lead to the outcomes  $(r_A, r_B)$ :

## **Experimental Falsification of Leggett's Nonlocal Variable Model**

Cyril Branciard,<sup>1</sup> Alexander Ling,<sup>2,3</sup> Nicolas Gisin,<sup>1</sup> Christian Kurtsiefer,<sup>2</sup> Antia Lamas-Linares,<sup>2</sup> and Valerio Scarani<sup>2</sup>

<sup>2</sup>Centre for Quantum Technologies, National University of Singapore, Singapore

<sup>3</sup>Temasek Laboratories, National University of Singapore, Singapore

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Bell's theorem guarantees that no model based on local variables can reproduce quantum correlations. Also, some models based on *nonlocal variables*, if subject to apparently "reasonable" constraints, may fail to reproduce quantum physics. In this Letter, we introduce a family of inequalities, which use a finite number of measurement settings, and which therefore allow testing Leggett's nonlocal model versus quantum physics. Our experimental data falsify Leggett's model and are in agreement with quantum predictions.

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Introduction. —Quantum physics provides a precise rule  $\rho$ to compute the probability that the measurement of A and B performed on two physical systems in the state  $|\Psi\rangle$  will

$$P_{Q}(r_{A}, r_{B}|A, B) = \langle \Psi | \mathcal{P}_{r_{A}} \otimes \mathcal{P}_{r_{B}} | \Psi \rangle$$
(1)

where  $\mathcal{P}_r$  is the projector on the subspace associated to the measurement result r. For entangled states, this formula predicts that the outcomes are correlated, irrespective of the distance between the two measurement devices. A natural explanation for correlations established at a distance is preestablished agreement: the two particles have left the source with some common information  $\lambda$ , called a local variable (LV), that allows them to compute the outcomes for each possible measurement; formally,  $r_A =$  $f_A(A, \lambda)$  and  $r_B = f_B(B, \lambda)$ . Satisfactory as it may seem a priori, this model fails to reproduce all quantum correlations: this is the celebrated result of John Bell [1], by now tested in a very large number of experiments. The fact that quantum correlations can be attributed neither to LV nor to communication below the speed of light is referred to as quantum nonlocality.

While nonlocality is a striking manifestation of quantum entanglement, the essence of quantum physics may be somewhere else [2]. For instance, nondeterminism is another important feature of quantum physics, with no a priori link with nonlocality. Generic theories featuring both nondeterminism and nonlocality have been studied, with several interesting achievements [3]; but it is not yet clear what singles quantum physics out. In order to progress in this direction, it is important to learn which other alternative models are compatible with quantum physics, and which are not. Bell's theorem having ruled out all possible LV models, we have to move on to models based on nonlocal variables (NLV). The first example of testable NLV model was the one by Suarez and Scarani [4], falsified in a series of experiments a few years ago [5]. A different such model was proposed more recently by Leggett [6]. This model supposes that the source emits product quantum states  $|\alpha\rangle \otimes |\beta\rangle$  with probability density

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 $\rho(\alpha, \beta)$ , and enforces that the marginal probabilities must be compatible with such states:

$$P(r_A|A) = \int d\rho(\alpha, \beta) \langle \alpha | \mathcal{P}_{r_A} | \alpha \rangle, \qquad (2)$$

$$P(r_B|B) = \int d\rho(\alpha, \beta) \langle \beta | \mathcal{P}_{r_B} | \beta \rangle.$$
(3)

The correlations however must include some nonlocal effect, otherwise this would be a (nondeterministic) LV model and would already be ruled out by Bell's theorem. What Leggett showed is that the simple requirement of consistency (i.e., no negative probabilities should appear at any stage) constrains the possible correlations, even nonlocal ones, to satisfy inequalities that are slightly but clearly violated by quantum physics. A recent experiment [7] demonstrated that state-of-the-art setups can detect this violation in principle. However, their falsification of the Leggett model is flawed by the need for additional assumptions because of the inequality they used [8], just as the original one by Leggett, supposes that data are collected from infinitely many measurement settings. In this Letter, we present a family of inequalities, which allow testing Leggett's model against quantum physics with a finite number of measurements. We show their experimental violation by pairs of polarization-entangled photons. We conclude with an overview of what has been learned and what is still to be learned about NLV models.

