Dynamical Control of Matter-Wave Tunneling in Periodic Potentials

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We report on measurements of dynamical suppression of interwell tunneling of a Bose-Einstein condensate (BEC) in a strongly driven optical lattice. The strong driving is a sinusoidal shaking of the lattice corresponding to a time-varying linear potential, and the tunneling is measured by letting the BEC freely expand in the lattice. The measured tunneling rate is reduced and, for certain values of the shaking parameter, completely suppressed. Our results are in excellent agreement with theoretical predictions. Furthermore, we have verified that, in general, the strong shaking does not destroy the phase coherence of the BEC, opening up the possibility of realizing quantum phase transitions by using the shaking strength as the control parameter.

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Quantum tunneling of particles between potential wells connected by a barrier is a fundamental physical effect. While typically quantum systems decay faster when they are perturbed, if the wells are shifted with respect to each other by a time-varying linear potential (e.g., by periodically shaking them back and forth), the tunneling rate can actually be reduced and, for certain strengths of the time-varying potential, even completely suppressed [1,2].

Modifications of the dynamics of quantum systems by applying periodic potentials have been investigated in a number of contexts including the renormalization of Landé g factors in atoms [3], the micromotion of a single trapped ion [4], and the motion of electrons in semiconductor superlattices [5]. In particular, theoretical studies of double-well systems and of periodic potentials have led to the closely related concepts of coherent destruction of tunneling and dynamical localization [1,6]. In the latter, tunneling between the sites of a periodic array is inhibited by applying an oscillating potential, e.g., by shaking the array back and forth (see Fig. 1), and, as a consequence, the tunneling parameter J representing the gain in kinetic energy in a tunneling event is replaced by $|J_{\mathrm{eff}}| < |J|$. In a number of experiments, signatures of this tunneling suppression have been observed [5,7,8], and recently dynamical localization and coherent suppression of tunneling have been demonstrated using light propagating in coupled waveguide arrays [9,10]. So far, however, an exact experimental realization of the intrinsically nonlinear Bose-Hubbard model [2] driven by a time-periodic potential has not been reported.

In this Letter, we report on the observation of the dynamical tunneling suppression predicted in Refs. [2,11] using Bose-Einstein condensates (BECs) in strongly driven periodic optical potentials [12]. In contrast to other systems, the parameters of such optical lattices—potential depth, lattice spacing, driving strength, and driving frequency—can be varied over a wide range. Also, our system allows us to observe the effects of the shaking both through the real-space expansion of the BEC in the optical lattice and by performing time-of-flight experiments, in which the phase coherence of the BEC can be measured and which allow us to verify that the tunneling suppression occurs in a phase-coherent way.

Furthermore, BECs have an intrinsic nonlinear on-site interaction energy (represented by U in Fig. 1), the interplay of which with the tunneling parameter J has been shown to lead to the Mott-insulator quantum phase transition for a critical value of the ratio U/J [13,14]. It has been theoretically predicted that, for a BEC in a shaken optical lattice, this ratio can be replaced by U/J_{eff} and, hence, that it should be possible to drive the system across the quantum phase transition by varying the shaking parameter [2,11]. In this work, we demonstrate the feasibility of the key ingredients of this scheme. In particular, we show that, when tunneling in the shaken lattice is completely suppressed, the phase coherence of the BEC is lost, in agreement with the physical picture of a sudden "switch-off" of the interwell coupling and a subsequent independent evolution of the local phases due to collisions between the atoms [15,16].

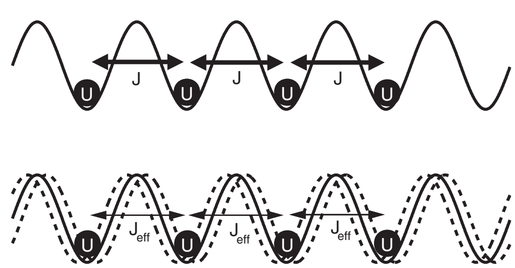

FIG. 1. Suppression of tunneling by strong driving. The dynamics of a Bose-Einstein condensate in a periodic potential is governed by the tunneling matrix element J and the on-site interaction energy U (above). If the potential is strongly shaken, tunneling between the wells is dynamically suppressed, leading to a renormalized tunneling matrix element J_{eff} (below) but leaving the interaction energy U unaffected.

