Strongly Resonant p-Wave Superfluids

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We study theoretically a dilute gas of identical fermions interacting via a p-wave resonance. We show that, depending on the microscopic physics, there are two distinct regimes of p-wave resonant superfluids, which we term "weak" and "strong." Although expected naively to form a paired superfluid, a strongly resonant p-wave superfluid is in fact unstable toward the formation of a gas of fermionic trimers. We examine this instability and estimate the lifetime of the p-wave molecules due to the collisional relaxation into trimers. We discuss consequences for the experimental achievement of p-wave superfluids in both weakly and strongly resonant regimes.

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Recently there has been considerable interest in trying to create a p-wave resonant superfluid experimentally [1–3]. The Bardeen-Cooper-Schrieffer (BCS) and Bose-Einstein condensation (BEC) regimes for such superfluids are not just different aspects of the same phase, as they are for the s-wave resonant superfluids, but rather are different phases. Thus the tuning from the BCS to BEC regime involves a phase transition [4,5] or sometimes a sequence of phase transitions [6,7]. Such a transition can even be topological in some cases [6,8–10]. If the superfluid is confined to two dimensions, the BCS phase will be topological and will support vortices with non-Abelian excitations [6,9].

In this Letter we show that resonant p-wave superfluids must be classified as two distinct types, with weak or strong Feshbach resonances (to be defined precisely later). The existing mean field theory of p-wave superfluids, worked out in [6,7,11], applies only to the case of weak Feshbach resonances. However, as we shall establish below, the p-wave resonance used in ongoing experiments on ^{40}K [2,3] is a strong resonance. It is therefore important to determine the properties of strongly resonant p-wave superfluids.

The full theory of strong p-wave resonances is yet to be constructed. Here we investigate an effect first noticed by Y. Castin and collaborators [12]: in the regime of strong p-wave resonances the fermions form trimer states with angular momentum (spin) 1. Superficially similar to Efimov states [13], these trimers are quite unusual. They are very strongly bound, with a binding energy largely independent of detuning from the resonance, as long as the detuning is not too large (but dependent on the strength of the resonance). Correspondingly, their size is of the order of the closed-channel bound molecular state, far smaller than the average interparticle separation. We find the critical value of the resonance's strength at which the trimers first appear and calculate their binding energy as a function of the resonance strength.

Thus if a BEC of strongly resonant p-wave molecules is created, one of its main channels of decay will be by molecular inelastic collisions, with two molecules turning into one atom and one trimer. We estimate the molecular lifetime due to this process and compare this with experimental observations [3]. We discuss limitations on the achievement of p-wave superfluids in both weak and strong resonances arising from this and other inelastic decay processes.

The theory developed here can be used to investigate the true ground state of a strongly resonant p-wave condensate. This is likely to be a gas of fermionic spin 1 trimers (or possibly of larger composite particles).

We consider a p-wave resonantly coupled superfluid, whose Hamiltonian is given by [6,7,11,14,15]

$$H = \sum_{p} \frac{p^2}{2m} \hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}} + \sum_{\mathbf{q},\mu} \left(\epsilon_0 + \frac{q^2}{4m} \right) \hat{b}^\dagger_{\mu\mathbf{q}} \hat{b}_{\mu\mathbf{q}} + \sum_{\mathbf{p},\mathbf{q},\mu} \frac{g(|\mathbf{p}|)}{\sqrt{V}} (\hat{b}_{\mu\mathbf{q}} p_\mu \hat{a}^\dagger_{\frac{\mathbf{q}}{2}+\mathbf{p}} \hat{a}^\dagger_{\frac{\mathbf{q}}{2}-\mathbf{p}} + \text{H.c.}). \quad (1)$$

Here $\hat{a}^\dagger$, $\hat{a}$ are the creation and annihilation operators of a spinless fermion (atom) with mass m, and $\hat{b}^\dagger_\mu$, $\hat{b}_\mu$ are the creation and annihilation operators of a bosonic diatomic molecule of spin 1 (the 3D vector index μ represents the projection of spin). This superfluid is controlled by four parameters. The first two are the detuning ϵ_0 and the overall particle number N, an expectation value of the operator $\hat{N} = \sum_p \hat{a}^\dagger_p \hat{a}_p + 2\sum_{\mu,\mathbf{q}} \hat{b}^\dagger_{\mu\mathbf{q}} \hat{b}_{\mu\mathbf{q}}$, or equivalently the Fermi energy $\epsilon_F = (6\pi^2\hbar^3 N/V)^{2/3}/(2m)$. The other two are contained in the coupling constant $g(|\mathbf{p}|)$. The physical origin of the dependence of g on $|\mathbf{p}|$ lies in the fact that the molecules have finite size, with g being proportional to the wave function of the molecule in the momentum space. This can be captured by choosing g to remain constant as long as $|\mathbf{p}| \ll \Lambda$ (which we denote simply by g) and quickly drop to zero if $|\mathbf{p}| \gg \Lambda$. Here

