



Analogue of Cosmological Particle Creation in an Ion Trap

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We study phonons in a dynamical chain of ions confined by a trap with a time-dependent (axial) potential strength and demonstrate that they behave in the same way as quantum fields in an expanding or contracting Universe. Based on this analogy, we present a scheme for the detection of the analogue of cosmological particle creation which should be feasible with present day technology. In order to test the quantum nature of the particle creation mechanism and to distinguish it from classical effects such as heating, we propose to measure the two-phonon amplitude via the 2nd red sideband transition and to compare it with the one-phonon amplitude (1st red sideband).

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Introduction.—The theory of quantum fields in curved space-times (see, e.g., [1]) comprises many fascinating and striking phenomena—one of them being the creation of real particles out of the (virtual) quantum vacuum fluctuations by a gravitational field. These effects include Hawking radiation given off by black holes as well as cosmological particle creation. A very similar mechanism—the amplification of quantum vacuum fluctuations due to the rapid expansion of the very early Universe—is (according to our standard model of cosmology) responsible for the generation of the seeds for cosmic structure formation. Hence, even though these effects are far removed from everyday experience, they are very important for the past and the future fate of our Universe.

Therefore, it would be desirable to render these phenomena accessible to an experimental investigation. Probably the most promising way for achieving this goal is to construct a suitable analogue which reproduces relevant features (such as the Hamiltonian) of quantum fields in curved space-times. Along this line of reasoning, proposals based on the analogy between phonons in dynamical Bose-Einstein condensates and quantized fields in an expanding or contracting Universe have been suggested [2]. Unfortunately, the detection of the created phonons in these systems is rather difficult (see also [3]).

On the other hand, the detection of single phonons in ion traps via optical techniques is already state of the art in current technology—which suggests the study of this setup instead. In this Letter, we shall derive the analogy between phonons in an axially time-dependent ion trap and quantum fields in an expanding or contracting Universe and propose a corresponding detection scheme for the analogue of cosmological particle creation. A similar idea has already been pursued in [4], but the proposal presented therein goes along with several problems, which will be discussed below [5].

Cosmological particle creation.—Let us start by briefly reviewing the basic mechanism of particle creation in an

expanding or contracting Universe. For simplicity, we consider a real massless scalar field ϕ . Its curved space-time action has the generic form [1] ($\hbar = c = 1$ throughout)

$$\mathcal{A} = \frac{1}{2} \int d^4x \sqrt{|g|} [(\partial_\mu \phi) g^{\mu\nu} (\partial_\nu \phi) - \zeta \mathfrak{R} \phi^2], \quad (1)$$

where $g^{\mu\nu}$ denotes the metric and $g = \det\{g_{\mu\nu}\}$ its determinant (Jacobian). In general, scalar fields could be coupled to the Ricci (curvature) scalar \mathfrak{R} via a dimensionless parameter ζ , but the main results remain valid for $\zeta = 0$. A spatially flat Universe can be described in terms of the Friedman-Robertson-Walker metric [6]

$$ds^2 = \alpha^6(t) dt^2 - \alpha^2(t) dr^2, \quad (2)$$

with the time-dependent scale parameter $\alpha(t)$ corresponding to the cosmic expansion/contraction. Here we have chosen a slightly unusual time-coordinate t related to the proper time τ via $d\tau = \alpha^3(t) dt$ in order to simplify the subsequent formulas. After a normal-mode expansion, the wave equation derived from (1) and (2) reads

$$\ddot{\phi}_k + [\alpha^4(t) k^2 + \zeta \alpha^6(t) \mathfrak{R}(t)] \phi_k = \ddot{\phi}_k + \Omega_k^2(t) \phi_k = 0; \quad (3)$$

i.e., each mode k just represents a harmonic oscillator with a time-dependent potential strength $\Omega_k^2(t)$. If the temporal variation of $\Omega_k(t)$ is much slower than the internal frequency ($\dot{\Omega}_k \ll \Omega_k^2$ and $\ddot{\Omega}_k \ll \Omega_k^3$, etc.), the quantum state will stay near the ground state (adiabatic theorem). However, if the external time dependence $\Omega_k(t)$ is too fast (nonadiabatic), the ground-state wave function cannot adapt to this change anymore and hence the evolution will transform the initial ground state into an excited state in general (cosmological particle creation). Because of the linearity of Eq. (3), the wave-function remains Gaussian (though with a different width) and hence the initial vacuum state $|0\rangle = |\psi(t \rightarrow -\infty)\rangle$ containing no particles