Our system consisting of a Bose-Einstein condensate inside a (sinusoidally) shaken one-dimensional optical lattice is approximately described [17] by the Hamiltonian

$$\hat{H}_0 = -J\sum_{\langle i,j\rangle}(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i) + \frac{U}{2}\sum_j \hat{n}_j(\hat{n}_j - 1) + K\cos(\omega t)\sum_j j\hat{n}_j, \quad (1)$$

where $\hat{c}_i^{(\dagger)}$ are the boson creation and annihilation operators on site i, $\hat{n}_i = \hat{c}_i^\dagger \hat{c}_i$ are the number operators, and K and ω are the strength and angular frequency of the shaking, respectively. The first two terms in the Hamiltonian describe the Bose-Hubbard model [13] with the tunneling matrix element J and the on-site interaction term U. The shaking of the lattice is expected to create a Floquet-quasienergy spectrum, in which the tunneling matrix element J is renormalized to an effective tunneling parameter [2]

$$J_{\mathrm{eff}} = J\mathcal{J}_0(K_0), \quad (2)$$

where $\mathcal{J}_0$ is the zeroth-order ordinary Bessel function and we have introduced the dimensionless parameter $K_0 = K/\hbar\omega$.

In our experiment, we created BECs of about 5×10^4 87-rubidium atoms using a hybrid approach in which evaporative cooling was initially effected in a magnetic time-orbiting potential trap and subsequently in a crossed dipole trap. The dipole trap was realized by using two intersecting Gaussian laser beams at 1030 nm wavelength and a power of around 1 W per beam focused to waists of 50 μm. After obtaining pure condensates of around 5×10^4 atoms, the powers of the trap beams were adjusted in order to obtain elongated condensates with the desired trap frequencies (≈ 20 Hz in the longitudinal direction and 80 Hz radially). Along the axis of one of the dipole trap beams, a one-dimensional optical lattice potential was then added by ramping up the power of the lattice beams in 50 ms (the ramping time being chosen so as to avoid excitations of the BEC). The optical lattices used in our experiments were created using two counterpropagating Gaussian laser beams ($\lambda = 852$ nm) with 120 μm waist and a resulting optical lattice spacing $d_L = \lambda/2 = 0.426$ μm. The depth V_0 of the resulting periodic potential is measured in units of $E_{\mathrm{rec}} = \hbar^2\pi^2/(2md_L^2)$, where m is the mass of the Rb atoms. By introducing a frequency difference $\Delta\nu$ between the two lattice beams (using acousto-optic modulators which also control the power of the beams), the optical lattice could be moved at a velocity $v = d_L\Delta\nu$ or accelerated with an acceleration $a = d_L\frac{d\Delta\nu}{dt}$. In order to periodically shake the lattice, $\Delta\nu$ was sinusoidally modulated with angular frequency ω, leading to a time-varying velocity $v(t) = d_L\Delta\nu_{\max}\sin(\omega t)$ and, hence, to an effective time-varying force in the lattice frame

$$F(t) = m\omega d_L\Delta\nu_{\max}\cos(\omega t) = F_{\max}\cos(\omega t). \quad (3)$$

The peak shaking force $F_{\max}$ is related to the shaking strength K in Eq. (1) by $K = F_{\max}d_L$, and hence

$$K_0 = \frac{K}{\hbar\omega} = \frac{md_L^2\Delta\nu_{\max}}{\hbar} = \frac{\pi^2\Delta\nu_{\max}}{2\omega_{\mathrm{rec}}}. \quad (4)$$

The spatial shaking amplitude $\Delta x_{\max}$ can then be written as $\Delta x_{\max} = (2/\pi^2)(\omega_{\mathrm{rec}}/\omega)K_0 d_L$, so for a typical shaking frequency $\omega/2\pi = 3$ kHz we have $\Delta x_{\max} \approx 0.5d_L$ at $K_0 = 2.4$.

