Theory of Fast Optical Spin Rotation in a Quantum Dot Based on Geometric Phases and Trapped States

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A method is proposed for the optical rotation of the spin of an electron in a quantum dot using excited trion states to implement operations significantly faster than those of most existing proposals. Key ingredients are the geometric phase induced by 2π hyperbolic secant pulses, use of coherently trapped states and use of naturally dark states. Our proposal covers a variety of quantum dots by addressing different parameter regimes. Numerical simulations with typical parameters for InAs self-assembled quantum dots, including their dissipative dynamics, give fidelities of the operations in excess of 99%.

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All-optically controlled spins in quantum dots (QDs) provide an attractive proposal for quantum-information processing (QIP) [1]. This proposal combines the merits of solid state, such as integrability with existing semiconductor technology, with the speed and high degree of control of lasers. Among the requirements for QIP [2], optical preparation [3,4] and measurement [5,6] of the spin qubit have been achieved, and the spin coherence time has been measured to be at least 3 μs [7]. At the heart of this QIP approach are optical spin rotations that constitute the quantum gates by which information is manipulated. This key step in the development of optical QIP with spins in QDs has not yet been demonstrated experimentally, although several proposals have been given [8–12]. Most of the latter employ some form of the adiabatic approximation (adiabatic elimination of an excited state [8,9], stimulated Raman adiabatic passage [10,11]), and thus they rely on adiabatic pulses that limit the speed of the gates. A method for fast optical spin rotations about a single axis (the optical axis) was proposed [12], but rotations about arbitrary axes are needed for QIP.

Here we present a proposal for arbitrary coherent rotations of a spin in a single QD without the use of an adiabatic approximation. We show that for realistic QD systems our method provides high fidelity gates operating significantly faster than other proposals. Our approach involves dark states, i.e., states which do not couple to the excited state by the incident laser due to polarization selection rules. These dark states can be the result of two phase locked (“coherent”) lasers, in which case they are known as coherently trapped states. The other element of our proposal is that the spin rotation is represented by the phase induced in the quantum state [12] by a laser with a hyperbolic secant temporal envelope, sech (σt) [13].

These sech pulses have the remarkable property of rendering the resulting time dependent Schrödinger equation for a two-level system analytically solvable. A pulse area can be defined independent of the detuning, an important asset for the celebrated phenomenon of self-induced transparency [14] for 2π pulses. Such pulses propagate virtually unattenuated and maintain their shape through resonant optical media, returning the individual two-level systems to their initial states. This is a highly attractive feature for the present QIP proposal because the information must be returned and stored in the spin subspace after it is manipulated. A less explored effect is that after the passage of the 2π sech pulse the state also acquires a geometric phase, for which we obtained an analytic expression [12]. This phase is global (an overall phase to the quantum state) for a two-level system. It becomes crucial when a third state that does not couple to the laser is present, and it is one of the key ingredients of the present proposal.

The energy levels and selection rules of the spin system in our approach are shown in Fig. 1. An external magnetic field B along the in-plane (x) direction defines the quantization axis. Then the energy eigenstates are the spin states along x, with Zeeman splitting $2\omega_e$. These eigenstates are linear combinations of the spin states along the QD growth axis, z.

The optical axis is along z. The intermediate optically excited states are the so-called trions, which are bound

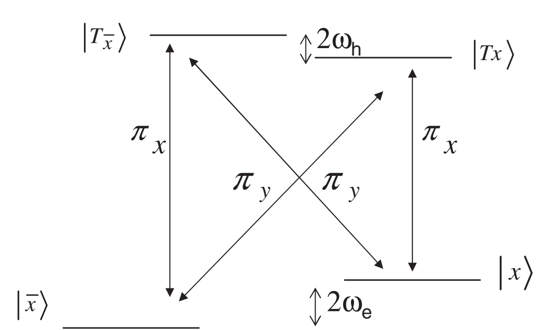

FIG. 1. Energy levels of the four-state system, comprised of the two electron spin eigenstates of σ_x, $\{|x\rangle, |\bar{x}\rangle\}$ and two trion spin states. Linearly polarized light π_x (π_y) induces only the indicated transitions.

states of an electron and an exciton, the latter created in the QD by the laser. The angular momentum of the trion is determined by the hole because the electrons are in the same orbital state of the QD, in a spin singlet.