*Theory.*—We restrict our theory to the case where the quantum degree of freedom under study is a qubit. We consider von Neumann measurements, that can be labeled by unit vectors in the Poincaré sphere  $S: A \rightarrow \vec{a}$  and  $B \rightarrow \vec{b}$ ; their outcomes will be written  $r_A, r_B \in \{+1, -1\}$ . Pure states of single particles can also be labeled by unit vectors  $\vec{u}, \vec{v}$  in S. Leggett's model requires [9]

$$P(r_A, r_B | \vec{a}, \vec{b}) = \int d\rho(\vec{u}, \vec{v}) P_{\vec{u}, \vec{v}}(r_A, r_B | \vec{a}, \vec{b})$$
(4)

with

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<sup>&</sup>lt;sup>1</sup>Group of Applied Physics, University of Geneva, Switzerland

$$P_{\vec{u},\vec{v}}(r_A, r_B | \vec{a}, \vec{b}) = \frac{1}{4} [1 + r_A \vec{a} \cdot \vec{u} + r_B \vec{b} \cdot \vec{v} + r_A r_B C(\vec{u}, \vec{v}, \vec{a}, \vec{b})].$$
(5)

The correlation coefficient  $C(\vec{u}, \vec{v}, \vec{a}, \vec{b})$  is constrained only by the requirement that (5) must define a probability distribution over  $(r_A, r_B)$  for all choice of the measurements  $\vec{a}, \vec{b}$ . Remarkably, this constraint is sufficient to derive inequalities that can be violated by quantum physics [6,8,10]. In the derivation of these inequalities, one defines two *orthogonal* planes in the Poincaré sphere:  $\Pi_j = \{\vec{a} \in S | \vec{a} \cdot \vec{n}_j = 0\}$  for  $\vec{n}_j \in S$  and  $\vec{n}_1 \cdot \vec{n}_2 = 0$ . For each unit vector  $\vec{a}_j \in \Pi_j$ , let us define  $\vec{a}_j^{\perp} = \vec{n}_j \times \vec{a}_j$ : the inequalities are then obtained in terms of coefficients  $E_j(\theta)$  which are the average over *all* directions  $\vec{a}_j$  of the correlation coefficient

$$C(\vec{a}_{j}, \vec{b}_{j}) = \sum_{r_{A}, r_{B}} r_{A} r_{B} P(r_{A}, r_{B} | \vec{a}_{j}, \vec{b}_{j})$$
(6)

with  $\vec{b}_j = (\cos\theta)\vec{a}_j + (\sin\theta)\vec{a}_j^{\perp}$  [11]. This is a problematic feature: such inequalities can be checked only by performing an infinite number of measurements or by adding the assumption of rotational invariance of the correlation coefficients  $C(\vec{a}, \vec{b})$ , as in [7]. It is thus natural to try and replace the average over all possible settings with an average on a discrete set. This is done by the following estimate: let  $\vec{w}$  and  $\vec{c}$  be two unit vectors, and let  $R_N$  be the rotation by  $\frac{\pi}{N}$  around the axis orthogonal to  $(\vec{w}, \vec{c})$ ; then it holds  $\frac{1}{N} \sum_{k=0}^{N-1} |(R_N^k \vec{c}) \cdot \vec{w}| \ge \frac{1}{N} \cot \frac{\pi}{2N} \equiv u_N$  [12]. Replacing the full average by this discrete average in the otherwise unchanged proofs [8,10], we obtain the following family of inequalities:

$$\begin{aligned} |E_1^N(\vec{a}_1,\varphi) + E_1^N(\vec{a}_1,0)| + |E_2^N(\vec{a}_2,\varphi) + E_2^N(\vec{a}_2,0)| \\ &\equiv L_N(\vec{a}_1,\vec{a}_2,\varphi) \le 4 - 2u_N \left| \sin\frac{\varphi}{2} \right| \end{aligned} \tag{7}$$

where

$$E_{j}^{N}(\vec{a}_{j},\theta) = \frac{1}{N} \sum_{k=0}^{N-1} C(\vec{a}_{j}^{k},\vec{b}_{j}^{k})$$
(8)

with  $\vec{b}_j = (\cos\theta)\vec{a}_j + (\sin\theta)\vec{a}_j^{\perp}$  and the notation  $\vec{c}^k = (R_{N,j})^k \vec{c}$  (the  $\frac{\pi}{N}$ -rotation is along  $\vec{n}_j$ ). This defines 2N and 4N settings for A and B, respectively.