After loading the BECs into the optical lattice, the frequency modulation of one of the lattice beams creating the shaking was switched on either suddenly or using a linear ramp with a time scale of a few milliseconds. Finally, in order to measure the effective tunneling rate $|J_{\mathrm{eff}}|$ between the lattice wells (where the modulus indicates that we are not sensitive to the sign of J, in contrast to the time-of-flight experiments described below), we then switched off the dipole trap beam that confined the BEC along the direction of the optical lattice, leaving only the radially confining beam switched on (the trap frequency of that beam along the lattice direction was on the order of a few hertz and hence negligible on the time scales of our expansion experiments, which were typically less than 200 ms). The BEC was now free to expand along the lattice direction through interwell tunneling, and its *in situ* width was measured using a resonant flash, the shadow cast by which was imaged onto a CCD chip. The observed density distribution was then fitted with one or two Gaussians.

In a preliminary experiment without shaking ($K_0 = 0$), we verified that, for our expansion times, the growth in the condensate width σ_x along the lattice direction was to a good approximation linear and that the dependence of $d\sigma_x/dt$ on the lattice depth (up to $V_0/E_{\mathrm{rec}} = 9$) followed the expression for $J(V_0/E_{\mathrm{rec}})$ in the lowest energy band [18]

$$J\left(\frac{V_0}{E_{\mathrm{rec}}}\right) = \frac{4E_{\mathrm{rec}}}{\sqrt{\pi}}\left(\frac{V_0}{E_{\mathrm{rec}}}\right)^{3/4} e^{-2\sqrt{V_0/E_{\mathrm{rec}}}}, \quad (5)$$

which is a good approximation for our range of lattice depths. This enabled us to confirm that $d\sigma_x/dt$ measured at a fixed time was directly related to J and, in a shaken lattice, to $|J_{\mathrm{eff}}(K_0)|$. We also verified that, for our parameters U and J, the condensate was not in the self-trapping regime [19]. The results of our measurements of $|J_{\mathrm{eff}}(K_0)/J|$ for various lattice depths V_0 and driving frequencies ω are summarized in Fig. 2. We found a universal behavior of $|J_{\mathrm{eff}}/J|$ that is in very good agreement with the Bessel-function rescaling of Eq. (2). We were able to measure $|J_{\mathrm{eff}}/J|$ for K_0 up to 12, albeit agreement with theory beyond $K_0 \approx 6$ was not as good, with the experimental values lying consistently below the theoretical curve. For the zeros of the $\mathcal{J}_0$ Bessel function at $K_0 \approx 2.4$ and 5.5, complete suppression of tunneling was observed (within

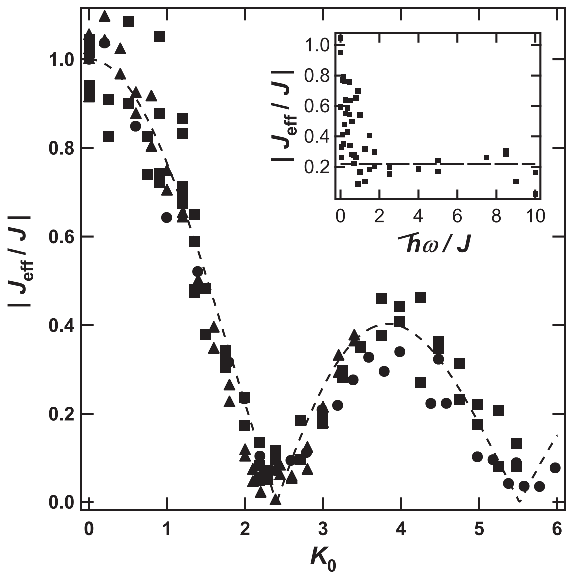

FIG. 2. Dynamical suppression of tunneling in an optical lattice. Shown here is $|J_{\rm eff}/J|$ as a function of the shaking parameter K_0 for $V_0/E_{\rm rec} = 6$, $\omega/2\pi = 1$ kHz (squares), $V_0/E_{\rm rec} = 6$, $\omega/2\pi = 0.5$ kHz (circles), and $V_0/E_{\rm rec} = 4$, $\omega/2\pi = 1$ kHz (triangles). The dashed line is the theoretical prediction. Inset: $|J_{\rm eff}/J|$ as a function of ω for $K_0 = 2.0$ and $V_0/E_{\rm rec} = 9$ corresponding to $J/h = 90$ Hz.

