# Zero Sound Mode in Normal Liquid $^3$He

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Inelastic x-ray scattering has been utilized to study the elementary excitations of normal liquid $^3$He at the temperature $T = 1.10 \pm 0.05$ K and saturated vapor pressure in the wave vector range $0.15 \leq Q \leq 3.15$ Å$^{-1}$. The present data provide direct information on the zero-sound mode in the mesoscopic wave vector range where it was expected to decay into particle-hole excitations. The obtained results show no evidence of such a decay: the zero-sound mode remains well defined in the whole explored wave number range, thus witnessing a continuous transition of the atom dynamics from the collective to the single particle regime similarly to what is usually found in simple liquids.

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Liquid $^3$He is the prototype of an uncharged Fermi liquid with a Fermi momentum $k_F = 0.78$ Å$^{-1}$ and a Fermi temperature $T_F = \epsilon_F/k_B = 5$ K at saturated vapor pressure, where $\epsilon_F$ is the Fermi energy and $k_B$ is the Boltzmann constant [1]. The recent books by Glyde [2] and Dobbs [3] review a large body of results available on this system.

The peculiar quantum nature of liquid $^3$He has specific signatures on its acoustic properties. In particular, the acoustic response of liquid $^3$He in the long wavelength limit was first discussed by Landau in his celebrated theory of interacting Fermi liquids [4,5]. This theory—strictly valid only for small wave numbers $Q \ll k_F$, small energies $\hbar\omega \ll \epsilon_F$ as well as low temperatures $T \ll T_F$—predicted that two distinct sound regimes could be accessed depending on whether the sound wave frequency would be low or high as compared to the collision frequency between excited quasiparticles. Landau called the two corresponding acoustic modes first- and zero-sound modes (FSM and ZSM), respectively, and proposed a description for the temperature and frequency dependence of the attenuation of the sound mode in both regimes. These predictions were confirmed by ultrasound experiments [6] which were then pushed up to frequencies of $\approx 1$ GHz [7].

Starting from the early 1960s, techniques of many body theory have been employed to extend the Landau quasiparticle theory to the finite momentum and energy range [1–3]. The corresponding excitations have been experimentally observed using the inelastic neutron scattering (INS) technique despite the difficulties related to the large absorption cross section of $^3$He. In a series of pioneering experiments [8–13] the main features characterizing the INS spectra of liquid $^3$He were clarified and described in terms of the ZSM arising from the coherent cross section and of the spin wave mode arising from the incoherent cross section. For values of the exchanged momentum $Q \geq 1.3$ Å$^{-1}$ these two excitations overlap to the point that it is very difficult, if not impossible, to disentangle the two contributions. Below this $Q$ value, the ZSM and the spin wave mode were found to be almost temperature independent in the range from 15 mK to 1.2 K.

Higher quality INS data were obtained more recently exploiting the enhanced performances of neutron spectrometers and optimized sample geometries [14–17]. These experiments confirmed the temperature independence of the finite-momentum acoustic response of the system (up to 1.4 K) and explored in addition its dependence on density. A large effort was dedicated as well to the study of the low-energy incoherent peak, that led to a $Q$-dependent effective mass model for the $^3$He quasiparticles [18] that was then successfully tested in recent high-resolution INS experiments [17]. Within this model a very accurate calculation has been worked out to derive the location of the particle-hole ($p$-$h$) band [17]. The ZSM is expected to be strongly damped and possibly to disappear on entering this band since it should rapidly decay into $p$-$h$ excitations (Landau damping); this should occur for $Q \geq 1.3$ Å$^{-1}$ [2,3], i.e., for $Q$ values where INS data do not provide clear results. The study of the interaction of the ZSM with $p$-$h$ excitations is therefore for the time being an intriguing theoretical issue with limited experimental support.