For a pure singlet state, the quantum mechanical prediction for  $L_N(\vec{a}_1, \vec{a}_2, \varphi)$  is

$$L_{\Psi^{-}}(\varphi) = 2(1 + \cos\varphi) \tag{9}$$

independent of *N* and of the choice of  $\vec{a}_1$ ,  $\vec{a}_2$  since the state is rotationally invariant. The inequality for N = 1 cannot be violated because  $u_1 = 0$  [13]. Already for N = 2, however, quantum physics violates the inequality; for  $N \to \infty$ ,  $u_N \to \frac{2}{\pi}$ , and one recovers the inequality derived in Ref. [8]. The suitable range of difference angles  $\varphi$  for probing a violation of the inequalities (7) can be identified from Fig. 1. The largest violation for an ideal singlet state would occur for  $|\sin\frac{\varphi}{2}| = \frac{u_N}{4}$ , i.e., at  $\varphi = 14.4^\circ$  for N = 2, increasing with N up to  $\varphi = 18.3^\circ$  for  $N \to \infty$ .

Experiment.—The experiment has to focus on different issues than when usual Bell inequalities are tested. On one hand, here there is no concern about spacelike separation, as the model under test is based on NLV. On the other hand, while the violation of a Bell inequality rules out LV models irrespective of which settings are ultimately used, the inequalities (7) require a precise control of the measurement settings [14]. We begin with a traditional parametric down conversion source [16] for polarization-entangled photon pairs with optimized collection geometry in single mode optical fibers [17] (Fig. 2). Light from a continuous-wave Ar-ion laser at 351 nm is pumping a 2 mm thick bariumbeta-borate crystal, cut for type-II parametric down conversion to degenerate wavelengths of 702 nm with a Gaussian spectral distribution of 5 nm (FWHM). We chose a pump power of about 40 mW to ensure both single frequency operation of the pump laser and to avoid saturation effects in the photodetectors. Collection of downconverted light into single mode optical fibers ensures a reasonably high polarization entanglement to begin with. In this configuration, we observed visibilities of polarization correlations of >98% both in the horizontal/vertical (HV) and  $\pm 45^{\circ}$  linear basis for polarizing filters located before the fibers. In order to avoid a modulation of the collection efficiency due to wedge errors in the wave plates, we placed subsequent polarization analyzing elements behind the fiber.

The projective polarization measurements for the different settings of the two observers were carried out using quarter wave plates, rotated by motorized stages by respective angles  $\alpha_{1,2}$ , and absorptive polarization filters rotated by angles  $\beta_{1,2}$  in a similar way with an accuracy of 0.1 °. This combination allows us to project on arbitrary elliptical polarization states. Finally, photodetection was done with passively quenched silicon avalanche diodes,



FIG. 1 (color online). Dependency of the combined correlation parameters  $L(\varphi)$  as a function of the separation angle  $\varphi$  for the quantum mechanical prediction for a pure singlet state, and bounds for nonlocal variable models assuming an averaging over various numbers of directions *N*.



FIG. 2 (color online). Experimental setup. Polarizationentangled photon pairs are generated in Barium-beta-borate (BBO) by parametric down conversion of light from an Ar ion pump laser (PL). After walk-off compensation ( $\lambda/2$ , CC), downconverted light is collected behind interference filters (IF) into birefringence-compensated (FPC) single mode optical fibers (SMF). Polarization measurements are carried out with a combination of a quarter wave plate ( $\lambda/4$ ) and polarization filters (PF) in front of photon counting detectors  $D_{1,2}$ . The measurement basis for each arm (1,2) is chosen by rotation of the wave plate and polarizing filter by angles  $\alpha_{1,2}$ ,  $\beta_{1,2}$  accordingly.

and photon pairs originating from a down conversion process were identified by coincidence detection. The compensator crystals (CC) and fiber birefringence compensation (FPC) were adjusted such that we were able to detect photon pairs in a singlet state.

