle_V$. A Hadamard gate H is applied to the qubit that transfers the state to $|\phi\rangle_V = (|0\rangle_V + |1\rangle_V)/\sqrt{2}$. By splitting a composite dipole trap in two parts [Fig. 2(a)], the flying qubit F is prepared in the state $|\psi\rangle_F = (|0\rangle_F + |1\rangle_F)/\sqrt{2}$. The flying qubit is then moved near to the vortex qubit [Fig. 2(b)] so that the trapped atom can collide with the unpaired Fermi atom, if any, in the vortex pair. As shown below, such a collision process yields a controlled phase gate, $\mathrm{CP}(\pi) \equiv \exp(i\pi n_V n_F)$, between the flying qubit and the vortex qubit, where $n_V = 0, 1$ is the number of unpaired Fermi atoms in the vortex pair and $n_F = 0, 1$ is the number of atoms in the flying qubit. It is easy to see that the gate $\mathrm{CP}(\pi)$ gives rise to the transformation, $|\psi\rangle_F|\phi\rangle_V \to [|0\rangle_F(|0\rangle_V + |1\rangle_V) + |1\rangle_F \times (|0\rangle_V - |1\rangle_V)]/2$, which can be transferred to an entangled state

$$|\Phi\rangle_{FV} = (|0\rangle_F|0\rangle_V + |1\rangle_F|1\rangle_V)/\sqrt{2}. \tag{1}$$

Two vortex qubits can be entangled by a projection measurement on the flying qubits of two entangled states $|\Phi\rangle_{F_1V_1}$ and $|\Phi\rangle_{F_2V_2}$. The dipole traps of the two flying qubits are spatially merged and the atom number is measured through fluorescence signals [Fig. 2(c)]. From the combined state $|\Phi\rangle_{F_1V_1}|\Phi\rangle_{F_2V_2}$, it is easy to deduce the probabilities for the three possible outcomes: one atom (50%), zero atoms (25%), and two atoms (25%). In the last two cases, the states of the vortex qubits are projected to $|0\rangle_{V1}|0\rangle_{V_2}$ and $|1\rangle_{V_1}|1\rangle_{V_2}$, respectively, and are not entangled. Therefore, in these cases the above procedure for creating the entangled states, $|\Phi\rangle_{F_1V_1}$ and $|\Phi\rangle_{F_2V_2}$, needs to be repeated. However, in the case where the measurement produces one atom, the quantum state of the two vortex qubits is projected to the expected entangled state

$$|\Psi\rangle_{V_1V_2} = (|0\rangle_{V1}|0\rangle_{V_2} + |1\rangle_{V_1}|1\rangle_{V_2})/\sqrt{2} \tag{2}$$

with additional simple qubit-flip gates. Note that the above entangled state can be created only with a 50% success probability.

The remaining problem for the entanglement generation is how to realize the controlled phase gate, $\mathrm{CP}(\pi)$, between the flying qubit and the vortex qubit. In Fig. 2(b), the center of the dipole trap, $\mathbf{r}_0(t) = z_0(t)\mathbf{e}_z$ (with the core of vortex 1 as origin) is adiabatically brought from a distance $d_0\mathbf{e}_z$ above the $z = 0$ plane to a distance zero, where the wave packets of the dipole trapped atom and the unpaired atom in the vortex pair (1, 2) overlap. The collision phases between the atoms are different for different quantum states of flying and vortex qubits with different energy,

$$E(i, j) = E_F(i) + E_V(j) + \Delta E_c(i, j), \tag{3}$$

where $i, j = 0, 1$ correspond to the quantum states $|0\rangle$ and

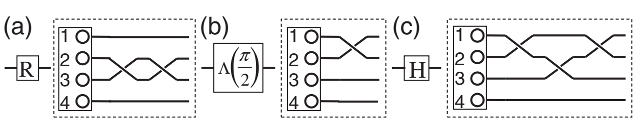

FIG. 1. (a) Single-qubit flip gate $R = -i\sigma_x$. (b) Single-qubit phase gate $\Lambda(\pi/2) = \mathrm{diag}(1, i)$. (c) Hadamard gate $H = \frac{1}{\sqrt{2}} \times \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

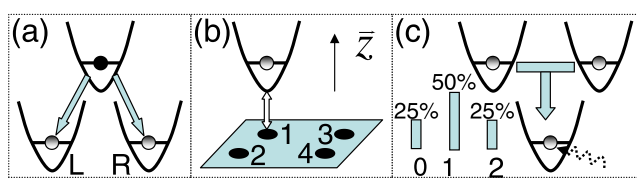

FIG. 2 (color online). (a) Construction of the flying qubit by splitting a composite dipole trap into two traps. (b) Realization of the gate $\mathrm{CP}(\pi)$ by controlled collisions of atoms. (c) Two flying qubits are merged into one and the number of atoms is measured through fluorescence signals to create entanglement between two topological qubits (see the text).