In an effort to provide more insight into this issue, an inelastic x-ray scattering (IXS) experiment has been performed on normal liquid $^3$He at $T = 1.10 \pm 0.05$ K and saturated vapor pressure in the wave vector range $0.15 \leq Q \leq 3.15$ Å$^{-1}$. In this $Q$-range x rays are characterized by an intrinsically coherent cross-section and only probe the dynamic structure factor, thus complementing INS results in the critical range for $Q \geq 1.3$ Å$^{-1}$. In this Letter the results of this experiment are reported which clarify that the ZSM in liquid $^3$He is well defined in the whole explored wave vector range, and that its dispersion curve does not enter the $p$-$h$ band. The atom dynamics of liquid $^3$He shows thus a continuous transition from the collective to the single-particle regime similarly to what is found for most liquids.

The IXS experiment was carried out at the inelastic x-ray scattering beam line ID16 at the European Synchrotron Radiation Facility (Grenoble, France). The beam line is described in detail elsewhere [19–21]. Five diced Si analyzers mounted in the 6.5 m long horizontal spectrometer were used to collect IXS spectra at five different $Q$ values at the same time. The spectrometer was operated at the Si(11,11,11) backscattering Bragg reflection, with an incident energy $E_i = 21.748$ keV, and an energy resolution characterized by a full width at half maximum (FWHM) of $1.58 \pm 0.05$ meV for all analyzers. The $Q$ resolution was set to $\approx 0.3$ nm$^{-1}$. The $^3$He sample (degree of purity higher than 99.99%) was 20 mm thick along the beam path. A $^4$He sorption stage mounted on top of a two-stages pulse tube cryogenerator was used to provide temperatures as low as 1.0 K as measured by a calibrated Ge resistor. The temperature of the $^3$He sample was kept at $1.10 \pm 0.05$ K during the whole experiment. The obtained IXS spectra are shown in Fig. 1.

The high-frequency dynamic structure factor $S(Q, \omega)$ of a liquid measured using INS or IXS usually consists of two components: the Brillouin doublet and a quasielastic peak. In classical liquids two main mechanisms can contribute to the quasielastic signal: constant-pressure entropy fluctuations and low-frequency density fluctuations [22]. The former gives rise to a quasielastic signal whose integrated intensity relative to that of the Brillouin doublet is the Landau-Placzek ratio $R = \gamma - 1$, where $\gamma$ is the constant-pressure to constant-volume specific heat ratio. Using the experimental value of $\gamma$ for $^3$He at $T = 1.1$ K [23] one obtains $R = 0.02$. The second mechanism gives rise to a quasielastic signal whose integrated intensity relative to the total spectral intensity, $f_q$, can be expressed as $f_q = 1 - (c_1/c_0)^2$, where $c_1$ and $c_0$ are the first and zero sound velocity, respectively. Using the known values of $c_1 = 187.6$ m/s and $c_0 = 194.6$ m/s [24] one obtains $f_q = 0.07$. Altogether, these two mechanisms should contribute a quasielastic signal somewhat smaller than 10% of the inelastic one. As a matter of fact, the spectra shown in Fig. 1 are dominated by the Brillouin peak, that disperses and broadens on increasing $Q$, and there is no clear evidence of any quasielastic signal. It remains an interesting question to understand whether the expected small quasielastic signal is actually buried under the tail of the inelastic excitation or whether the expressions used above are somehow inappropriate for the case of liquid $^3$He. In any event, it is a reasonable approximation to describe the measured spectra in terms of a single inelastic excitation.

A simple model able to describe the measured spectra is the damped harmonic oscillator (DHO) function [25]:

$$S(Q, \omega) = \frac{[n(\omega) + 1]}{\pi} \frac{4Z(Q)\omega\Gamma(Q)}{[\omega^2 - \Omega(Q)^2]^2 + 4\omega^2\Gamma(Q)^2}, \tag{1}$$

where $\Omega$ is the maximum of the longitudinal current, $(\omega/Q)^2 S(Q, \omega)$; $2\Gamma$ is the FWHM of the Brillouin peak; $Z(Q)$ is an intensity factor; and $n(\omega)$ is the Bose factor $n(\omega) = [\exp(\frac{\hbar\omega}{k_B T}) - 1]^{-1}$. Figure 1 shows the DHO model functions convoluted to the instrumental resolution best