After birefringence compensation of the optical fibers, we observed the corresponding polarization correlations between both arms with a visibility of 99.5  $\pm$  0.2% in the HV basis, 99.0  $\pm$  0.2% in the  $\pm$ 45° linear basis, and 98.2  $\pm$  0.2% in the circular polarization basis. Typical count rates were 10 100 s<sup>-1</sup> and 8000 s<sup>-1</sup> for single events in both arms, and about 930 s<sup>-1</sup> for coincidences for orthogonal polarizer positions. We measured an accidental coincidence rate using a delayed detector signal of 0.41  $\pm$  0.07 s<sup>-1</sup>, corresponding to a time window of 5 ns.

The two orthogonal planes we used in the Poincaré sphere included all the linear polarizations for one, and HV linear and circular polarizations for the other. That way, we intended to take advantage of the better polarization correlations in the "natural" basis HV for the down conversion crystal. Each of the 4*N* correlation coefficients  $C(\vec{a}, \vec{b})$  in (7) and (8) was obtained from four settings of the polarization filters via

$$C(\vec{a}, \vec{b}) = \frac{n_{\vec{a}, \vec{b}} + n_{-\vec{a}, -\vec{b}} - n_{-\vec{a}, \vec{b}} - n_{\vec{a}, -\vec{b}}}{n_{\vec{a}, \vec{b}} + n_{-\vec{a}, -\vec{b}} + n_{-\vec{a}, \vec{b}} + n_{\vec{a}, -\vec{b}}}$$
(10)

from the four coincident counts  $n_{\pm \vec{a},\pm \vec{b}}$  obtained for a fixed integration time of T = 4 s each. For N = 2, 3, and 4, we carried out the full generic set of 8, 12, and 16 setting

groups, respectively, with each  $E_j^N(0)$  containing a HV analyzer setting.

The measured values of *L* corresponding to inequalities for N = 2, 3, and 4 are shown in Fig. 3, together with the corresponding bounds (7) and the quantum expectation for a pure singlet state (9). The corresponding standard deviations in the results were obtained through usual error propagation assuming Poissonian counting statistics and independent fluctuations on subsequent settings. For N =2, we already observe a clear violation of the NLV bound; the largest violation was obtained for N = 4 with about 17 standard deviations above the NLV bound. As expected, the experimental violation increases with growing number of averaging settings *N*. Selected combinations of  $(N, \varphi)$ violating NLV bounds are summarized in Table I.

Our results are well-described assuming residual colored noise in the singlet state preparation [18]. In the inset of Fig. 3, we check that  $L_{expt}(\varphi) = L_{expt}(-\varphi)$  as it should. If this symmetry would be broken, e.g., by imprecise alignment of the polarizers, the violation may be overestimated, whence the importance of consistency checks.



FIG. 3 (color online). Experimental results for the observed correlation parameters  $L_N$  (dots), the quantum mechanical prediction for a pure singlet state (curved lines, dashed lines), and the bounds for the nonlocal variable models (almost straight lines). In all cases, our experiment exceeds the NLV bounds for appropriate difference angles  $\varphi$ .

TABLE I. Selected values of L violating the NLV bounds  $L_{\text{NLV}}$  for different averaging numbers N.

Ν	arphi	$L_{\rm NLV}$	$L_{ m expt} \pm \sigma$	$L_{\rm expt} - L_{\rm NLV}$
2	12.5°	3.8911	$3.9127 \pm 0.0033$	$6.45\sigma$
2	15°	3.8695	$3.8970 \pm 0.0036$	$7.59\sigma$
2	17.5°	3.8479	$3.8638 \pm 0.0042$	$3.83\sigma$
3	12.5°	3.8743	$3.9140 \pm 0.0027$	$14.77\sigma$
3	15°	3.8493	$3.8930 \pm 0.0030$	$14.58\sigma$
3	17.5°	3.8243	$3.8608 \pm 0.0034$	$10.67\sigma$
3	20°	3.7995	$3.8400 \pm 0.0036$	$11.15\sigma$
4	12.5°	3.8686	$3.9091 \pm 0.0024$	$17.01\sigma$
4	15°	3.8424	$3.8870 \pm 0.0026$	$16.84\sigma$
4	17.5°	3.8164	$3.8656 \pm 0.0029$	$17.11\sigma$