The hole has a total angular momentum of $3/2$, with the $m_j = \pm 1/2$ ("light hole") doublet separated by more than 20 meV from the $m_j = \pm 3/2$ states due to confinement. We can therefore safely ignore the light hole levels. The $m_j = \pm 3/2$ ("heavy hole") states behave like a pseudospin. In the presence of a magnetic field along x the trion energy eigenstates, $|T_x\rangle$, $|T_{\bar{x}}\rangle$, are the symmetric and antisymmetric linear combinations of the heavy hole trions quantized along z. $2\omega_h$ denotes their Zeeman splitting. The transitions $|x\rangle \to |T_x\rangle$ and $|\bar{x}\rangle \to |T_{\bar{x}}\rangle$ ($|x\rangle \to |T_{\bar{x}}\rangle$ and $|\bar{x}\rangle \to |T_x\rangle$) are coupled by linearly polarized light π_x (π_y) [4]. We note that due to the spin antipairing of the two electrons there is no net electron-hole exchange interaction in the trion and thus the selection rules are not affected by anisotropic exchange [15,16].

Our proposal is based on the observation that from the four-level system of Fig. 1, different two-level systems can be selected by appropriate laser polarization without the need for frequency selectivity. In our approach all four states participate in the rotation scheme. For each of the decoupled two-level systems, a 2π sech pulse will be used to induce the desired phase in the ground state. It is well known that given arbitrary rotations about two axes any rotation can be implemented as a composite rotation. Here we design rotations about z and x by arbitrary angles and compose general rotations from them.

(a) *Rotations about z.*—We showed earlier [12] that broadband circularly polarized pulses (here determined by the bandwidth $\beta_z \gg 2(\omega_e + \omega_h)$) allow the four-level system in Fig. 1 to be treated as a two-level system (plus two uncoupled levels): During the ultrafast pulse the spin and trion precessions are effectively "frozen." The phase gates of Ref. [12] are used for z rotations. The angle of rotation is

$$\phi_z = 2\arctan(\beta_z/\Delta), \tag{1}$$

where Δ is the detuning. The fidelity and purity of the z operations both increase as the pulses become faster. For a subpicosecond pulse, the fidelity was found to be as high as 99.99%.

(b) *Rotations about x.*—As illustrated in Fig. 1 by choosing linearly polarized light π_x, we reduce the four-level system to two independent two-level systems, $\{|x\rangle, |T_x\rangle\}$ and $\{|\bar{x}\rangle, |T_{\bar{x}}\rangle\}$. A π_x linearly polarized 2π sech pulse with bandwidth β_x is used here, which acts on both transitions and which is detuned by Δ_1 (Δ_2) for transition $|x\rangle \leftrightarrow |T_x\rangle$ ($|\bar{x}\rangle \leftrightarrow |T_{\bar{x}}\rangle$). Each two-level system acquires a phase [as in Eq. (1)] determined by its detuning. The relative phase determines the angle of rotation about the x axis, $\phi_x = \phi_1 - \phi_2$, which is

$$\phi_x = 2\arctan\frac{\beta_x(\Delta_1 - \Delta_2)}{\Delta_1\Delta_2 + \beta_x^2}. \tag{2}$$

From Eq. (2) a π rotation requires $\Delta_1\Delta_2 < 0$. This can be understood intuitively: If the laser is detuned positively (or negatively) for *both* transitions, the phases will have the same sign, so there is an upper limit to the obtainable angle of rotation. Centering the laser frequency between the two transitions gives rotation angles up to π if β_x is chosen properly. Therefore for a π rotation there is an upper bound to the bandwidth β_x, which makes rotations about x slower than those about z.

