Observation of Accelerating Airy Beams

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We report the first observation of Airy optical beams. This intriguing class of wave packets, initially predicted by Berry and Balazs in 1979, has been realized in both one- and two-dimensional configurations. As demonstrated in our experiments, these Airy beams can exhibit unusual features such as the ability to remain *diffraction-free* over long distances while they tend to *freely accelerate* during propagation.

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In 1979 Berry and Balazs made an important observation within the context of quantum mechanics: they theoretically demonstrated that the Schrödinger equation describing a free particle can exhibit a nonspreading Airy wave packet solution [1]. Perhaps the most remarkable feature of this Airy packet is its ability to *freely accelerate* even in the absence of any external potential. As first noted in Ref. [1], in one dimension (1D), this Airy packet happens to be unique, e.g., it is the only nontrivial solution (apart from a plane wave) that remains invariant with time [1,2].

Over the years, nonspreading or nondiffracting wave configurations have been systematically investigated in higher dimensions (2D and 3D), particularly in the areas of optics and atom physics [3–6]. What makes the analogy between these two seemingly different disciplines possible is the mathematical correspondence between the quantum mechanical Schrödinger equation and the paraxial equation of diffraction [7]. In terms of experimental realization, optics has thus far provided a fertile ground in which the properties of such nonspreading localized waves can be directly observed and studied in detail. Perhaps the best known example of such a 2D diffraction-free optical wave is the so-called Bessel beam first suggested and observed by Durnin *et al.* [3]. This work sparked considerable theoretical and experimental activity and paved the way toward the discovery of other interesting nondiffracting solutions [4,5]. We note that, even though at first sight, the aforementioned propagation-invariant beams may appear dissimilar, they in fact share common characteristics. First, they are all generated from an appropriate conical superposition of plane waves [3–5]. Even more importantly, all these solutions are known to convey infinite power, a direct outcome of their nondiffracting nature. Of course, in practice, all these nonspreading beams are normally truncated by an aperture (because of lack of space and power) and as a result they tend to diffract during propagation [8]. Yet, if the geometrical size of the limiting aperture greatly exceeds the spatial features of the ideal propagation-invariant fields, the diffraction process is considerably "slowed down" over the intended propagation distance and hence for all practical purposes these beams are called "diffraction-free" [9]. We emphasize that no localized 1D propagation-invariant beam can be synthesized through conical superposition.

In this Letter we report the first observation of 1D and 2D accelerating diffraction-free Airy beams [1,10]. These beams, in contrast to the already known families of nondiffracting fields, are also possible in 1D and do not result from conical superposition. Our experiments demonstrate that even though the Airy beams are exponentially truncated (convey finite power) they still exhibit their key characteristics [10]. More specifically, they resist diffraction while their main intensity maxima or lobes tend to accelerate during propagation along parabolic trajectories. This behavior persists over long distances in spite of the fact that the center of gravity of these wave packets remains constant (an outcome of Ehrenfest's theorem) and diffraction eventually takes over [1,7,10,11]. The observed propagation dynamics are in good agreement with theory.

To examine the behavior of optical Airy wave packets, we invoke the normalized paraxial equation of diffraction (potential-free Schrödinger equation) [10]:

$$i\frac{\partial \phi}{\partial \xi} + \frac{1}{2}\frac{\partial^2 \phi}{\partial s^2} = 0, \tag{1}$$

where ϕ is the electric field envelope, $s = x/x_0$ represents a dimensionless transverse coordinate, x_0 is an arbitrary transverse scale, $\xi = z/kx_0^2$ is a normalized propagation distance, and $k = 2\pi n/\lambda_0$. As first shown in Ref. [1], Eq. (1) admits the following Airy nondispersive solution,

$$\phi(\xi, s) = \mathrm{Ai}(s - (\xi/2)^2)\exp(i(s\xi/2) - i(\xi^3/12)). \tag{2}$$

Clearly, at the origin $\phi(0, s) = \mathrm{Ai}(s)$. Equation (2) clearly shows that the intensity profile of this wave remains invariant during propagation while it experiences constant acceleration. The term $(\xi/2)^2$ in Eq. (2) describes this ballistic trajectory. Figure 1(a) depicts the diffraction-free propagation of such an accelerating Airy wave packet as a function of distance ξ. An alternative interpretation of this interesting result was given by Greenberger through the principle of equivalence [12]. More specifically, he remarked that a stationary Airy wave packet associated with a quantum mechanical particle in a constant gravita-