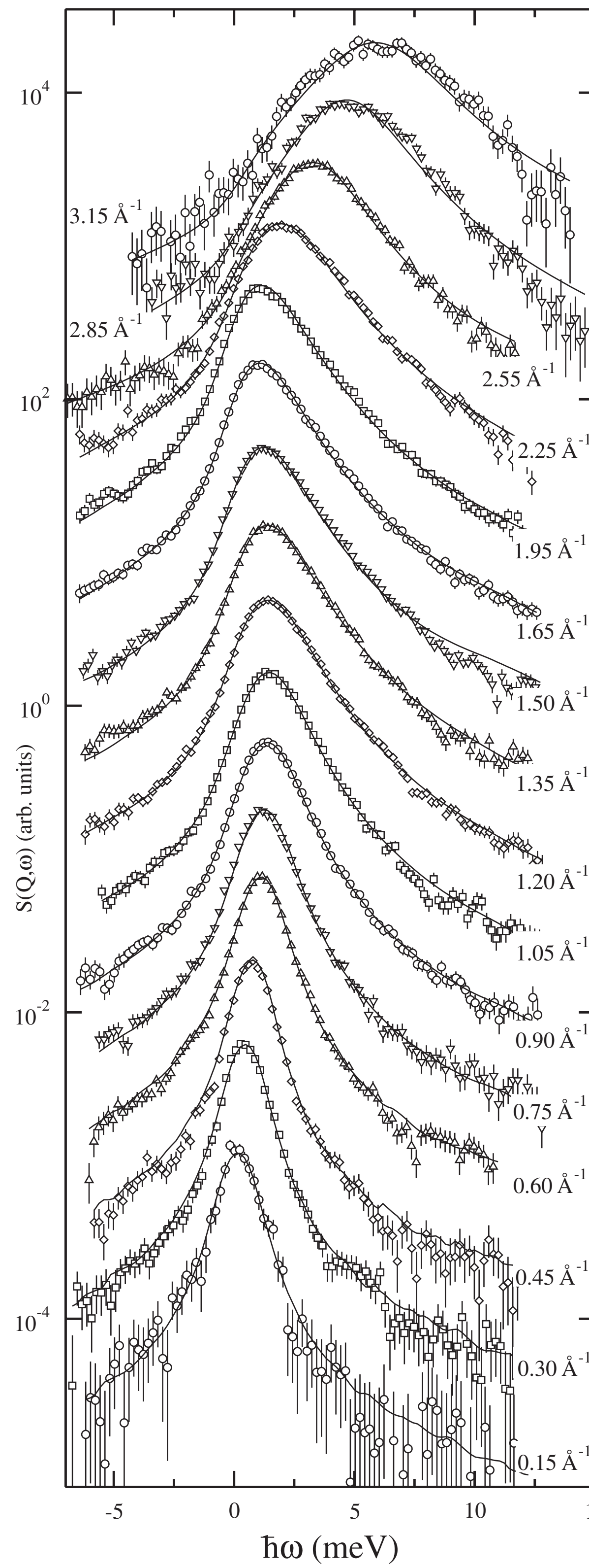


FIG. 1. Inelastic x-ray scattering spectra of liquid $^3$He at $T = 1.10 \pm 0.05$ K and saturated vapor pressure with the best fitting damped harmonic oscillator model functions at the indicated $Q$ values. For clarity reasons each spectrum has been multiplied by a factor of 3 with respect to the preceding one in the figure.

fitting the measured spectra. The DHO model can be fitted very well to the experimental data for $Q \leq 2.55$ Å$^{-1}$, while different models like the additive approach (AA) with a Gaussian approximation of the momentum distribution function, $n(p)$, or the convolution approach (CA) with a suitable model for the $n(p)$ [26–29] turn out to be more appropriate at $Q = 2.85$ Å$^{-1}$ and above; at 2.55 Å$^{-1}$ both the DHO and AA models can be used equally well to fit the data. In what follows only the results of the DHO fits to the spectra up to $Q = 3.15$ Å$^{-1}$ will be discussed. The spectrum measured for $Q = 1.35$ Å$^{-1}$ is shown in some more detail in Fig. 2 along with the instrumental function, the best DHO fit to the data and the corresponding unconvoluted DHO function; the residual of the fit is shown as well in standard deviation units to emphasize the goodness of the fit.