(c) *Rotations about arbitrary axis.*—Finally, by combining the above rotations about x and z we can implement any rotation. For example, rotations about y can be realized by $R_y(\phi) = R_z^\dagger(\pi/2)R_x(\phi)R_z(\pi/2)$.

The imperfections in these gates come predominately from trion decay and from spin precession during z rotations [17]. We have taken these effects into account in our calculations of the fidelity by numerical solution of the Liouville equation for the density matrix. The fidelity is a measure of how well the target gate is implemented and is defined as $\mathcal{F}(U) = \overline{|\langle\Psi|U^\dagger U_{\rm id}|\Psi\rangle|^2}$, where $U_{\rm id}$ is the target operation, U is the actual operation, and the average is taken over all input spin states [18]. The purity of an operation is $\mathcal{P} = \overline{{\rm Tr}\rho^2}$, where ρ is the spin density matrix after the rotation.

In our calculations of the fidelity and purity of gates we have used parameters appropriate for self-assembled InAs QDs: $\omega_e = 0.062$ meV, $\omega_h = 0.039$ meV (corresponding to $B \sim 8$ T) [4,19], $\beta_x = 0.073$ meV for π-rotations, $\beta_x = 0.113$ meV for $\pi/2$-rotations, $\beta_z = 4$ meV, and trion lifetime $\tau_t = 900$ ps [20]. Typical gate fidelities, listed for some rotations in Table I along with the purities, are on the order of 99.5%. We note that mixing between heavy and light holes will simply modify the polarization selection rules; thus it will not result in losses, since it can be straightforwardly incorporated in our proposal [9,12].

In Fig. 2 we show the elements of the spin density matrix for rotation $R_y(\pi) = R_z^\dagger(\pi/2)R_x(\pi)R_z(\pi/2)$, which is our slowest operation, acting here on the initial spin state $|\bar{x}\rangle$. The z rotations are so fast that on this time scale they appear as vertical steps at $t_1 \simeq 5$ ps and $t_2 = t_1 + 2\pi/\omega_e$. The total gate is also fast, about 68 ps. The spin vector, constructed as $\vec{S} = {\rm Tr}(\vec{\sigma}\rho_s)$, is shown in a Bloch

TABLE I. Fidelity and purity of selected rotations of spin for InAs QD parameters.

$R_n(\phi)$	Fidelity	Purity
$R_x(\pi/2)$	99.49%	98.89%
$R_z(\pi/2)$	99.99%	99.97%
$R_y(\pi)$	99.28%	98.57%
$R_y(\pi/2)$	99.45%	98.8%

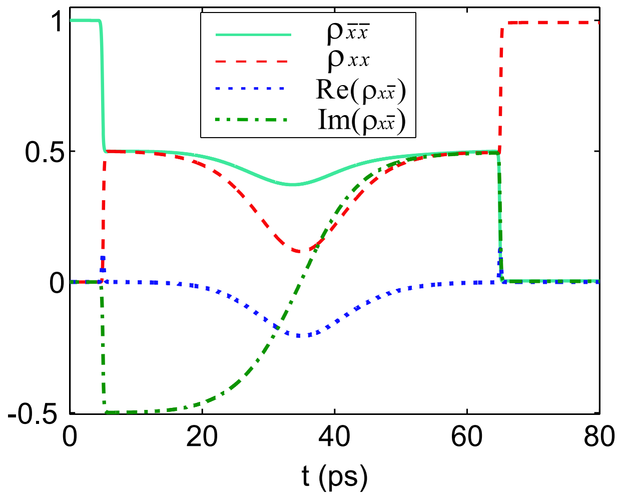

FIG. 2 (color online). Matrix elements of spin density matrix, ρ, for the rotation $R_y(\pi)$ acting on $|\bar{x}\rangle$. The dashed (red) line is ρ_{xx}, the solid (cyan) line is $\rho_{\bar{x}\bar{x}}$, the dotted (blue) line is $\mathrm{Re}(\rho_{x\bar{x}})$, and the dash-dotted (green) line is $\mathrm{Im}(\rho_{x\bar{x}})$, and ρ is in the interaction picture (free spin precession is removed). The duration of this composite gate is about 68 ps.

sphere in Fig. 3. Here $\vec{\sigma}$ is the spin operator and ρ_s is the spin density matrix in the Schrödinger picture.