$R_e \sim \hbar/\Lambda$ is the physical (closed-channel) size of the molecules. In this Letter we take $g(|\mathbf{p}|) = g\Theta(\Lambda - p)$ (Θ is equal to 1 or 0 depending on whether its argument is positive or negative). We believe the specific choice of the functional dependence of g on p does not significantly affect our results, with arguments to that end given below.

Two dimensionless parameters can be constructed out of g, ϵ_F, and Λ, namely $\gamma = m^{5/2}g^2\sqrt{\epsilon_F}/\hbar^3$ and $c_2 = m^2g^2\Lambda/(3\pi^2\hbar^3)$. In order to observe universal (short distance physics independent) behavior, the interparticle separation ($\sim\hbar/\sqrt{m\epsilon_F}$) must be kept much bigger than R_e; thus $\gamma \ll c_2$. γ and c_2 control the perturbative expansion of (1) in powers of the coupling g. It is customary, when analyzing Eq. (1), to apply a mean field approximation whose validity is based on the smallness of g. Strictly speaking, both γ and c_2 must be small in order for the mean field approximation employed in the original publications investigating Eq. (1) [6,7] to be valid. γ depends on the interparticle separation and can be made small simply by reducing the particle density. c_2, however, depends solely on the physics of the Feshbach resonance which led to Eq. (1); its value is fixed by the atomic type and Feshbach resonance involved, so it cannot be continuously controlled.

One terms the superfluids with $\gamma \ll 1$ as those with narrow Feshbach resonances, while the ones with $\gamma \gg 1$ are the broad Feshbach resonance superfluids [6,11,16–19]. Likewise, we will term the $c_2 \gg 1$ resonances as the strong p-wave Feshbach resonances, while those with $c_2 \ll 1$ are weak resonances. The experimentally observed p-wave resonances are typically narrow. The analysis carried out in Ref. [11] showed that the p-wave Feshbach resonance in ^{40}K used in Refs. [2,3] was strong. The derivation relied on the scattering amplitude of two atoms calculated in Refs. [6,11]: $f(k) = k^2/(-1/v + k_0k^2/2 - ik^3)$, where $v = -mg^2/[6\pi\hbar\omega_0(1 + c_2)]$ is the effective volume, controlled by the physical detuning ω_0 (related to ϵ_0 by $\omega_0 = [\epsilon_0 - mg^2\Lambda^3/(9\pi^2\hbar^2)]/[1 + c_2]$), and $k_0 = -4\Lambda(1 + c_2)/(\pi c_2)$ is a parameter similar to the effective range of s-wave scattering (having, however, the dimensions of inverse length). Once k_0 and Λ are known (numerically or experimentally), c_2 can also be found.

The narrow and weak p-wave resonances have been thoroughly investigated in prior publications. It is therefore imperative to consider the narrow and strong resonances. The main idea behind the analysis is based on the fact that fluctuational corrections to the mean field come from two distinct regions in momentum space, p of order $\hbar/l$, where l is interparticle spacing, and p of order Λ. The former capture the many-body physics of Eq. (1) and are small as long as γ is small. The latter come from high momenta and energies at which no real particles propagate. Thus this contribution, controlled by c_2, is essentially few-body, equivalent to solving a few-body Schrödinger equation.

We now turn our attention to the physical consequences of strong resonances. The main consequence is the existence of a bound state of three atoms when c_2 exceeds a certain threshold. To show this, we calculate the scattering amplitude of one atom and one molecule. This is given by a sequence of diagrams depicted in Fig. 1. These diagrams are identical to the ones studied in the context of the s-wave BCS-BEC crossover [20,21].