In standard IXS experiments the position of the quasielastic signal is used to fix the zero value of the exchanged energy scale; in the present case this operation requires a more complex procedure. Specifically, the $\omega = 0$ position has been fixed by binding the ratio of the first to zeroth moment of the model used to describe the $S(Q, \omega)$ to the corresponding exact sum-rule value:

$$\frac{\int_{-\infty}^{\infty} \hbar\omega S(Q,\omega)d\omega}{\int_{-\infty}^{\infty} S(Q,\omega)d\omega} = \frac{\hbar^2 Q^2}{2MS(Q)}, \tag{2}$$

where $M$ is the atomic mass and $S(Q)$ is the structure factor. In this procedure, the $S(Q)$ data from Ref. [30] have been used. The $x$ scale in Figs. 1 and 2 has been fixed as a result of this procedure.

In Fig. 3 the relevant parameters of the spectral shape analysis based on the DHO model are reported: the dispersion curve, $\Omega(Q)$, and the damping $2\Gamma(Q)$ for the ZSM. INS data for $^3$He at $T = 0.12$ K from Ref. [15] are reported as well in Fig. 3 together with the particle-hole band proposed in Ref. [18]. The dispersion curve here obtained for $^3$He closely resembles those typically obtained for simple liquids, i.e., a linear dependence of $\Omega(Q)$ on $Q$ followed by a bending with a minimum roughly corresponding to the maximum of the static structure factor (dotted line in the lower panel of Fig. 3) and finally by a definite increase towards the single-particle limit. The damping in turn increases with $Q$ roughly as $Q^2$ up to the maximum of the static structure factor where it reaches a plateau followed by a clear increase on approaching the single-particle limit. The present results for both the ZSM dispersion and its width are in very good agreement with the INS data up to $Q = 0.8$ Å$^{-1}$. In particular, the very weak temperature dependence of the ZSM parameters in the low-$Q$ range—already observed in previous neutron studies—is here confirmed. This observation indicates that the Landau model [4,5] cannot be simply extrapolated to

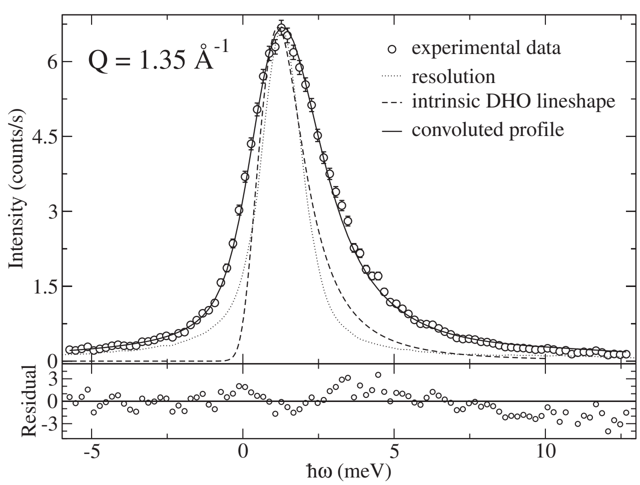


FIG. 2. Experimental IXS spectrum of liquid $^3$He for $Q = 1.35$ Å$^{-1}$ along with the energy resolution, the best DHO fit to the data, and the unconvoluted DHO function. The lower panel reports the residual of the fit in standard deviation units.