This gate scheme is especially attractive for experiment in these systems because it uses only simple, nonadiabatic sech pulses. In this parameter regime it does not require phase locking of the lasers, which is a significant experimental simplification. It requires the ability to perform Rabi oscillations between spin and trion, which has been demonstrated for InAs QDs [19], and thus it is accessible with state of the art technology. The only approximation is that the spin is considered to be frozen during the pulse for z rotations; i.e., $\beta_z \gg 2(\omega_e + \omega_h)$. This requirement can be satisfied for the widely used ultrafast lasers and the current InAs QDs.

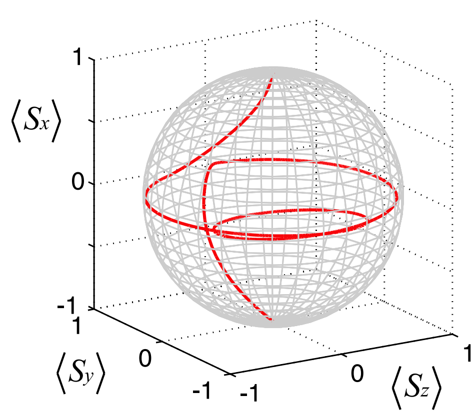

FIG. 3 (color online). Bloch sphere representation of the composite spin rotation $R_y(\pi)$ acting on the initial spin state $|\bar{x}\rangle$; i.e., $\langle S_x\rangle = -1$.

For QDs with larger Zeeman splittings the pulse bandwidths discussed above can become too large for practical purposes. Then our approach can be modified. We utilize coherent population trapping (CPT), in which a superposition of two levels, each having nonzero dipole coupling to an excited state, is dark to an (appropriately chosen) coherent combination of laser pulses. This well-known phenomenon in the optics of atoms [21] has been demonstrated in a semiconductor [22] for a similar system to the one we study. Here we use frequency selectivity to isolate a three-level system from the four levels of Fig. 1. We choose narrow-band pulses, $\beta \ll 2(\omega_e - \omega_h)$, where β is the bandwidth. For concreteness we spectrally focus the pulses to pick out the three-level Λ system $\{|x\rangle, |\bar{x}\rangle, |T_x\rangle\}$. The two transitions are addressed separately by polarization selectivity. The total laser field is

$$\vec{E} = E_x f_x(t) e^{i\omega_x t}\hat{x} + e^{i\alpha} E_y f_y(t) e^{i\omega_y t}\hat{y} + \text{c.c.} \qquad (3)$$

For a coherently trapped state the two pulses should have the same detuning (two-photon resonance), i.e., $\omega_{x,T_x} - \omega_x = \omega_{\bar{x},T_x} - \omega_y \equiv \Delta$, and the same temporal envelope $f_x(t) = f_y(t) \equiv f(t)$. Then the new spin states, $|B\rangle$ and $|D\rangle$ (bright and dark, respectively), are related to the basis states $\{|x\rangle, |\bar{x}\rangle\}$ through the unitary transformation

$$\mathcal{T} = \begin{bmatrix} \cos\vartheta & -e^{i\alpha}\sin\vartheta \\ e^{-i\alpha}\sin\vartheta & \cos\vartheta \end{bmatrix}. \qquad (4)$$

Here ϑ is defined by $\tan\vartheta = E_y/E_x$, and the matrix element between $|B\rangle$ and $|T_x\rangle$ is $V_{B,T_x} = \Omega_o f(t) e^{i\Delta t}$, where $\Omega_o = \sqrt{\Omega_x^2 + \Omega_y^2}$ [23] and Ω_x (Ω_y) is the Rabi frequency of the transition with polarization π_x (π_y). Now we choose the envelope to be $f(t) = \mathrm{sech}(\beta t)$. We require the population to return to the spin subspace after the passage of the pulse; i.e., we require the total pulse acting on the bright state to be a 2π pulse, which gives $\Omega_o = \beta$.