The atoms propagate with the free propagator $G(\mathbf{p}, \omega) = 1/[\omega - p^2/(2m) + i0]$, while to find the molecular propagator one needs to calculate its self-energy :

$$D_{\mu\nu}(\mathbf{q}, \omega) = \delta_{\mu\nu}\Bigg/\Bigg[(1 + c_2)\left(\omega - \frac{q^2}{4m} - \omega_0 + i0\right) + c_2\frac{\sqrt{m}}{\Lambda}\left(\frac{q^2}{4m} - \omega - i0\right)^{3/2} \times \arctan\left(\frac{\Lambda}{\sqrt{q^2/4 - m\omega}}\right)\Bigg]. \quad (2)$$

(In these and subsequent expressions we set $\hbar = 1$ for clarity.) Each loop in the diagrams in Fig. 1 is linearly divergent. It is this divergence, occurring at momenta $p \sim \Lambda$ and controlled by c_2, which we would like to capture. To do so, we study the atom-dimer scattering problem with the following kinematics: a boson of spin μ and 4-momentum $(\mathbf{0}, \kappa + E_3)$ scatters off a fermion with 4-momentum $(\mathbf{0}, 0)$. The outgoing particles are a boson with spin ν and 4-momentum $(\mathbf{q}, q_0 + \kappa + E_3)$ and a fermion with $(-\mathbf{q}, -q_0)$. Here $\kappa(\omega_0)$ is an implicit function of the detuning such that the bosonic propagator $D(\mathbf{q}, q_0 + \kappa)$ has a pole as $\mathbf{q}, q_0 \to 0$. $E_3 \le 0$ is the energy at which we are looking for a bound state. The scattering T matrix has the following general form:

$$T_{\mu\nu}(\mathbf{p}, p_0) = T_1(p, p_0)\delta_{\mu\nu} + T_2(p, p_0)p_\mu p_\nu/p^2 \quad (3)$$

and the scattering length a_{bf} is related to $T_1(0, 0)$ (evaluated at $E_3 = 0$) as $a_{\mathrm{bf}} = \frac{m}{3\pi}T_1(0, 0)$.

The integral equation for the T matrix is derived analogously to the s-wave problem [20,21] and is

$$T_{\mu\nu}(\mathbf{p}, p_0) = -\frac{2}{1 + c_2}G(\mathbf{p}, p_0 + \kappa + E_3)p_\mu p_\nu g(|\mathbf{p}|)g(|\mathbf{p}|/2) - 4i\int\frac{d^4q}{(2\pi)^4}T_{\mu\alpha}(\mathbf{q}, q_0)D(\mathbf{q}, q_0 + \kappa + E_3)G(-\mathbf{q}, -q_0) \times G(\mathbf{p} + \mathbf{q}, p_0 + q_0 + \kappa + E_3)(p + q/2)_\alpha(q + p/2)_\nu g(|\mathbf{p} + \mathbf{q}/2|)g(|\mathbf{q} + \mathbf{p}/2|). \quad (4)$$

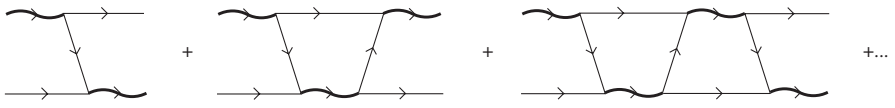

FIG. 1. The diagrams whose sum gives the scattering amplitude between an atom and a molecule.

The factor $1 + c_2$ is the inverse residue of the bosonic propagator. $T_{\mu\nu}(\mathbf{q}, q_0)$ is analytic in the upper halfplane of q_0 and thus we may integrate out q_0, setting $q_0 \to -q^2/2m$. To solve the integral equation we then let $p_0 \to -p^2/2m$. For simplicity we define $T_i(p, -p^2/2) \equiv T_i(p)$. Measuring momenta in units of the cutoff, energies in units of Λ^2/m, and the T matrix itself in units of $1/(m\Lambda)$ we find the integral equation

$$T_j(p) = 6\pi^2 \frac{c_2}{1+c_2} \frac{p^2}{p^2 - \kappa - E_3} \Theta(1-p)\delta_{2j} - 3c_2 \int_0^2 q^2 dq D(q, -q^2/2 + \kappa + E_3) a_{ji}(p, q) T_i(q). \quad (5)$$

The coefficients $a_{ji}(p, q)$ are given by an integration over directions of $\mathbf{q}$:

$$a_{ji}(p, q) = \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \frac{\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}_{jk} \left(\frac{\delta_{\mu\nu}}{\frac{p_\mu p_\nu}{p^2}}\right)_k \left(\frac{\delta_{\mu\nu}}{\frac{q_\mu q_\alpha}{q^2}}\right)_i}{\kappa + E_3 - p^2 - q^2 - \mathbf{p}\cdot\mathbf{q} + i0} (p + q/2)_\alpha (q + p/2)_\nu g(|\mathbf{p} + \mathbf{q}/2|) g(|\mathbf{q} + \mathbf{p}/2|)/g^2. \quad (6)$$