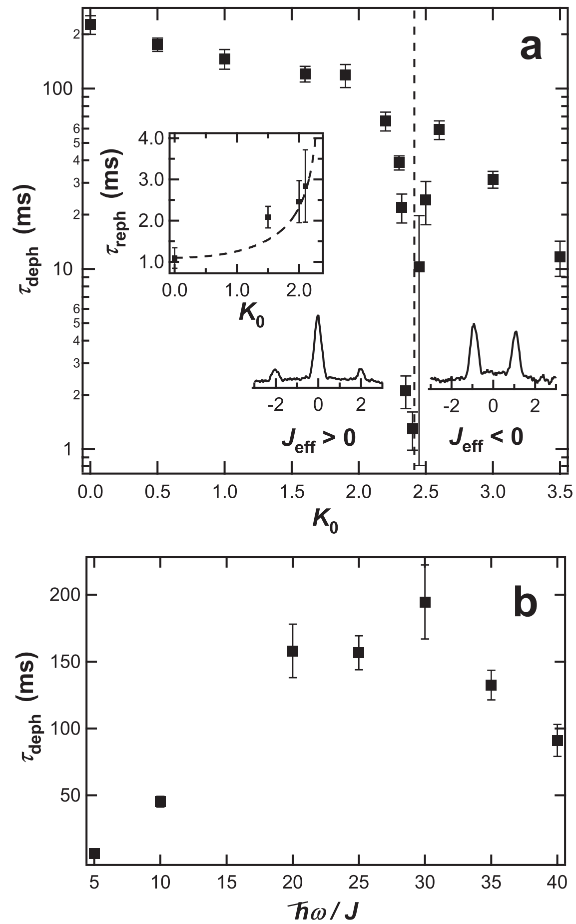

FIG. 3. Phase coherence in a shaken lattice. (a) Dephasing time $\tau_{\rm deph}$ of the condensate as a function of K_0 for $V_0/E_{\rm rec} = 9$ and $\omega/2\pi = 3$ kHz. The vertical dashed line marks the position of $K_0 = 2.4$ dividing the regions with $J_{\rm eff} > 0$ (left) and $J_{\rm eff} < 0$ (right). In both regions, a typical (vertically integrated) interference pattern without final acceleration to the zone edge is shown (the x axis is scaled in units of the recoil momentum $p_{\rm rec} = h/d_L$.) Inset: Rephasing time after dephasing at $K_0 = 2.4$ and subsequent reduction of K_0. (b) Dephasing time as a function of $\hbar\omega/J$ for $K_0 = 2.2$.

our experimental resolution, we could measure a suppression by at least a factor of 25).

We also checked the behavior of $|J_{\rm eff}/J|$ as a function of ω for a fixed value of $K_0 = 2$ (see inset in Fig. 2) and found that, over a wide range of frequencies between $\hbar\omega/J \approx 0.3$ and $\hbar\omega/J \approx 30$, the tunneling suppression works, although for $\hbar\omega/J \lesssim 1$ we found that $|J_{\rm eff}(K_0)/J|$ deviated from the Bessel function near the zero points, where the suppression was less efficient than expected. In the limit of large shaking frequencies ($\omega/2\pi \gtrsim 3$ kHz, to be compared with the typical mean separation of $\approx$15 kHz between the two lowest energy bands at $V_0/E_{\rm rec} = 9$), we observed excitations of the condensate to the first excited band of the lattice. In our *in situ* expansion measurements, these band excitations (typically less than 30% for $K_0 > 3$ and less than 10% for $K_0 < 3$) were visible in the condensate profile as a broad Gaussian pedestal below the near-Gaussian profile of the ground-state condensate atoms. From the widths of those pedestals, we inferred that $|J_{\rm eff}/J|$ of the atoms in the excited band also followed the Bessel-function rescaling of Eq. (2) and that the ratios of the tunneling rates in the two bands agreed with theoretical models.