We still have freedom in choosing the bandwidth β and the detuning Δ. The bandwidth is constrained by the requirement of frequency selectivity, $\beta \ll 2(\omega_e - \omega_h)$. After the passage of the pulse the bright state has picked up a phase relative to the dark state given by $\phi = 2\arctan(\frac{\beta}{\Delta})$. This phase is the angle by which any spin state not parallel to $|B\rangle$ (or, equivalently, $|D\rangle$) is rotated. The axis of rotation is that defined by $|B\rangle$. By varying the laser parameters, ϑ and α, we vary the composition of the bright/dark basis, i.e., vary the axis of rotation. Thus we have designed an arbitrary spin rotation $R_n(\phi) = e^{-i\phi\hat{n}\cdot\vec{\sigma}/2}$, where $\vec{\sigma}$ is the spin operator. The axis of rotation is $\hat{n} = (\cos\vartheta, \sin\vartheta\sin\alpha, \sin\vartheta\cos\alpha)$. This axis is the same as that of the adiabatic scheme of Kis and Renzoni [10] where CPT also is used. However, there an auxiliary lower level was required for the population to be stored during the gate. This requirement limits their scheme to physical systems where such an extra level is available. In our case,

TABLE II. Fidelity and purity of selected rotations for CdSe QD parameters.

$R_n(\phi)$	Fidelity	Purity
$R_z(\pi/2)$	98.84%	97.8%
$R_z(\pi)$	97.56%	95.34%
$R_x(\pi/2)$	98.42%	97%
$R_{xz}(\pi/2)$	99.2%	98.78%

no auxiliary levels are needed, and the same pulse creates the trapped state and induces the geometric phase.

The main sources of dissipation in the CPT-based scheme are nonresonant transitions to the higher trion state $|T_{\bar{x}}\rangle$ and spontaneous emission of the trion [17]. To show their effects on the gates we have made numerical simulations of the fidelity. For illustration purposes, we choose parameters appropriate for self-assembled CdSe quantum dots as an example of a physical system with relatively large Zeeman splittings: $\omega_e = 0.4$ meV, $\omega_h = 0.05$ meV (corresponding to $B \sim 8$ T) [24], $\beta \sim 0.025$ meV, and trion lifetime $\tau_t = 580$ ps [25]. We find that for gate durations of about 100 ps typical fidelities range between 98% and 99%, and they are shown in Table II.

We have examined the effect of deviations from a sech pulse shape and found our scheme to be quite robust for pulses of total area 2π. The fidelity decreases by less than 1% when the pulse shape is 80% sech and 20% of other common pulse envelopes [26] even when the bandwidth of the admixture is varied (we varied it up to 20%). Chopping off the tails of the sech pulse causes only a 1–5% fidelity loss when 15% of the sech is removed.

In conclusion, we have developed a method of realizing the key step of optical spin rotations for QIP in realistic QDs based on the geometric phases induced by hyperbolic secant pulses of area 2π. This scheme should be accessible with current experimental capabilities, and it yields fast (1–100 ps) rotations because it does not employ the adiabatic approximation. The fidelities of these rotations for typical QD systems are high, up to 99.99%. We note that our method provides an exact solution to rotations in a three-level Λ-type system and may prove useful in other areas of physics that involve interaction of coherent radiation with quantum systems such as atoms [27].

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[27] An additional advantage of our scheme is that the use of 2π sech pulses may allow for potential manipulation by the same pulse of a logical qubit comprised by several physical qubits located along the optical path. Then, as in self-induced transparency, the different qubits would interact with a single reshaped pulse. We note that numerical simulations [G. Panzarini, U. Hohenester, and E. Molinari, Phys. Rev. B **65**, 165322 (2002)] imply that self-induced transparency is possible in an array of QDs.