$|1\rangle$, respectively. $E_F(0)=0$ and $E_F(1)=\int d^3\mathbf{r}\alpha^*(\mathbf{r}-\mathbf{r}_0(t))[-\hbar^2\nabla^2/2m_F+V_F(\mathbf{r}-\mathbf{r}_0(t))]\alpha(\mathbf{r}-\mathbf{r}_0(t))+E_g$ are the energies of the flying qubit in the states $|0\rangle$ and $|1\rangle$, respectively. $V_F(\mathbf{r}-\mathbf{r}_0(t))$ is the harmonic potential of the dipole trap, and $\alpha(\mathbf{r}-\mathbf{r}_0(t))$ is the ground state wave function of the dipole trapped atom with mass m_F. E_g is the interaction energy between the dipole trapped atom and the paired BCS condensate. The second term $E_V(j)$ corresponds to the energy of the fermionic state in the vortex cores near the dipole trap. Because these states are the solutions of the Bogoliubov–de Gennes (BdG) equations with eigenvalue zero, $E_V(j)=0$ for $j=0,1$ [6].

The last term describes the collision energy [17] between dipole trapped atoms and unpaired Fermi atoms and is nonzero only if both the flying qubit and the vortex qubit are in the occupied state,

$$\Delta E_c(1,1)=\frac{g}{2}\int d^3\mathbf{r}|\alpha(\mathbf{r}-\mathbf{r}_0(t))|^2\delta n_V(\mathbf{r}). \tag{4}$$

Here g is the collision interaction strength and $\delta n_V(\mathbf{r})$ denotes the changes of the atom density from the BCS condensate density. Using the standard harmonic trap wave function for $\alpha(\mathbf{r}-\mathbf{r}_0(t))$ and the wave function for the zero-energy mode obtained from the solution of the BdG equations, we find

$$\Delta E_c(1,1)=\hbar\Omega_\pm\exp[-z_0^2(t)/\bar{a}^2], \tag{5}$$

where $\bar{a}^2=a_D^2+a_V^2$, with a_D and a_V the oscillation lengths for harmonic trapping potentials along the z direction of the dipole trap and the superfluid, respectively. $\hbar\Omega_\pm$ is the characteristic energy scale for the collision interaction, which is determined by the collision interaction strength g as well as the overlap between the wave functions of the dipole trapped atoms and unpaired fermions in the vortex pair $(1,2)$. Note that the occupied zero-energy fermionic state of the vortex pair $(1,2)$ is as likely to contain extra atoms (corresponds to Ω_+) as to miss atoms (corresponds to Ω_-) compared to the BCS condensate, which yields $\Omega_+=-\Omega_-$.

The state-dependent energy (3) yields a dynamic phase

$$\varphi(i,j)=\varphi_F(i)+\phi_c(i,j), \tag{6}$$

where $\varphi_F(i)=\frac{1}{\hbar}\int_{-\tau}^{\tau}E_F(i)dt$ is the single-qubit phase that can be incorporated in the definition of the flying qubit, the collision phase $\phi_c(i,j)=\frac{1}{\hbar}\int_{-\tau}^{\tau}\Delta E_c(i,j)dt$, and $\mp\tau$ denote the time when the dipole trap center $\mathbf{r}_0(t)$ moves from and back to the initial place $d_0\mathbf{e}_z$. Assuming that $\mathbf{r}_0(t)$ varies adiabatically as $z_0(t)/d_0=\eta[\exp(t^2/\tau_r^2)-1]/[1+\eta\exp(t^2/\tau_r^2)]$ with the parameter $\eta=\exp(-\tau_i^2/\tau_r^2)$, the controlled collision phase is

$$\theta\equiv\phi_c(1,1)=\Omega_\pm\tau_r\int_{-\bar{\tau}}^{\bar{\tau}}\exp\left[-\Upsilon\eta\frac{e^{\bar{t}^2}-1}{1+\eta e^{\bar{t}^2}}\right]d\bar{t}, \tag{7}$$

where $\Upsilon=d_0^2/\bar{a}^2$ and time in the above integration has been scaled by τ_r. With a set of parameters for ^{6}Li, $a_D=a_V=0.4$ μm, $d_0=10a_D=4$ μm, $\tau_r=\tau_i=3.57/\Omega$, $\tau=10\tau_r$, s-wave scattering length $a_s\sim 53$ nm, and the vortex core size $\xi\sim 1$ μm, we estimate $\Omega_\pm\sim\pm 2\pi\times 6.6$ kHz, $\tau\sim 0.86$ ms and the phase $\theta=\pm\pi$ [i.e., $\exp(i\theta)=-1$]. Therefore, the controlled phase gate CP(π) can be realized.