$\forall_k \hat{a}_k |0\rangle = 0$ evolves into a squeezed state

$$\begin{aligned} |\psi(t \rightarrow \infty)\rangle &= \prod_k \exp\{\xi_k \hat{a}_k^\dagger \hat{a}_{-k}^\dagger - \text{H.c.}\} |0\rangle \\ &= |0\rangle + \sum_k \xi_k |1_k, 1_{-k}\rangle + \mathcal{O}(\xi_k^2), \end{aligned} \quad (4)$$

which does contain pairs of particles $|1_k, 1_{-k}\rangle = |k, -k\rangle$ with opposite k (momentum conservation). The squeezing parameter ξ_k for each mode is determined by the solution of Eq. (3) and thus by the external time dependence $\Omega_k(t)$, i.e., the rapidity of the cosmic evolution $\alpha(t)$. It governs the number of created particles per mode via

$$\langle \hat{n}_k \rangle = \langle \psi(t \rightarrow \infty) | \hat{a}_k^\dagger \hat{a}_k | \psi(t \rightarrow \infty) \rangle = \sinh^2(|\xi_k|). \quad (5)$$

Ion-trap analogue.—Assuming a strong radial confinement of the ions, we consider their axial motion only. In a time-dependent harmonic axial potential described by the oscillator frequency $\omega_{\text{ax}}(t)$, the position q_i of the i th ion obeys the equation of motion

$$\ddot{q}_i + \omega_{\text{ax}}^2(t) q_i = \gamma \sum_{j \neq i} \frac{\text{sign}(i-j)}{(q_i - q_j)^2}, \quad (6)$$

where the factor γ encodes the strength of the Coulomb repulsion between the ions. Assuming a static situation initially, the classical solution to the above equation can be obtained via the scaling ansatz $q_i(t) = b(t)q_i^0$, where q_i^0 are the initial static equilibrium positions, leading to the evolution equation for the scale parameter $b(t)$

$$\ddot{b}(t) + \omega_{\text{ax}}^2(t)b(t) = \frac{\omega_{\text{ax}}^2(0)}{b^2(t)}. \quad (7)$$

In order to treat the quantum fluctuations of the ions (leading to the quantized phonon modes), let us split the full position operator $\hat{q}_i(t)$ for each ion into its classical trajectory $b(t)q_i^0$ and quantum fluctuations $\delta\hat{q}_i(t)$ via $\hat{q}_i(t) = b(t)q_i^0 + \delta\hat{q}_i(t)$. Since these fluctuations $\delta\hat{q}_i(t)$ are very small for ions, we may linearize the full equation of motion (6)

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{\text{ax}}^2(t) \right) \delta\hat{q}_i = \frac{1}{b^3(t)} \sum_j M_{ij} \delta\hat{q}_j, \quad (8)$$

with a time-independent matrix M_{ij} arising from the Coulomb term in (6). Diagonalization of this matrix (normal-mode expansion) yields the phonon modes

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{\text{ax}}^2(t) + \frac{\omega_\kappa^2}{b^3(t)} \right) \delta\hat{q}_\kappa = 0, \quad (9)$$

labeled by κ . The time-independent eigenvalues $\omega_\kappa^2 \geq 0$ of the matrix M_{ij} determine the phonon frequencies. The lowest mode is the center-of-mass mode corresponding to a simultaneous (rigid) motion of the ions. Since the ion distances do not change, the Coulomb term does not contribute in this situation $\omega_\kappa^2 = 0$. The next mode is the breathing mode with $\omega_\kappa^2 = 2\omega_{\text{ax}}^2(0)$. Comparing Eqs. (7)

and (9), we see that this mode exactly corresponds to the scaling ansatz; i.e., the ion chain expands or contracts linearly. Hence this is the only mode which can be excited classically (for a purely harmonic potential). I.e., without imperfections such as heating, phonons in the other modes can only be created by quantum effects.