Overview and perspectives. -Let us now set this work in a broader context. Assuming that the various loopholes in experimental violations of Bell's inequality are of technical and not of fundamental nature, any mechanism that reproduces quantum correlations must be nonlocal. For instance, a possible assumption is that the source produces independent particles, which later exchange some kind of "communication" (which cannot be used to send classical information). Because this communication would need to travel faster than light, its speed would be frame dependent. The model could have a preferred frame ("quantum ether"), in which case signaling could be defined consistently [19]; or a frame defined by the measuring devices, in which case the model would depart from quantum predictions when the devices are set in relative motion [4,5]. There are also NLV models that do reproduce quantum predictions exactly. Explicit examples are Bohmian Mechanics [20] and, for the case of two qubits, the Toner-Bacon model [21]. Both are deterministic. Now, in Bohmian mechanics, if the first particle to be measured is A, then assumption (2) can be satisfied, but assumption (3)cannot. This remark sheds clearer light on the Leggett model, where both assumptions are enforced: the outcomes of particle B are required to reproduce the expected local statistics, but also to take nonlocal information into account to generate the correlations.

As a conclusion, it must be said that the broad goal sketched in the introduction, namely, to pinpoint the essence of quantum physics, has not been reached yet. However, Leggett's model and its conclusive experimental falsification reported here have added a new piece of information towards this goal.

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*Note added.*—Paterek *et al.* have independently worked on the same line of thought, and have presented data that violate the N = 2 inequality [15].

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- [11] This step is taken after (27) in [8], before (8) in [10]. The derivation of the original inequalities goes through the same step between (3.9) and (3.10) in [6].
- [12] Proof: let  $\tilde{\xi}$  be the angle between  $\vec{w}$  and  $\vec{c}$ , and  $\xi = (\tilde{\xi} \frac{\pi}{2}) \mod \frac{\pi}{N}$ , such that  $\xi \in [0, \frac{\pi}{N}]$ : then it holds  $\sum_{k=0}^{N-1} |(R_N^k \vec{c}) \cdot \vec{w}| = \sum_{k=0}^{N-1} |\cos(\tilde{\xi} + \frac{k\pi}{N})| = \sum_{k=0}^{N-1} \sin(\xi + \frac{k\pi}{N}) = \sin\xi + Nu_N \cos\xi \ge Nu_N$  as announced.
- [13] Actually, the data measured on a singlet state for N = 1, as in [7], can be reproduced by the explicit NLV Leggetttype model presented in [8]. Indeed, the validity condition for that NLV model is that there exists unit vectors  $\vec{u}, \vec{v}$  in the Poincaré sphere, such that, for all pairs of observables  $\vec{a}, \vec{b}$  measured in the experiment, one has  $|\vec{a} \cdot \vec{b} \pm \vec{u} \cdot \vec{a}| \le$  $1 \mp \vec{v} \cdot \vec{b}$  (Eq. (10) of [8]) or, equivalently,  $|\vec{a} \cdot \vec{b} \pm \vec{v} \cdot \vec{b}| \le$  $1 \mp \vec{u} \cdot \vec{a}$ . Now, for the case N = 1, one would measure four sets of observables  $\vec{a}_j, \vec{b}_j = (\cos\theta)\vec{a}_j + (\sin\theta)\vec{a}_j^{\perp}$  in planes j = 1, 2 and for  $\theta = 0$ ,  $\varphi$ . Then for  $\vec{u} = -\vec{v}$  orthogonal to both  $\vec{a}_1^{\perp}$  and  $\vec{a}_2^{\perp}$  and whatever  $\theta$ , one has  $|\vec{a}_j \cdot \vec{b}_j \pm \vec{v} \cdot \vec{b}_j| = |\cos\theta \mp \vec{u} \cdot [(\cos\theta)\vec{a}_j + (\sin\theta)\vec{a}_j^{\perp}]| = |\cos\theta(1 \mp \vec{u} \cdot \vec{a}_j)| \le 1 \mp \vec{u} \cdot \vec{a}_j$  as required.
- [14] In our experiment, as well as in those performed in Vienna [7,15], the detection loophole is obviously still open, and there is a memory loophole, since data have been collected in a sequential way.
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