Violation of Bell's inequality.—The entangled state $|\Psi\rangle_{V_1V_2}$ between two remote vortex qubits can be used to test the violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality, a variant of the Bell inequality [18]. Violation of the CHSH inequality would establish the quantum nonlocality between the two vortex qubits. A schematic diagram of this test is given in Fig. 3. The test requires one to measure the vortex qubits along four different directions: $A_1=\sigma_z^{V_1}\otimes I^{V_2}$, $A_2=\sigma_x^{V_1}\otimes I^{V_2}$, $B_1=-I^{V_1}\otimes(\sigma_z^{V_2}+\sigma_x^{V_2})/\sqrt{2}$, and $B_2=I^{V_1}\otimes(\sigma_z^{V_2}-\sigma_x^{V_2})/\sqrt{2}$. After the measurements, two parties at V_1 and V_2 need to communicate their results through the classical channel. After repeated measurements, the statistical average $L=\langle A_1B_1\rangle+\langle A_2B_2\rangle+\langle A_2B_1\rangle-\langle A_1B_2\rangle$ is evaluated. The quantum nonlocality of the entangled state yields $L=2\sqrt{2}$, which violates the CHSH inequality for local realism, $L\leq 2$ [18].

The above four measurements correspond to measuring the two vortex qubits in four different bases which are eigenstates of the respective operators, A_1: V_1 on $\{|0\rangle_{V_1},|1\rangle_{V_1}\}$; A_2: V_1 on $\{(|0\rangle_{V_1}+|1\rangle_{V_1})/\sqrt{2},(|0\rangle_{V_1}-|1\rangle_{V_1})/\sqrt{2}\}$; B_1: V_2 on $\{a|0\rangle_{V_2}+b|1\rangle_{V_2},b|0\rangle_{V_2}-a|1\rangle_{V_2}\}$; B_2: V_2 on $\{a|0\rangle_{V_2}-b|1\rangle_{V_2},b|0\rangle_{V_2}+a|1\rangle_{V_2}\}$, where $a=\cos(\pi/8)$, $b=\sin(\pi/8)$. In the experiment, A_1 is a fusion measurement of the number of unpaired atoms in the vortices [6]. Measurements A_2, B_1, and B_2 can be implemented by first applying suitable single-qubit operations to the qubits to transfer their measurement bases to $\{|0\rangle_V,|1\rangle_V\}$, following by fusion measurement A_1. The corresponding single-qubit operations are A_2: H; B_1: $H\Lambda(e^{i\pi/4})H\Lambda^2(\pi/2)$; B_2: $H\Lambda(e^{i\pi/4})H\Lambda^2(-\pi/2)$, where $\Lambda(e^{i\pi/4})=\mathrm{diag}(1,e^{i\pi/4})$ is a single-qubit phase gate.

Universal quantum gates.—It is well known that a set of quantum gates [10,14]

$$H,\qquad \Lambda[\exp(i\pi/4)],\qquad \Lambda(\sigma_z) \tag{8}$$

are sufficient to simulate any quantum circuit, where $\Lambda(\sigma_z)=\mathrm{diag}(1,1,1,-1)$ is the two-qubit controlled

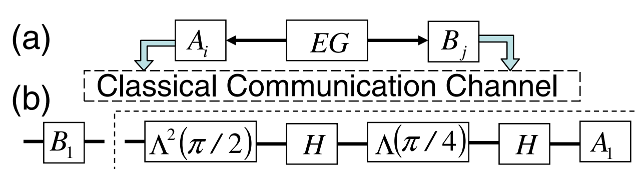

FIG. 3 (color online). (a) Testing the violation of the CHSH inequality. (b) The realization of the B_1 measurement in (a).

phase gate between two vortex qubits. Among these three gates, the Hadamard gate H can be implemented using the topological braiding operations. The single-qubit phase gate $\Lambda(e^{i\pi/4})$ can be realized by bringing two vortices 1 and 2 of a qubit close together [14], which yields a relative dynamic phase $\varkappa = \int_{-T_p}^{T_p} \Delta E_z(t) dt$ between two states $|0\rangle_V$ and $|1\rangle_V$, with $\Delta E_z(t)$ as the energy splitting induced by the tunneling between two vortices and $2T_p$ as the total tunneling period. A properly chosen tunneling process with $\varkappa = \pi/4$ yields the gate $\Lambda(e^{i\pi/4})$.