Comparing Eqs. (3) and (9) and identifying ϕ_k with $\delta\hat{q}_\kappa$, we observe a strong similarity: The wave number k^2 in (3) directly corresponds to ω_κ^2 in (9) and the scale factors $\alpha(t)$ and $b(t)$ enter in a similar way. However, an expanding Universe is analogous to a contracting ion chain and vice versa. In the mode-independent terms, the axial trap frequency ω_{ax} acts like the Ricci scalar \mathfrak{R} . Interestingly, both are related to the second time-derivatives of the corresponding scale factors $b(t)$ and $\alpha(t)$.

In view of the formal equivalence of Eqs. (7) and (9), we obtain the same effects as in cosmology—in particular, the mixing of creation and annihilation operators

$$\hat{a}_\kappa(t \rightarrow \infty) = \alpha_\kappa \hat{a}_\kappa(0) + \beta_\kappa \hat{a}_\kappa^\dagger(0), \quad (10)$$

which can be expressed in terms of the Bogolubov coefficients satisfying $|\alpha_\kappa|^2 - |\beta_\kappa|^2 = 1$. Note that the above relation is just the operator representation of the squeezing transformation in Eq. (4) with $|\beta_\kappa| \leftrightarrow \sinh(|\xi_k|)$.

Detection scheme.—In the following we describe how to realize the proposed experiment by applying operations closely related to those implemented on qubit-ions in quantum information processing [7]. We focus on initializing the system, simulating the nonadiabatic expansion of space, performing the readout of the final state (particle- or phonon-number distribution) and distinguishing it from a classically describable outcome, for example, caused by thermal heating. To perform a first realization, we will confine one single earth alkaline atomic ion to the axis of a linear radio-frequency trap [8] similar to that described in [9,10]. The required simulation basis can be composed by a $^2S_{1/2}$ electronic ground-state level of $^{25}\text{Mg}^+$, here the state $|F=3; m_f=3\rangle = |\downarrow\rangle$, and the associated harmonic oscillator levels $|n\rangle$ related to the axial harmonic confinement, as depicted in Fig. 1. At the start of each experiment, the ion will be laser cooled close to the ground state of the axial (external) motion and optically pumped into the electronic (internal) state $|\downarrow\rangle$ [11]. Then we will decrease adiabatically the axial confinement and subsequently reset it nonadiabatically to its initial value (as already proposed in [12] in another context). Since the ground-state wave function of the ion cannot adapt to the restored stiff confinement (nonadiabatic case), it will oscillate symmetrically around the minimum of the final trapping potential [13], i.e., without populating odd motional states. As shown above, this nonclassical oscillation is to be described via a squeezed state (see also [14]) depicted in Fig. 2.

We propose, in addition to established schemes described in [14] or [15], for example, an alternative method to distinguish classical noise (such as the initial thermal

