Time Dilation of a Bound Half-Fluxon Pair in a Long Josephson Junction with a Ferromagnetic Insulator

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The fluxon dynamics in a long Josephson junction with a ferromagnetic insulating layer is investigated. It is found that the Josephson phase obeys a double sine-Gordon equation involving a bound π fluxon solution, and the internal oscillations of the bound pair acting as a clock exhibit Lorentz reductions in their frequencies regarded as a relativistic effect in the time domain, i.e., time dilation. This is the complement to the Lorentz contraction of fluxons with no clock. A possible observation scheme is also discussed.

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The Lorentz transformation in space-time plays a key role in the special theory of relativity, resulting in counterintuitive phenomena far from our everyday experience such as length contraction when an object is travelling close to the speed of light. The Lorentz contraction is difficult to confirm experimentally for energetic reasons related to macroscopic objects;, i.e., macroscopic objects cannot be accelerated to a sufficiently high velocity for the Lorentz contraction to be observed. Only a few examples have been demonstrated in specific systems using Earth's rotation [1–3]. An elementary excitation with a quantum unit of magnetic flux in long Josephson junctions, known as a fluxon, obeys sine-Gordon equations involving the Lorentz invariance. The length contraction of fluxons in Josephson junctions was shown in experiments by recording the voltage pulse created by the fluxon motion [4] and by imaging the contraction of the fluxon-antifluxon collision region in an annular Josephson junction with increasing fluxon velocity by employing a low-temperature scanning electron microscope [5].

However, a single fluxon cannot exhibit another relativistic effect in the time domain, namely, time dilation where moving clocks run slowly, because it does not have its own clock. As with any vibrating system, an internal degree of freedom in the system can serve as a moving clock counting time intervals. One candidate is a bound pair consisting of a fluxon and an antifluxon, and known as a breather. The breather moves along a Josephson transmission line with the internal oscillation in which the fluxon and the antifluxon are exchanging their positions. This internal oscillation acts as a clock. However, the breather is inadequate for testing this effect because it is too fragile as regards external perturbations [6]. In particular, the breather disappears even in a small dissipation because it is topologically equivalent to the vacuum. This feature makes it extremely difficult to observe the effect of time dilation. Another possibility is the π-π kink pair solution in a double sine-Gordon equation (DSGE) in long isolated Josephson junctions [7]. This kink pair has a stable internal oscillation below the lower edge of the continuum phonon band [8]. Although this internal oscillation actually attenuates in the presence of dissipation, it is possible to excite the oscillation again on demand because the kink pair itself with a topological twist cannot decay into vacuum. However, it is difficult to observe π-π kink oscillations in isolated systems with a noncontact measurement. In this Letter, we propose a new type of Josephson transmission line that obeys a DSGE, namely, a long Josephson junction with a ferromagnetic insulator, in order to explore the time dilation of a bound pair of half fluxons through their internal oscillations. We also discuss an experimental scheme for observing time dilation in such a system.

In the past decade, considerable attention has been directed towards Josephson π junctions characterized by a minimum Josephson coupling energy at a phase difference of π, in relation to the physics of a Cooper pair with a finite momentum. The crossover between 0 and π junction states was demonstrated in superconductor-ferromagnet-superconductor (SFS) junctions as a function of temperature [9] and barrier thickness [10]. In the vicinity of the crossover, it is conjectured that a second order component of Cooper-pair tunneling, i.e., $\sin(2\phi)$ with ϕ being the phase difference across the junction, becomes dominant in the current-phase relation [11–14]. This might significantly change the dynamics of the phase difference across the junction through the current-phase relation in a long Josephson junction.