A controlled phase gate $\Lambda(\sigma_z)$ between two arbitrary vortex qubits can be realized *deterministically* provided one has been able to create the entangled state $|\Psi\rangle$ between two vortex qubits. Considering two vortex qubits G and Q (with the constituent vortices G_1, G_2, Q_1, Q_2, etc.), we note that $\Lambda_{GQ}(\sigma_z) = \Lambda_G(\pi/2)\Lambda_Q(\pi/2) \times \exp(i\pi\gamma_{G_1}\gamma_{G_2}\gamma_{Q_1}\gamma_{Q_2}/4)$, where the last term involves interaction among four vortices. The four-vortex operator can be implemented using one additional vortex pair (γ_{W_1}, γ_{W_2}) (initially prepared in state $|0\rangle$) by noting that [19]

$$\exp(i\pi\gamma_{G_1}\gamma_{G_2}\gamma_{Q_1}\gamma_{Q_2}/4) = 2U_{\mu\nu}P_\mu^{(2)}P_\nu^{(4)}, \qquad (9)$$

where $P_\pm^{(2)} = (1 \mp i\gamma_{Q_1}\gamma_{W_1})/2$ and $P_\pm^{(4)} = (I \pm \gamma_{G_1}\gamma_{G_2}\gamma_{Q_2}\gamma_{W_1})/2$ are nondestructive measurements which project the state of the vortices to the eigenstates of the operators $-i\gamma_{Q_1}\gamma_{W_1}$ and $\gamma_{G_1}\gamma_{G_2}\gamma_{Q_2}\gamma_{W_1}$. $U_{\mu\nu}$ are corresponding braiding operations for different measurement results $\{\mu\nu\}$.

$P_\pm^{(2)}$ can be realized via a basis transformation method. We exchange the vortices γ_{Q_1} and γ_{W_1} to transfer two eigenstates of $-i\gamma_{Q_1}\gamma_{W_1}$ to $\{|00\rangle_{QW}, |11\rangle_{QW}\}$ or $\{|10\rangle_{QW}, |01\rangle_{QW}\}$, depending on the total topological charge of the four vortices γ_{Q_1}, γ_{Q_2}, γ_{W_1}, and γ_{W_2}. We then apply a fusion measurement on the vortex pair (γ_{W_1}, γ_{W_2}) to determine whether the state is $|0\rangle_W$ or $|1\rangle_W$, which correspond to the eigenvalues $+1$ or -1 of the projection measurements $P_\pm^{(2)}$. After the fusion measurement, the vortex pair (γ_{W_1}, γ_{W_2}) is recreated in the state $|0\rangle_W$. If the result of the fusion measurement is the state $|1\rangle_W$, this state is recovered by applying a single-qubit-flip operator R. Vortices γ_{Q_1} and γ_{W_1} are exchanged again to transfer the states back to the eigenstates of $-i\gamma_{Q_1}\gamma_{W_1}$. With this basis transformation method, $P_\pm^{(2)}$ can be performed nondestructively.

However, such a basis transformation method does not work for the measurements $P_\pm^{(4)}$ because they involve eigenstate measurements of four vortices. Recent work [10] showed mathematically that $P_\pm^{(4)}$ can be realized deterministically using the auxiliary entangled state $|\Psi\rangle$, for which we provide a prescription in this Letter, coupled with the braiding operations and the fusion measurements. Here we refer to Ref. [10] for the mathematical details of this measurement. Note that the measurement $P_\pm^{(4)}$ can be *deterministically* implemented, although $|\Psi\rangle$ in our scheme can be generated only with a 50% success probability. This is because $|\Psi\rangle$ can be prepared using off-line procedures that are not involved in the measurement process. In addition, pairs with nonperfect entanglement can be purified to pairs of nearly perfect entanglement through off-line purification processes. Therefore, the controlled phase gate $\Lambda(\sigma_z)$ can be implemented with a high accuracy because the remaining processes involve only the braiding operations and the fusion measurements.

In summary, we proposed and analyzed a scheme to generate entanglement between two topological vortex qubits in a $p_x + ip_y$ atomic superfluid with the assistance of external flying qubits. We showed how to test the violation of Bell's inequality using the obtained entanglement. Finally, we showed how to deterministically implement a set of universal quantum gates in the chiral p-wave superfluid, which had hitherto remained a major conceptual problem, using the entanglement created between two topological qubits.

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