The scattering length a_{bf} is found by solving Eq. (5) at $E_3 = 0$. The binding energy of the trimer corresponds to a pole in the T matrix and thus to a solution of the homogeneous integral equation at a specific value of E_3. Figure 2(a) shows how the scattering length is negative for a weak Feshbach resonance, becoming more negative and diverging at $c_2 \approx 3.3$. This is the strength of the resonance at which the bound trimer appears, as illustrated in Fig. 2(b). As $c_2 \to \infty$ the scattering length saturates at $a_{\mathrm{bf}} \approx 1.9/\Lambda$ and the binding energy at $E_3 \approx -0.11\Lambda^2/m$.

We note that at large c_2 the large binding energy of the trimer indicates that the physics at large momenta, sensitive to the specific choice of $g(|\mathbf{p}|)$, plays a role in its formation. Yet for the values of c_2 close to the threshold of the appearance of the trimer, its binding energy is small and thus it is insensitive to that choice [although the critical value of c_2, being defined in terms of R_e, itself depends on the choice of $g(|\mathbf{p}|)$]. As c_2 is increased further, we find it extremely unlikely that trimers would disappear due to some particular choice of g or due to some additional short distance physics not taken into account by Eq. (1). Indeed, to disappear, the trimer would need first to lower its energy, going back to the regime where the short distance physics is not important. Additionally, we have checked that several different choices of g, representing realistic molecular wave functions, give the same qualitative result.

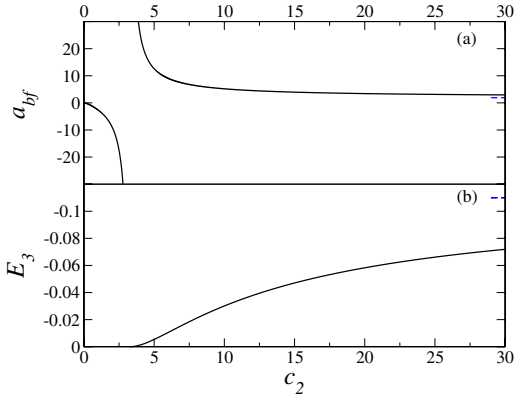

FIG. 2 (color online). (a) Scattering length a_{bf} in units of $\hbar/\Lambda$ and (b) binding energy E_3 of the trimer in units of Λ^2/m, both as functions of c_2. Here, detuning has been set to zero. The large c_2 limit is indicated.

The existence of the bound trimer state for large c_2 raises the possibility of an inelastic decay channel in which two dimers collide to leave a trimer and an unbound atom (with large relative velocity). (Henceforth we use the term "dimer" to refer to a molecule of two atoms, to distinguish this clearly from a trimer.) In a nondegenerate gas of dimers, these inelastic losses will cause the density of dimers n_d to decay as

$$\frac{dn_d}{dt} = -\alpha_{\mathrm{dd}} n_d^2, \quad (7)$$

with $\alpha_{\mathrm{dd}} = 2\frac{\hbar}{m}\langle k_i \sigma_{\mathrm{in}}(k_i)\rangle$ where the average is over the relative momenta of the incident dimers, k_i, and $\sigma_{\mathrm{in}}(k_i) = \int |f_{\mathrm{in}}|^2 d\Omega \frac{k_f}{k_i}$, with f_{in} the inelastic scattering amplitude into a final momentum k_f. Arguments similar to the ones presented above for dimer-atom scattering show that $|f_{\mathrm{in}}|^2 \sim R_e^2$. Thus, for large c_2, such that the trimer binding energy is $-E_3 \sim \hbar^2/(mR_e^2)$ and is large compared to the incident kinetic energy, one finds