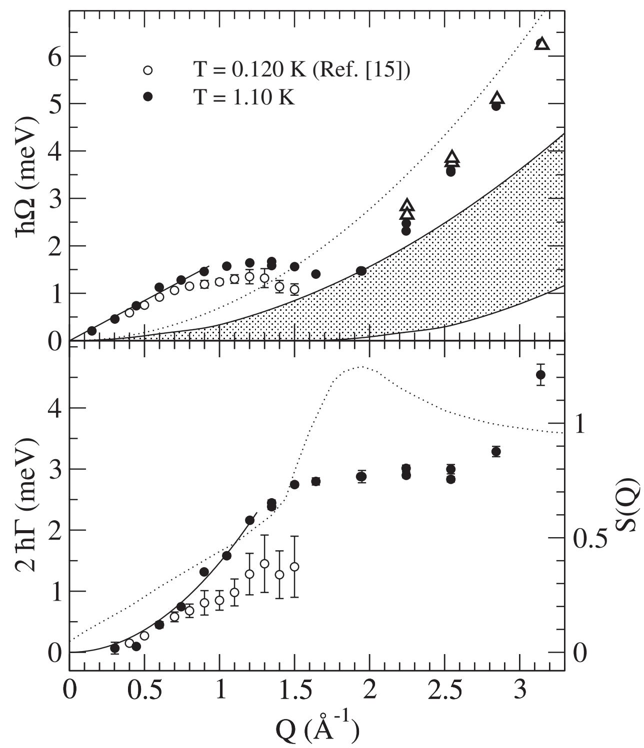


FIG. 3. Upper panel: dispersion of the zero-sound mode. Solid circles: present results obtained using the DHO line shape ansatz; open triangles: present results obtained using the additive approach with Gaussian $n(p)$; open circles: INS results from Ref. [15] ($T = 0.12$ K). The shaded area represents the particle-hole band calculated according to Ref. [18]; the full line at low-$Q$ emphasizes the initial linear dependence of the dispersion curve; the dotted line represents the recoil energy of $^3$He which corresponds as well to the dispersion curve in the $Q \to \infty$ limit (impulse approximation). Lower panel: intrinsic Brillouin widths, same symbols as above. The full line shows the initial quadratic dependence on $Q$ of the FWHM; the dotted line (right $y$ axis) reports the experimental $S(Q)$ from Ref. [30].

predict the correct value for the width as measured by INS or IXS: in this model, in fact, both the FSM and the ZSM have a strong temperature dependence, as confirmed by ultrasound experiments [6] at low frequencies. The weak temperature dependence of the ZSM width here reported rather suggests a nondynamic origin for the ZSM damping in the high-frequency range probed by INS and IXS, as commonly found in disordered systems [31]. The peculiar quantum nature of normal liquid $^3$He seems thus to play a minor role for the high-frequency ZSM.

It has to be observed in Fig. 3 that the present dispersion curve lies above the one obtained with INS at $Q$ values around $Q = 1.5$ Å$^{-1}$, and that a similar situation holds for $2\Gamma(q)$ as well. At $Q \approx 1.5$ Å$^{-1}$ INS $^3$He spectra suffer from the superposition of the coherent and incoherent signal which leads to the overlap of the ZSM peak and the spin wave one; at the same time, the progressive departure of the spectral line shape from the DHO ansatz may introduce systematic uncertainties in the present IXS results due to the above described data reduction which is sensitive to the model that is used. However, and independently of possible minor systematic errors, the most important finding here is that the $^3$He ZSM peak does not undergo any major change in the wave vector range from 1.5 to 2.0 Å$^{-1}$, where it was expected to enter the particle-hole excitation band and be heavily damped. Actually, the IXS dispersion curve reported in Fig. 3 bends at $Q = 2$ Å$^{-1}$ and avoids crossing the $p$-$h$ band [18] (which should be in addition smeared out owing to the relatively high temperature of the present measurement). Therefore, one can infer from the present data that the Landau damping scenario simply does not occur.