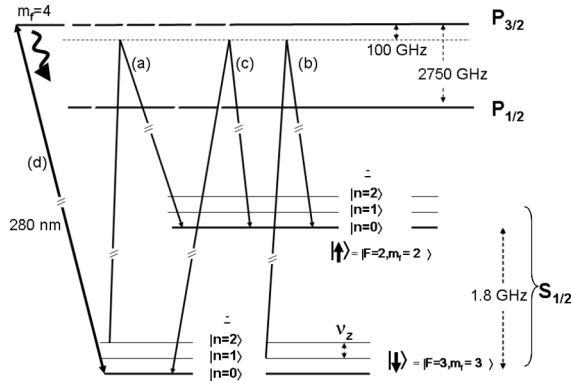


FIG. 1. Schematic of the relevant energy levels (not to scale) of one $^{25}\text{Mg}^+$ ion. Shown are the ground-state hyperfine levels supplying the two internal states ($|\downarrow\rangle$ and $|\uparrow\rangle$) and the first three equidistant harmonic oscillator levels $|n\rangle$, related to the harmonic axial confinement in a linear ion trap. Typically, the energy splitting of the motional levels and the Zeeman shift induced by an external magnetic field are of the same order of magnitude within 1–10 MHz, therefore much smaller than the Hyperfine splitting of 1.8 GHz, the fine structure splitting of 2750 GHz and the optical transition frequency of the order of 10^{15} Hz. We depict the resonant transition state sensitive detection named (d) and the relevant types of off resonant (≈ 100 GHz) two-photon stimulated Raman transitions (a), (b), and (c) described in the text.

distribution or heating during the process) from a squeezed state generated by quantum effects considered here. In order to readout the final motional state, we will first couple it (via suitable lasers) to two internal states of the ion. Besides the electronic ground state $|F=3; m_f=3\rangle = |\downarrow\rangle$, the second internal state to span a two-level system (analogous to a qubit) is implemented via a second hyperfine state of $^{25}\text{Mg}^+$, $|F=2, m_f=2\rangle \equiv |\uparrow\rangle$, separated from the state $|\downarrow\rangle$ by the hyperfine splitting $\omega_0 \approx 2\pi \times 1.8$ GHz. We will accomplish the coupling of the two internal states $|\downarrow\rangle$, $|\uparrow\rangle$ and the motional states $|n\rangle$ via two-photon stimulated Raman transitions [7] requiring two laser beams ($\lambda \approx 280$ nm), with wave vector difference $\Delta\mathbf{k} = \mathbf{k}_2 - \mathbf{k}_1$ aligned along the trap axis z ($|\Delta k| = \sqrt{2} \times 2\pi/\lambda =$

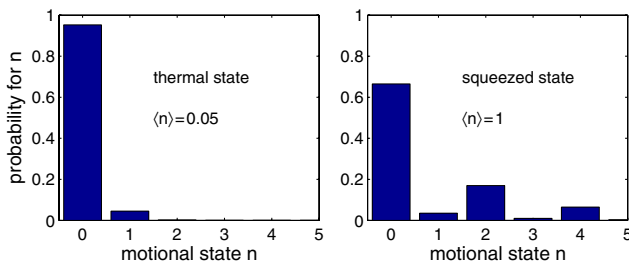


FIG. 2 (color online). Initial and final population of the motional states $|n\rangle$. Left: experimentally realized thermal state with $\langle \hat{n} \rangle = 0.05$. Right: simulated spectrum of the squeezed thermal state with $\langle \hat{n} \rangle = 1$. The predominant population of even motional states emphasizes the nonclassical character of this state.

$2\pi/\lambda_{\text{eff}}$). Via detuning the frequency difference $\omega_2 - \omega_1$ from the hyperfine splitting $\omega_o \pm 2\pi m\nu_z$ by integer multiples m of the axial trapping frequency ν_z , we may drive the carrier-transition ($m = 0$) or the first- ($m = 1$) and second- ($m = 2$) side band transitions, respectively. Note that the spectral resolution of the two-photon stimulated Raman transition is independent on the natural line width $\Gamma = 2\pi \times 43$ MHz of the resonant transition—but proportional to the inverse of the Rabi frequency, adjusted via the intensities or detuning of the laser beams allowing for the resolution of the individual motional states separated by $\nu_z \ll \Gamma$. In order to measure the population of the motional state $|n = 2\rangle$, we will drive a sequence of transitions (cf. Fig. 1), synthesized by a second-side band (a) transition ($|\downarrow, n = 2\rangle \rightarrow |\uparrow, n = 0\rangle$) followed by a carrier (c) transition ($|\uparrow, n = 0\rangle \rightarrow |\downarrow, n = 0\rangle$). The final readout (d) described below is internal-state dependant and provides us with the population in state $|\downarrow\rangle$. After the sequence (a),(c),(d) of transitions this is almost exclusively equivalent to the population of the motional state $|n = 2\rangle$ (because the probability of even higher excitations $|n \geq 3\rangle$ is expected to be much smaller and their Rabi frequencies are also different). This result can then be compared with the outcome after a first red-side band (b) transition ($|\downarrow, n = 1\rangle \rightarrow |\uparrow, n = 0\rangle$) followed by a carrier (c) transition ($|\uparrow, n = 0\rangle \rightarrow |\downarrow, n = 0\rangle$), providing the probability of motional excitation $|n = 1\rangle$. As soon as we deduce a higher probability for motional state $|n = 2\rangle$ than for motional state $|n = 1\rangle$, we show the incompatibility with classical effects such as a thermal distribution and get strong evidence for the nonclassical effect of squeezing [16].