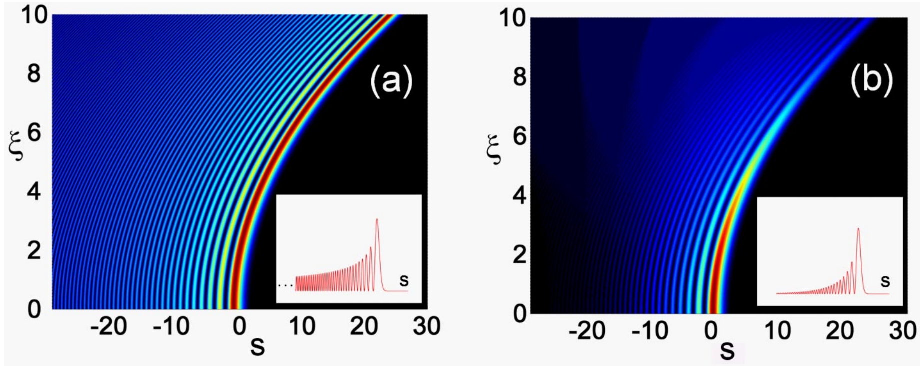

FIG. 1 (color online). Propagation dynamics of (a) a diffraction-free Airy wave and (b) a finite-energy Airy packet when $a = 0.05$. The corresponding input intensities of these beams are shown in the insets.

tional field will be perceived as accelerating upwards by a free-falling observer in whose frame of inertia gravitational forces are absent. As also indicated in [1], this accelerating behavior is by no means in conflict with Ehrenfest's theorem which describes the motion of the center of gravity of a wave packet [1,7]. This is because the Airy beam is not square integrable ($\int \mathrm{Ai}^2(x)dx \to \infty$) and thus its center of mass cannot be defined [1,13].

The properties of finite-energy (power) Airy beams were recently investigated theoretically within the context of optics [10]. One possible way to realize such beams is to introduce an exponential aperture function, i.e., let $\phi(0, s) = \mathrm{Ai}\,(s)\exp(as)$ [10], where a is a positive parameter so as to ensure containment of the infinite Airy tail. Typically, $a \ll 1$ so that the resulting wave packet closely resembles the intended Airy function [see inset of Fig. 1(b)]. By directly integrating Eq. (1) we find [10]:

$$\phi(\xi, s) = \mathrm{Ai}\,(s - (\xi/2)^2 + ia\xi)\exp(as - (a\xi^2/2) - i(\xi^3/12) + i(a^2\xi/2) + i(s\xi/2)). \quad (3)$$

The Fourier transform $\Phi_0(k)$ of this finite norm wave packet is proportional to $\Phi_0(k) \propto \exp(-ak^2)\exp(ik^3/3)$ [10]. From this latter equation, one can readily deduce that the angular Fourier spectrum of this truncated Airy beam is Gaussian and involves a cubic phase (k^3) resulting from the Fourier transform of the Airy function itself. This particular form of the spectrum has important implications in terms of experimentally synthesizing this truncated version of Airy packets. As a result, this wave can be generated from a broad Gaussian beam through a Fourier transformation provided that a cubic phase is imposed.

Figure 1(b), shows the propagation dynamics of a finite-energy Airy wave packet when $a = 0.05$. As clearly seen, for $a \ll 1$ the beam still displays all the interesting characteristics of the ideal Airy packet. During propagation, it remains quasi-invariant over several diffraction lengths while again the intensity features tend to "freely accelerate". For this case, the beam behaves as if it was almost ideal [see Fig. 1(a)] for an appreciable distance until diffraction eventually takes over. We note that here, the term "acceleration" must be cautiously used since the center of mass of a finite-energy Airy packet can be defined and in fact remains invariant with distance [1,11]. Yet, as depicted in Fig. 1(b), for small aperture factors the local intensity features still move on a parabolic trajectory and thus accelerate within the beam.