In conclusion, IXS spectra of $^3$He at $T = 1.10$ K which directly provide the dynamic structure factor $S(Q, \omega)$ have been measured in the exchanged wave vector range $0.15 \leq Q \leq 3.15$ Å$^{-1}$. These results agree with the previous INS measurements in the $Q$ range up to $Q = 0.8$ Å$^{-1}$, while for higher $Q$ up to 1.5 Å$^{-1}$ the agreement between the present IXS and the previous INS data gets progressively worse. Above 1.5 Å$^{-1}$ the existing INS data cannot any longer be exploited owing to the overlap of the coherent and incoherent peaks; the present IXS data represent the first experimental results available for liquid $^3$He in the $Q$ range from 2.5 to 3.15 Å$^{-1}$. The present IXS data do not show any sign of decay of the ZSM into $p$-$h$ excitations: a well-defined mode is observed in the whole explored range whose linewidth does not vary much in the $Q$ range between 1.5 and 2.0 Å$^{-1}$ where the ZSM was expected to enter the $p$-$h$ band. As a matter of fact, the present measurements indicate that the ZSM does not possibly even enter the $p$-$h$ band in the whole $Q$ range up to the single-particle regime, thus showing a behavior very similar to the one commonly observed in liquids.

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[1] P. Nozières and D. Pines, *The Theory of Quantum Liquids* (Perseus, New York, 1999), Vol. 1.
[2] H. R. Glyde, *Excitations in Liquid and Solid Helium* (Clarendon, New York, 1994).
[3] E. R. Dobbs, *Helium Three* (Oxford University, Oxford, 2000).
[4] L. D. Landau, Sov. Phys. JETP **3**, 920 (1957).
[5] L. D. Landau, Sov. Phys. JETP **5**, 101 (1957).
[6] W. R. Abel, A. C. Anderson, and J. C. Wheatley, Phys. Rev. Lett. **17**, 74 (1966).
[7] B. E. Keen, P. W. Mattews, and J. Wilks, Proc. R. Soc. A **284**, 125 (1965).
[8] R. Scherm *et al.*, J. Phys. C **7**, L341 (1974).
[9] W. G. Stirling *et al.*, J. Phys. C **9**, 1643 (1976).
[10] K. Sköld *et al.*, Phys. Rev. Lett. **37**, 842 (1976).
[11] K. Sköld and C. A. Pelizzari, J. Phys. C **11**, L589 (1978).
[12] P. A. Hilton *et al.*, Z. Phys. B **30**, 107 (1978).
[13] P. A. Hilton *et al.*, J. Phys. C **13**, L295 (1980).
[14] R. Scherm *et al.*, Phys. Rev. Lett. **59**, 217 (1987).
[15] B. Fåk *et al.*, J. Low Temp. Phys. **97**, 445 (1994).
[16] B. Fåk *et al.*, J. Low Temp. Phys. **110**, 417 (1998).
[17] H. R. Glyde *et al.*, Phys. Rev. B **61**, 1421 (2000).
[18] B. Fåk and H. R. Glyde, Phys. Rev. B **55**, 5651 (1997).
[19] R. Verbeni *et al.*, J. Synchrotron Radiat. **3**, 62 (1996).
[20] C. Masciovecchio *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. B **111**, 181 (1996).
[21] C. Masciovecchio *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. B **117**, 339 (1996).
[22] J. Boon and S. Yip, *Molecular Hydrodynamics* (Dover, Mineola, 1991).
[23] H. L. Laquer, S. G. Sydoriak, and T. R. Roberts, Phys. Rev. **113**, 417 (1959).
[24] I. Rudnick, J. Low Temp. Phys. **40**, 287 (1980).
[25] B. Fåk and B. Dorner, Physica (Amsterdam) **234–236B**, 1107 (1997).
[26] V. F. Sears, Phys. Rev. B **30**, 44 (1984).
[27] H. R. Glyde, Phys. Rev. B **50**, 6726 (1994),
[28] R. T. Azuah *et al.*, Phys. Rev. B **56**, 14620 (1997).
[29] H. R. Glyde, R. T. Azuah, and W. G. Stirling, Phys. Rev. B **62**, 14337 (2000).
[30] F. K. Achter and L. Meyer, Phys. Rev. **188**, 291 (1969).
[31] G. Ruocco *et al.*, Phys. Rev. Lett. **83**, 5583 (1999).