We now turn to the phase coherence of the BEC in the shaken lattice, which was made visible by switching off the dipole trap and lattice beams and letting the BEC fall under gravity for 20 ms. This resulted in an interference pattern whose visibility reflected the condensate coherence [20]. In the region between the first two zeros of the Bessel function, where $\mathcal{J}_0 < 0$, we found an interference pattern [see Fig. 3(a)] that was shifted by half a Brillouin zone. This shift can be interpreted as an inversion of the curvature of the (quasi)energy band at the center of the Brillouin zone when the effective tunneling parameter is negative. We then quantified the visibility $\mathcal{V} = (h_{\rm max} - h_{\rm min})/(h_{\rm max} + h_{\rm min})$ of the interference pattern after shaking the condensate in the lattice for a fixed time between 1 and $\approx$ 200 ms and finally accelerating the lattice to the edge of the Brillouin zone. In the expression for $\mathcal{V}$, $h_{\rm max}$ is the mean value of the condensate density at the position of the two interference peaks, and $h_{\rm min}$ is the condensate density in a region of width equal to about 1/4 of the peak separation centered about the halfway point between the two peaks. For a perfectly phase-coherent condensate, $\mathcal{V} \approx 1$, whereas for a strongly dephased condensate, $\mathcal{V} \approx 0$. For $K_0 \lesssim 2.2$, the BEC phase coherence was maintained for several tens of milliseconds, demonstrating that the tunneling could be suppressed by a factor of up to 10 over hundreds of shaking cycles without significantly disturbing the BEC.

This result is expressed more quantitatively in Fig. 3(a). Here the condensate was held in the lattice ($V_0/E_{rec} = 9$), and the shaking was switched on suddenly at $t = 0$ (we found no significantly different behavior when K_0 was linearly ramped in a few milliseconds). Thereafter, $\mathcal{V}$ was measured as a function of time, and the decay time constant τ_{deph} of the resulting near-exponential function was extracted. Apart from a slow overall decrease in τ_{deph} for increasing K_0, a sharp dip around $K_0 = 2.4$ is visible. In this region, $|J_{eff}/J| < 1/20$ and, hence, $|J_{eff}/h| \lesssim 10$ Hz, which for our experimental parameters is comparable to the on-site interaction U/h (we checked that the widths of the on-site wave functions and hence U were independent of K_0 by analyzing the side peaks in the interference pattern). This means that neighboring lattice sites are effectively decoupled and the local phases evolve independently due to interatomic collisions, leading to a dephasing of the array [14,16]. By increasing the dipole trap frequency (and hence U), we verified that τ_{deph} decreases as expected. We also studied a rephasing of the BEC when, after an initial dephasing at $K_0 = 2.4$, the value of the shaking parameter was reduced below 2.4. The time constant τ_{reph} of the subsequent rephasing of the condensate (mediated by interwell tunneling and on-site collisions) increased with decreasing J_{eff} [see the inset in Fig. 3(a), where we compare τ_{reph} with the inverse of the generalized Josephson frequency $\omega_{\text{Josephson}}^{-1} \propto J_{eff}^{-1/2}$ predicted by the two-well model [16,21]].

Finally, we investigated the dependence of τ_{deph} on the shaking frequency ω [see Fig. 3(b)]. Interestingly, while the tunneling suppression as observed *in situ* works even for $\hbar\omega/J \approx 1$, in order to maintain the phase coherence of the condensate, much larger shaking frequencies are needed. Indeed, for our system there exists an optimum shaking frequency of $\hbar\omega/J \approx 30$.

In summary, we have measured the dynamical suppression of tunneling of a BEC in strongly shaken optical lattices and found excellent agreement with theoretical predictions. Our results show that the tunneling suppression occurs in a phase-coherent way and can, therefore, be used as a tool to control the tunneling matrix element while leaving the on-site interaction energy unchanged (in contrast to the usual technique of increasing the lattice depth, which changes both) and without disturbing the condensate. This might ultimately lead to the possibility of controlling quantum phase transitions by strong driving of the lattice. In this context, it will be important to investigate the question of adiabaticity when dynamically changing the shaking parameter. Furthermore, our system also opens up other avenues of research such as the realization of exact dynamical localization using discontinuous shaking waveforms [8,22] or tunneling suppression in superlattices [23].

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Note added in proof.—Similar results have been obtained in an array of double wells by Kierig and co-workers [24].

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