In order to study experimentally the propagation dynamics of finite-energy Airy wave packets we exploit the fact that the Fourier transform of the function $\mathrm{Ai}\,(s)\exp(as)$ is a Gaussian beam modulated with a cubic phase. An air-cooled argon-ion continuous-wave laser operating at 488 nm emits a linearly polarized, high quality Gaussian beam that is subsequently collimated to a width of 6.7 mm (FWHM). This broad Gaussian beam is then reflected from the front facet of a computer-controlled liquid crystal spatial light modulator (SLM). This SLM is used to impose the cubic-phase modulation ($-20\pi.. + 20\pi$ in 2 cm) that is necessary to produce the Airy beam. In order to generate the one-dimensional Airy wave packet, a converging cylindrical lens with a focal length of $f = 1.2$ m is placed at a distance f in front of the SLM phase array. After the SLM, the Fourier transform of this phase-modulated Gaussian beam is then obtained at a distance $d = f = 1.2$ m behind the lens. The Airy beam produced is then

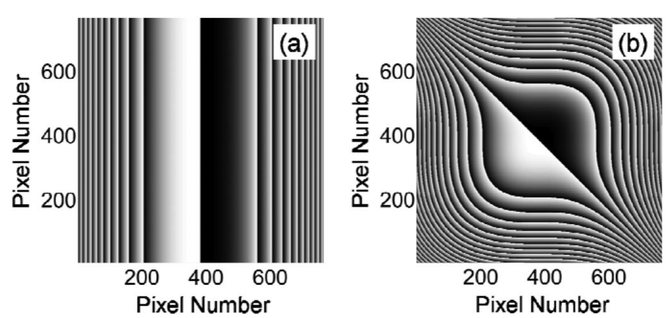

FIG. 2. Phase masks used to generate (a) 1D and (b) 2D-Airy beams. The cubic phase is "wrapped" between $[0, 2\pi]$. In the gray scale pattern, black corresponds to 0 and white to 2π radians.

imaged on a carefully aligned CCD camera through a 5 × microscope objective. The propagation dynamics of these beams are then recorded as a function of propagation distance by translating the imaging apparatus. Figures 2(a) and 2(b) show the phase masks used to generate the 1D and subsequently 2D Airy beams, respectively.

Figure 3(a) depicts the intensity profile of a 1D exponentially truncated Airy beam at the origin ($z = 0$). In our experiment, $x_0 = 53$ μm and $a = 0.11$. Figures 3(b) and 3(c) show the corresponding intensity profiles of this Airy packet at $z = 10$, 20 cm, respectively. As expected, the beam remains almost diffraction-free while its main lobe tends to quadratically accelerate. Our measurements show that the spatial FWHM width of the main lobe (containing in this case more than 70% of the total beam energy) remains almost invariant up to a distance of approximately 25 cm and retains its original value of ≈ 90 μm. It is worth noting that this occurs in free space and is by no means a result of some optical nonlinearity [14]. Figures 3(d)–3(f) depict the corresponding expected theoretical behavior of this same Airy packet at these same distances—in good agreement with experiment. Note that a Gaussian beam of this size would have diffracted at least 6–7 times in this same distance, Fig. 3(g) and 3(h). In addition, had the cubic phase not been imposed on the initial broad wave front, the resulting Gaussian beam would have expanded ×24 in 25 cm. What was also clearly demonstrated in our experiment was the transverse acceleration of the local intensity maxima, Fig. 4. This parabolic trajectory is a result of acceleration and is well described by the theoretical relation $x_d \cong \lambda_0^2 z^2/(16\pi^2 x_0^3)$, as long as the beam remains quasi-diffraction-free and before diffraction effects take over. The solid line in Fig. 4 corresponds to this latter analytical expression. As these results indicate, after 30 cm of propagation the beam experiences a deflection of 820 μm—comparable to the total size of the packet ($\approx$ first 10 lobes of the Airy beam). Again, we emphasize that the acceleration observed here refers to the local intensity features of the packet. In all cases, the center of gravity $\langle s \rangle$ of this wave remains invariant [1,11] since $d\langle s \rangle/d\xi = (i/2)\int(\phi_s^*\phi - \phi_s\phi^*)ds$ is constant.