Consider two identical superconductors separated by a thin ferromagnetic insulator with thickness a lying in the xy plane as shown in Fig. 1. The dimension w in the y

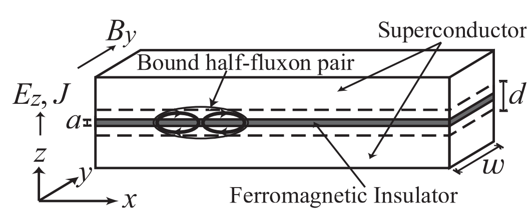

FIG. 1. Schematic diagram of a long Josephson junction with a ferromagnetic insulator.

direction is assumed to be much smaller than the Josephson penetration depth, and thus the junction is considered to be one-dimensional [5]. Starting as usual from Maxwell equations that takes account of the spatiotemporal dependence of the phase difference ϕ across the junction due to electric field E_z and magnetic field B_y through the relations $E_z = (\hbar/2ea)\partial_t\phi$ and $B_y = (\hbar/2ed)\partial_x\phi$, with $\partial_t = \partial/\partial t$ and $\partial_x = \partial/\partial x$, we find

$$(\hbar/2e\mu d)\partial_x^2\phi = J + (\hbar\epsilon/2ea)\partial_t^2\phi, \tag{1}$$

where ϵ is the dielectric constant of the ferromagnetic insulator. In contrast to a usual long Josephson junction, the current density in a long Josephson junction with a ferromagnetic insulator is now given as [11–14]

$$J = J_c(\lambda\sin\phi + \sin 2\phi), \tag{2}$$

where J_c is the amplitude of the current density for the second harmonic and λ is the ratio of the amplitude between the first and second harmonics. When $\lambda \simeq 0$, J_c becomes the Josephson critical current density. Substituting Eq. (2) into Eq. (1), we obtain

$$\partial_x^2\phi - \partial_t^2\phi = \lambda\sin\phi + \sin 2\phi, \tag{3}$$

where the coordinate x is normalized by $\sqrt{2}\lambda_J$ where $\lambda_J = \sqrt{\hbar/4e\mu dJ_c}$ is the Josephson penetration depth for $\lambda = 0$ and the time t is normalized by $\sqrt{2}\omega_J^{-1}$, where $\omega_J = \bar{c}/\lambda_J$ is the frequency of the Josephson plasma oscillation with $\bar{c} = \sqrt{a/d\epsilon\mu}$. The phase difference in an SFS hybrid junction thus obeys a DSGE, and constitutes one of the main results in this Letter.

A DSGE is not integrable and exact soliton solutions do not exist. However, it is possible to find a topologically stable 2π-kink solution for a range of parameter λ, i.e., $\lambda > 0$. The potential energy density is expressed as $U_J(\phi) = \lambda(1 - \cos\phi) + \frac{1}{2}(1 - \cos 2\phi)$. Here, the energy is normalized by $E_0 = 2\sqrt{2}\varepsilon_J w\lambda_J$, where $\varepsilon_J = \hbar J_c/4e$ is the specific Josephson coupling energy per unit area for $\lambda = 0$. Since this potential has minima at $\phi = 0 (\mathrm{mod}\ 2\pi)$ for $\lambda > 0$, there are 2π-kink solutions where the phase changes from 0 to $\pm 2\pi$ as x passes from $-\infty$ to ∞. The shape of the 2π kink is determined by the competition between the potential energy density $U_J(\phi)$ and the elastic energy density, $U_E(\phi) = 1/2(\partial_x\phi)^2$, which measures the rigidity of the Josephson phase difference.

For $0 < \lambda < 2$, the potential energy density U_J also has an extra local minimum at $\phi = \pi$, in addition to the $\phi = 0$ minima. This minimum leads to a new situation where the 2π kink splits into two separate π kinks owing to the gain of the Josephson energy U_J against the elastic energy U_E. The equilibrium separation between two π kinks is determined by the competition between the gain of U_J around $\phi = \pi$ and the loss of U_E by the additional change in the phase difference. With a moving π-π kink pair, these compete in time, resulting in a new type of dynamics. The π-π kink pair exhibits an internal oscillation related to the relative oscillations of two π kinks around the equilibrium separation [15,16]. Figure 2 shows a typical profile of phase difference ϕ during the π-π kink oscillation.