$$\alpha_{\mathrm{dd}} \sim \frac{\hbar}{m} R_e. \quad (8)$$

It is instructive to compare this result with the inelastic decay constants into deep bound states for s-wave dimers, formed from (two-component) fermions or from bosons with s-wave scattering length a. Close to the s-wave resonance, $a \gg R_e$, and the decay constant (8) is much smaller than that expected for bosons, $\alpha_{\mathrm{dd}}^{s\text{-boson}} \sim \hbar a/m$, but is larger than that for s-wave dimers of fermions, $\alpha_{\mathrm{dd}}^{s\text{-fermion}} \sim \frac{\hbar R_e}{m}(R_e/a)^{2.55}$ [22]. The suppressed decay of s-wave dimers of fermions is explained in Ref. [22] as an effect of the Pauli principle, reducing the probability to find three atoms within a length scale R_e. In a p-wave dimer the two atoms have a probability of order unity to be inside the centrifugal barrier, at a separation of order R_e. Taking this feature of the p-wave dimers into account, simple estimates lead to $\alpha_{\mathrm{dd}} \sim \hbar R_e/m$ for decay into trimers, consistent with the result (8) from the T matrix calculation. In addition to this channel, there are inelastic channels—active for both weak and strong resonances—involving decay into deep dimer states. Applying the same simple estimates, one finds that the inelastic decay constants for

dimer-dimer and dimer-atom scattering are also $\alpha_{dd} \sim \alpha_{da} \sim \hbar R_e/m$.

In recent experimental work [3] a gas of p-wave Feshbach dimers was created in ^{40}K. Unfortunately the lifetime of the dimers was observed to be quite short, about 2 ms. While ^{40}K can suffer losses through dipolar relaxation (an effect expected to be absent for p-wave resonances in other fermionic systems, for example ^{6}Li), Ref. [3] found that the lifetime was shorter than that predicted for dipolar relaxation alone. Additional losses could arise from inelastic collisions of the dimers. This mechanism would imply a density dependence of the decay rate; this dependence has not, as yet, been established experimentally.

Within the above considerations, we expect the decay rate of dimers via relaxation into deep trimers or dimers under inelastic collisions (with other dimers or with unbound atoms) to be of order $\Gamma_{in} \sim \frac{\hbar R_e}{m} n$, where n is the density of atoms or dimers with which a given dimer can collide. Taking $n \simeq 7 \times 10^{12}$ cm^{-3} (the atomic density in the experiments of Ref. [3]) we find $\Gamma_{in} \sim 10$ Hz. This estimate is more than 1 order of magnitude smaller than the additional decay rate required to account for the observations of Ref. [3]. However, we note that the prefactor to the estimate is uncertain. In view of this uncertainty, and in view of the lack of clear evidence of a density dependence in the experiment, it remains an open issue whether the dimer lifetime in Ref. [3] is limited by inelastic collisions.

Our analysis has important consequences for the possibilities to achieve superfluid phases close to a p-wave resonance. On the BEC side of the resonance, our calculations show that the elastic dimer-dimer scattering amplitude is $f_{el} \sim R_e$. Consequently, the elastic scattering rate is of order $\Gamma_{el} \sim \frac{\hbar R_e}{m} n_d (k_i R_e)$, which is typically much smaller than the inelastic decay rate, $\Gamma_{in} \sim \frac{\hbar R_e}{m} n_d$. (For a BEC of dimers, k_i is small compared to the inverse particle spacing, $1/l$, so $k_i R_e \lesssim R_e/l \ll 1$.) It is therefore unlikely that a BEC of dimers can undergo sufficient elastic collisions to thermalize before inelastic losses deplete the gas. On the other hand, on the BCS side of the resonance, thermalization can proceed at a much faster rate, and will be limited by the rate of hybridization of the dimers with the unbound atoms (this is the rate at which pairs of atoms can exchange their relative momentum). Assuming the densities of dimers and atoms to be comparable, $n_d \sim n_a \equiv n$, one finds that the hybridization rate, as set by the width of the resonance, is $\Gamma_{hyb} \sim \frac{\gamma}{1+c_2} \frac{\epsilon_F}{\hbar}$, which leads to $\Gamma_{hyb}/\Gamma_{in} \sim \frac{c_2}{1+c_2}$. Thus, provided the resonance is not very weak (c_2 very small), the rate of hybridization is parametrically the same as Γ_{in}, and the system may thermalize before inelastic losses deplete the gas. Thus our results show that it is on the BCS side of a strong resonance that one has the best opportunity to attain a thermalized p-wave superfluid phase. Finally, we note that the limitations we have described in this paragraph, arising from decay into deep bound states, could be eliminated in an "optical Feshbach" scheme in which the particles are coupled to a deep closed-channel molecule. In this case, it is important that the resonance be sufficiently weak in order also to eliminate inelastic decay processes into the trimer states that always exist for strong resonances.

Finally, we remark that after this Letter became available as a preprint, a paper [23] appeared where similar conclusions were reached.

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