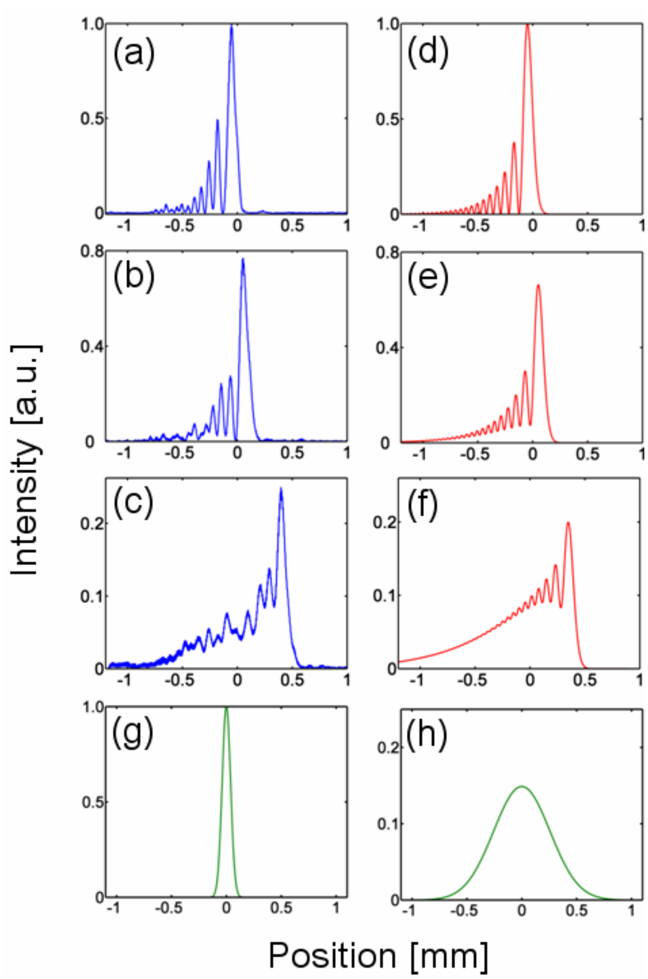

FIG. 3 (color online). Observed intensity cross sections of a planar Airy beam at (a) 0 cm, (b) 10 cm, and (c) 20 cm. Corresponding theoretical plots for these same distances (d)–(f). (g) A Gaussian beam having the same intensity FWHM as the first Airy lobe. (h) Corresponding diffraction profile after 25 cm of propagation.

Similarly we have also considered 2D Airy beams satisfying the paraxial equation of diffraction in two dimensions. The case of an ideal 2D Airy packet was first suggested by Besieris *et al.* [15]. In this case a 2D SLM phase pattern [Fig. 2(b)] was imposed on the Gaussian beam and was then Fourier transformed through a spherical lens. By doing so we were able to produce finite-energy Airy wave packets of the form $\phi = \mathrm{Ai}(x/x_0)\mathrm{Ai}(y/y_0) \times \exp(x/w_1)\exp(y/w_2)$. The evolution diffraction dynamics of these latter 2D field configurations can be readily solved by separation of variables using the result of Eq. (3) [10]. The intensity distribution of such a wave is shown in Fig. 5(a), when $w_1 = w_2$, corresponding to an x-y trunca-

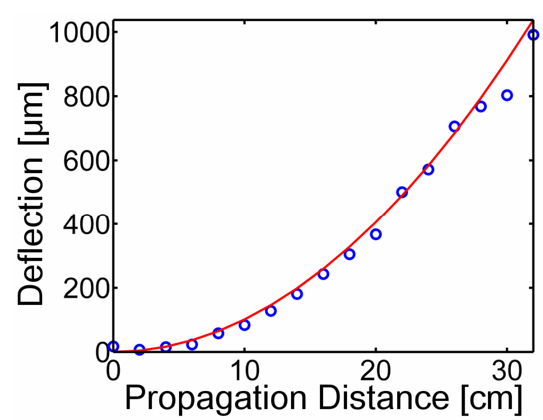

FIG. 4 (color online). Transverse acceleration of an Airy beam when $a = 0.11$ as a function of distance. Circles mark experimental results while the solid line represents the expected theoretical deflection.