The eigenfrequency of the π-π kink oscillation can be estimated by using the collective coordinate method [16]. We assume the solution to DSGE in the form

$$\phi = 2\tan^{-1}e^{\Gamma[x+(R/2)]} + 2\tan^{-1}e^{\Gamma[x-(R/2)]}, \tag{4}$$

where $\Gamma = \sqrt{2+\lambda}$ is the slope of each π kink and R denotes the separation between two π kinks, which can be regarded as a collective coordinate that describes π-π kink oscillation. Substituting Eq. (4) into the Hamiltonian of the DSGE, the potential energy and the inertial mass for the collective coordinate R are obtained as

$$V(R) = 2\Gamma\left[1 + \frac{\Gamma R}{\sinh(\Gamma R)}\right] + \frac{4}{\Gamma}\left[1 - \frac{\Gamma R}{\sinh(\Gamma R)}\right] \times \coth^2(\Gamma R/2) + 2\lambda R\coth(\Gamma R/2), \tag{5}$$

$$M(R) = \Gamma\left(1 - \frac{\Gamma R}{\sinh\Gamma R}\right). \tag{6}$$

The equilibrium separation R_0 can be determined from the relation $\cosh^2(\Gamma R_0/2) = 1 + 2/\lambda$. Thus, the π-π kink oscillation can be regarded as the oscillation of a particle with a mass depending on its position around the equilibrium position R_0 on the anharmonic potential shown in Fig. 3. In a harmonic approximation we obtain the eigenfrequency $\omega_0(\lambda) = \sqrt{V''(R_0)/M(R_0)}$, where the prime denotes the differentiation with respect to R. This frequency of the π-π kink oscillation serves as a clock to measure the time.

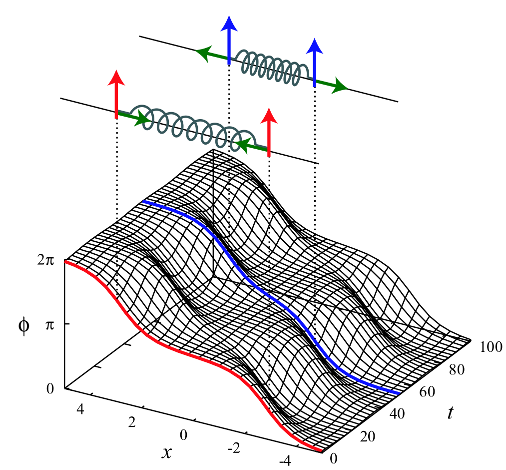

FIG. 2 (color online). Phase profile during the π-π kink oscillation. Vertical (red) arrows denote the positions of π kinks. Expansion and contraction of springs express the deviation from equilibrium separation between π kinks symbolically and horizontal (green) arrows denote the directions of restoring forces.

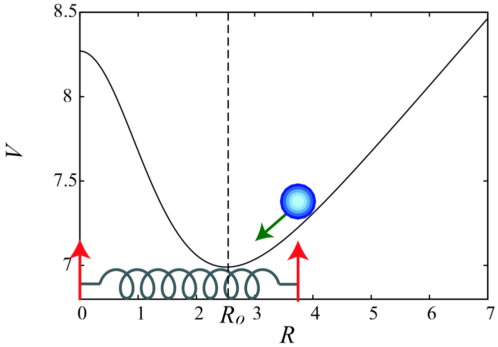

FIG. 3 (color online). Interaction potential between π kinks.

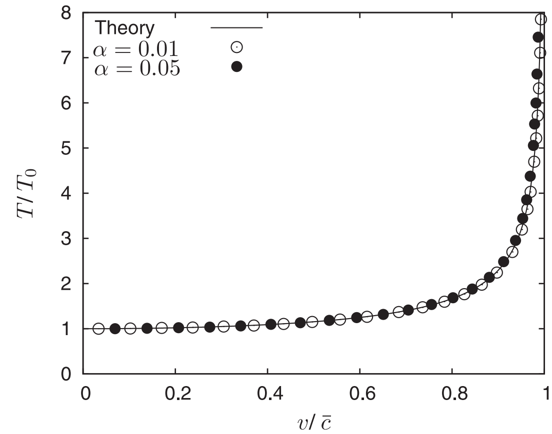

FIG. 4. Velocity dependence of the π-π kink oscillation period normalized by the period at $v = 0$ for $\lambda = 0.25$. The points are obtained by numerical simulations for different strengths of dissipation, while the solid curve is the theoretical expression of the time dilation, i.e., Eq. (7).