Finally, we have to readout the internal state efficiently. To this end, we apply an additional resonant laser beam (d), tuned to a cycling transition [7], coupling only state $|\downarrow\rangle$ resonantly to the $P_{3/2}$ level and providing spontaneous emission at rates of >10 MHz. This allows us to distinguish the “bright” $|\downarrow\rangle$ from the “dark” $|\uparrow\rangle$ state with high accuracy, even at a low detection efficiency (due to the restricted solid angle, etc.).

Envisioned results.—The above mentioned state of the art techniques allow the cooling of the axial motion close to the ground state $\langle \hat{n} \rangle \approx 0.05$ [8] (see also [10,11]) and to optically pump into the down state $|\downarrow\rangle$ with 99% [8] or even higher fidelity [10,11]. First experiments show a possible nonadiabatic variation of the axial motional frequency ν_z between 200 kHz and ≥ 2 MHz with a related rise time of the order of 1 μs , which is sufficiently fast compared to the oscillation period of the lower frequency. Numerical simulations (based on measured temporal variation curves) indicate that we should be able to transfer approximately 20% of the motional state population from the ground state $|n = 0\rangle$ into state $|n = 2\rangle$, which corresponds to a squeezed state with $\langle \hat{n} \rangle \approx 1$. Starting with a thermal distribution with $\langle \hat{n} \rangle \approx 0.05$ instead of the exact ground state $|n = 0\rangle$, there will also be a small final population (a few percent) of the state $|n = 1\rangle$; see Fig. 2. However, this

residual effect will be significantly smaller than the $|n = 2\rangle$ population such that the signatures of squeezing can be measured as described above [16].

Conclusions.—Since the state of the art fidelities for the carrier and sideband transitions as well as the state sensitive detection exceed 99% [10,11], the initialization and measurement of the system can be provided with high accuracy. In order to benefit from these operational fidelities, the main task will be to minimize classical disturbances. For example, we have to carefully balance the applied voltages for confinement during their nonadiabatic changes to prevent classical excitation of the axial motional mode. In comparison to quantum information processing with trapped ions, the requirements for the present proposal may be a bit easier to achieve because the duration of the experiment will be short (around 3 ms) compared to the inverse of the thermal heating rate for motional quanta inside the trap (≤ 0.005 quanta/ms [8]) and because the thermal and the squeezed motional spectra show fundamentally different characteristics; see Fig. 2. It should also be emphasized that it is impossible to energetically resolve individual motional states with pulse durations short compared to the inverse of their frequency difference (see also [4,5]). This impossibility in resolution is related to the Heisenberg uncertainty principle $\Delta E \Delta t \geq \hbar/2$ that allows the creation of pairs of particles (phonons) out of the vacuum (ground) state in first place. Increasing the system towards 8 modes (ions) might be feasible by this proposal with state of the art techniques [17], further scaling might benefit from the technical progress driven by the attempts of the quantum information community.

Apart from experimentally testing the analogue of cosmic particle creation [18] (which might ultimately allow the study of the impact of decoherence and interactions, etc.), the investigation of nonadiabatic switching of trapping potentials and its influence on the quantum state of motion might also shed light on possible problems in schemes where the fast shuttling of ions in a multiplex trap architecture is required for scaling towards a universal quantum computer.

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