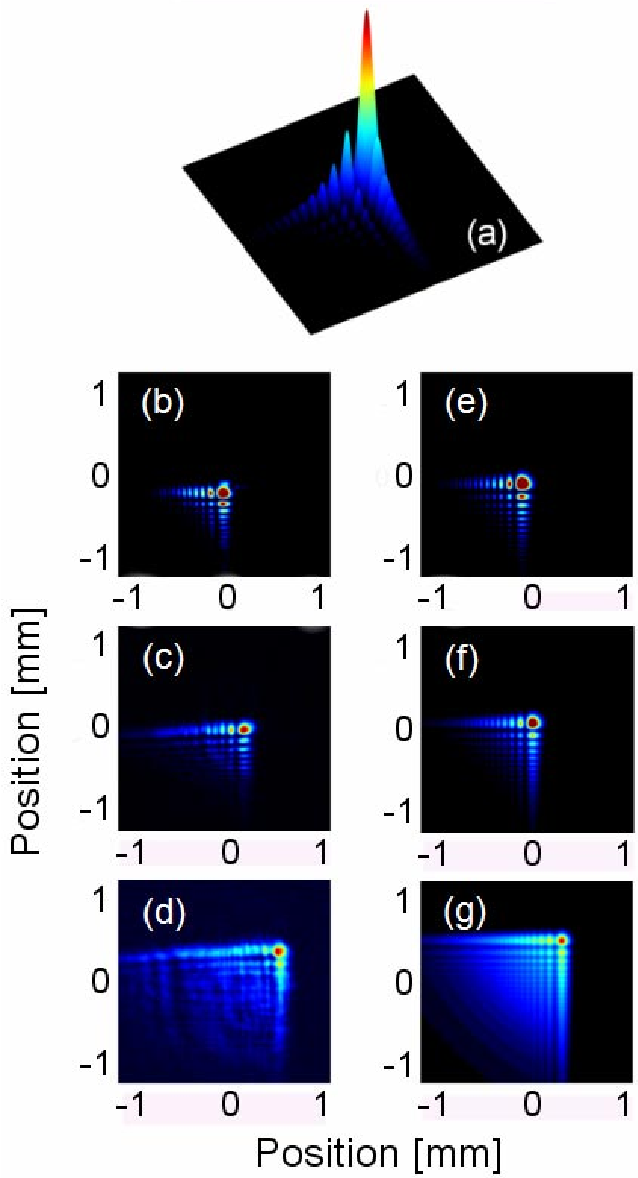

FIG. 5 (color online). (a) A schematic of a 2D Airy packet. Observed intensity distribution of a 2D Airy beam at (b) $z = 0$ cm, (c) $z = 10$ cm, and (d) $z = 20$ cm. Corresponding theoretical results at these same distances (e)–(g).

tion factor of $a = 0.11$. In this case, approximately 50% of the energy resides in the main intensity lobe at the corner. In general, the flexibility in separately adjusting the x-y parameters allows one to control the transverse acceleration vector of this novel 2D nondiffracting beam. In our experiments we considered beams with equal scales in x-y, and thus the acceleration occurred along the 45° axis. For the pattern generated, $x_0 = 53\ \mu$m and the aperture factor is $a = 0.11$. As in the 1D case, our experimental results indicate that this 2D beam propagates almost diffraction-free up to a distance of 25 cm. The main lobe keeps its spot size (90 μm) up to a distance of ~25 cm and the beam moves on a 2D parabolic trajectory with $x_d = y_d$. The diffraction dynamics of these 2D Airy beams are shown in Figs. 5(b)–5(g).

We would like to point out that 1D Airy wave packets can also be synthesized in the temporal domain using dispersive elements [16]. This may lead to the first observation of dispersion-free Airy pulses in optical fibers, in both the normal and anomalous dispersion regime [17]. The use of Airy nondiffracting beams for particle manipulation [18] or in nonlinear media [14,19,20] may be another fruitful direction.

In conclusion, we have reported the first observation of Airy optical wave packets. As demonstrated in our experiments, these Airy beams can exhibit unusual features such as the ability to remain diffraction-free over long distances while they tend to freely accelerate during propagation.

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