Since the DSGE is also Lorentz invariant, the period of the π-π kink oscillation should increase as the velocity of the pair increases as a consequence of the relativistic time delay

$$T/T_0 = \{1 - (v/\bar{c})^2\}^{-1/2}, \tag{7}$$

where T is the period of π-π kink oscillation at the finite velocity v, and T_0 is the period at $v = 0$. However, in an actual junction, dissipation always exists and destroys Lorentz invariance. Scott studied the effects of dissipation on the time dilation of a moving breather and showed that the frequency suffers Lorentz reduction, while the damping of oscillation is independent of velocity [17]. If this is also the case for π-π kink oscillation, a π-π kink pair can be used in testing the relativistic time dilation.

To confirm the time dilation of a traveling kink pair, we performed a numerical simulation by using a standard difference approximation with the perturbed DSGE,

$$\partial_t^2\phi - \partial_x^2\phi + \alpha\partial_t\phi + \lambda\sin\phi + \sin2\phi = I_b, \tag{8}$$

where $\alpha = 1/\omega_J R_s C$ is a damping coefficient where R_s is the shunt resistance and C is the capacitance of the junction. I_b is the direct bias current which keeps the velocity of the π-π kink pair.

Figure 4 shows the numerical results for T/T_0 as a function of the terminal velocity v of a pair, which is determined by the balance between damping (α) and driving (I_b). Here we take $\lambda = 0.25$ and assume that an initial bound pair is expressed by Eq. (4) with appropriate separations. The solid curve is simply the prediction obtained from the special theory of relativity, i.e., the relativistic time delay, described by Eq. (7). From this, we can see that our numerical results for different strengths of dissipation are in good agreement with the theoretical predictions. In addition, the values of T_0 obtained from a numerical simulation are about 9.44 for $\alpha = 0.01$ and 9.46 for $\alpha = 0.05$, which are close to the value of 9.24 estimated from $\omega_0(0.25)$. For larger α, we have seen the agreement with the theory in the lower velocity region where $\alpha T \lesssim 1$, which means underdamping. These results indicate that the frequency of the π-π kink oscillation can be considered to be almost independent of α. Thus, the π-π kink oscillation is a promising candidate for use in testing the relativistic time dilation in superconducting quantum circuits.

Now let us discuss a possible scheme for observing the effect of the time dilatation of a π-π kink oscillation in a long SFS junction. In general, it is difficult to monitor the real-time dynamics of a π-π kink oscillation directly, while it might be possible to determine the eigenfrequency of a propagating bound pair by using the resonance effect. The bound pair exhibits a forced oscillation under an external alternating current, leading to excitation of the π-π kink pair. In particular, resonance occurs when the frequency of the external current matches the frequency of the π-π kink oscillation. With resonance, the amplitude of the oscillation becomes large, and thus achieve a distinguishable value compared with the equilibrium width of the kink pair. It may be possible to observe the π-π kink pair by using an existing experimental technique such as low-temperature scanning electron microscopy [5], namely, by imaging the collision region between a π-kink pair and an anti-π-kink pair in an annular SFS Josephson junction. Thus, we can measure the frequency of the π-π kink oscillation by detecting the total width of a π-π kink pair as a function of the frequency of the applied oscillating current.

To study the resonance behavior, we performed a numerical simulation based on Eq. (8) including external alternating currents $I_{ex}\sin\omega t$ together with the direct bias current I_b, where I_{ex} and ω are the normalized amplitude and frequency, respectively. The parameters we use in the simulations are $\lambda = 0.25$, $I_{ex} = 0.15$ and $\alpha = 0.1$.

Figure 5 shows the amplitude of the oscillation of the separation between π kinks as a function of the normalized period of external alternating current $2\pi/\omega T_0$ with $T_0 = 9.54$ and the terminal velocity of a pair $v/\bar{c}$. The amplitude

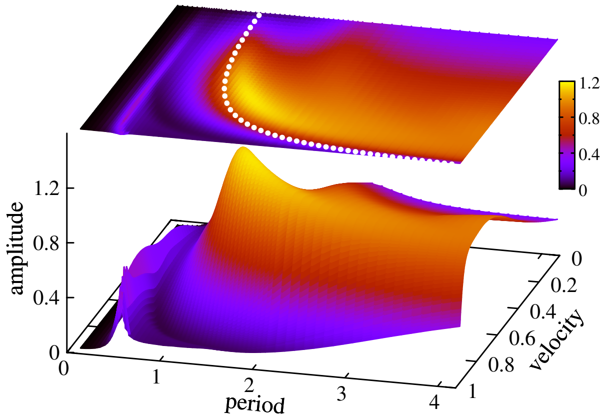

FIG. 5 (color online). Amplitude of the π-π kink oscillation as a function of the period of external alternating current and the velocity of the bound fluxon pair. The (white) dotted line denotes the relation of the relativistic time dilation Eq. (7).

is estimated from the half value of the difference between the maximum and minimum separation of the kink pair, and is normalized by the width of a moving π kink with velocity v, i.e., $\sqrt{2}\lambda_J\sqrt{1-(v/\bar{c})^2}/\Gamma$. We can easily find the resonance of the π-π kink oscillation as indicated by a (white) dotted line. This line shows the Lorentz dilation expressed by Eq. (7). In addition, there is another resonance with $1/2$ harmonic that has a period twice that of eigen oscillation as a result of a nonlinear effect, called nonlinear resonance [18]. As seen in Fig. 3, a nonlinear interaction is induced between π kinks when the amplitude of the π-π kink oscillation becomes large because of an increase in the external field amplitude. In contrast to a linear interaction yielding a harmonic resonance, the nonlinear interaction gives rise to a subharmonic resonance at a double eigenfrequency.The $1/2$-harmonic resonance corresponds to the cubic nonlinearity of the asymmetric potential shown in Fig. 3. There is also another resonance that is almost independent of the pair velocity around a period of 0.5. This resonance corresponds to a homogeneous Josephson plasma oscillation whose period $T_{\rm JP}$ is determined by the curvature of U_J around $\phi = 0$ minima, i.e., $T_{\rm JP}/T_0 = 2\pi/\sqrt{U_J''(0)}T_0 = 2\pi/\sqrt{\lambda+2}T_0 \simeq 0.44$, where the prime denotes the differentiation with respect to ϕ.

Finally, let us roughly estimate the parameters. In the vicinity of the 0-π crossover, Josephson critical current would be quite small, say, 2 orders of magnitude smaller than that of usual junction [12,14], and thus $\lambda_J \sim 100$ μm, $\omega_J \sim 10$ GHz and $\alpha \sim 0.1$. The amplitude of π-π kink oscillation at the resonance is of the order of 10–100 μm, which is within the range of spatial resolution of the experiment [5].

In summary, we have studied fluxon dynamics in a long Josephson junction with a ferromagnetic insulator. In such a system, the Josephson current-phase relation differs from that of a conventional junction, resulting in half-integer fluxons that obey a double sine-Gordon equation. The bound pair of the half-fluxons exhibits an internal oscillation that is unique to this system, and could be detected through this oscillation by using the resonance effect. This might provide evidence for the existence of a second harmonic component in the current-phase relation of an SFS hybrid junction. In addition, we demonstrated numerically the time dilation of the bound half-fluxon pair by detecting the Lorentz reduction in its frequency as a function of pair velocity. This is the complement to the Lorentz contraction. Moreover, a quantized π-π kink pair might also provide an application for Josephson-based quantum computers as a new type of *mobile* qubit using Josephson π states [19].

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