Black-Hole No-Hair Theorems for a Positive Cosmological Constant

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We extend all known black-hole no-hair theorems to space-times endowed with a positive cosmological constant Λ. Specifically, we prove that static spherical black holes with $\Lambda > 0$ cannot support scalar fields in convex potentials and Proca-massive vector fields in the region between the black hole and the cosmic horizon. We also demonstrate the existence of at least one type of quantum hair, and of exactly one charged solution for the Abelian Higgs model. Our method of proof can be applied to investigate other types of quantum or topological hair on black holes in the presence of a positive Λ.

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The no-hair conjecture states that gravitational collapse reaches a stationary final state, characterized by a small number of parameters. The part of this that has been rigorously proved, called the no-hair theorem [1–3], deals with the uniqueness of stationary black-holes, which are characterized by mass, angular momentum, and charges corresponding to long-range gauge fields. In particular, static black-holes do not support external fields corresponding to scalars in convex potentials, Proca-massive gauge fields [4], or even gauge fields which have become massive via the Abelian Higgs mechanism [5,6].

All these theorems assume, in addition to stationarity, asymptotic flatness, which requires a vanishing cosmological constant. The stress-energy tensor then must vanish at infinity, which means that all matter fields must approach their vacuum values. However, recent observations suggest a strong possibility that the Universe is equipped with a positive cosmological constant, $\Lambda > 0$ [7,8]. If this is so, there should be a cosmic horizon of size $\sim 1/\sqrt{\Lambda}$, and proofs of uniqueness of black-holes become suspect. Even if a black-hole forms as the final state of gravitational collapse, its horizon will be inside the cosmic horizon. There is no global timelike Killing vector outside the black-hole horizon. Further, the stress-energy tensor need not vanish at infinity, nor even at the cosmic horizon, so boundary conditions for the fields are not obvious.

Price's theorem [9], which may be thought of as a perturbative no-hair theorem, was proved for $\Lambda > 0$ some years ago [10] for massless small fluctuations. But no version of a theorem about the existence of static matter fields has been established for $\Lambda > 0$. Here we establish classical no-hair theorems for various different fields, and also extend one known case of quantum hair, on static black-hole space-times with $\Lambda > 0$. Our method involves a paradigm shift—we consider only the region between the black-hole horizon and the cosmic horizon, and ignore the asymptotic behavior of both the metric and the matter fields. In fact, we do not use the equations for the metric at all, beyond assuming the existence of a cosmic horizon. We find that it is possible to extend most of the known no-hair theorems to black holes in a Universe with $\Lambda > 0$. We also find one clear exception, that of the Abelian Higgs model.

We will consider the various no-hair conjectures in a black-hole space-time endowed with a positive cosmological constant, which leads to the existence of a cosmic horizon. By a static black hole with $\Lambda > 0$ we will mean a space-time with at least two horizons, between which there is a timelike Killing vector ζ^μ satisfying $\zeta_{[\mu}\nabla_\nu\zeta_{\lambda]} = 0$. Then ζ^μ is orthogonal to a spacelike hypersurface Σ, which is assumed to be spherically symmetric. The norm $\lambda(r) = \sqrt{-\zeta^\mu\zeta_\mu}$ vanishes at two values $r_H < r_C$ of the radial coordinate r, thus dividing the manifold into three regions. The region $r < r_H$ contains a space-time singularity. The points of this region do not lie to the past of Σ (for which $r_H < r < r_C$), while the points of Σ do not lie to the past of the region $r > r_C$. We are not concerned with the world outside the cosmic horizon; so the asymptotic behavior of the metric will not be relevant to our calculations. In particular we do not assume the metric to be asymptotically de Sitter.

The various no-hair theorems will be taken to be statements about the corresponding classical fields on the spacelike hypersurface Σ between the two horizons. We will not look for solutions, only prove general statements about their existence. The crucial ingredient for these proofs is that the squared norm of the stress-energy tensor is bounded at each horizon. This is dictated by Einstein's equation, $G_{\mu\nu} = 8\pi T_{\mu\nu} - \Lambda g_{\mu\nu}$; if the stress-energy tensor $T_{\mu\nu}$ has unbounded norm at any point, the norm of the Einstein tensor $G_{\mu\nu}$ must also become unbounded there, giving rise to a curvature singularity at that point. Since the horizons are assumed to be regular, i.e., only coordinate singularities, it follows that the Einstein tensor and hence the stress-energy tensor must have bounded norm at both horizons. Similar arguments show that the norm of the stress-energy tensor must be static; i.e., its Lie derivative must vanish along the vector field ζ^μ. Generally we will say that the stress-energy tensor is bounded, or static, when we actually mean its norm has those properties.

Although the calculations are on a spacelike hypersurface Σ orthogonal everywhere to a timelike Killing vector, it is convenient to use covariant notation without resorting to explicit coordinates. Let $\Pi^{\mu}_{\mu'} = \delta^{\mu}_{\mu'} + \lambda^{-2}\zeta^{\mu}\zeta_{\mu'}$ denote the projection tensor which projects vectors to Σ and let $\tilde{\nabla}_{\mu}$ denote the induced connection on Σ. Then for a rank p antisymmetric tensor Ω whose Lie derivative with respect to ζ^{μ} vanishes,

$$\tilde{\nabla}_{\alpha}(\lambda\omega^{\alpha\mu\ldots\nu}) = \lambda(\nabla_{\alpha}\Omega^{\alpha\mu'\ldots\nu'})\Pi^{\mu}_{\mu'}\ldots\Pi^{\nu}_{\nu'}, \quad (1)$$

where ω is the Σ-projection of Ω. This is essentially the statement that the four divergence of Ω is the same as its three divergence when both Ω and the metric are time independent. All our proofs will be based on this result.

Let us start with the example of a real scalar field ϕ in a potential $V(\phi)$. The equation of motion for ϕ is

$$\nabla_{\mu}\nabla^{\mu}\phi = \frac{\partial V(\phi)}{\partial\phi} = V'(\phi). \quad (2)$$

A nonvanishing $V(\phi)$ enters the stress-energy tensor, so it follows that $\mathcal{L}_{\zeta}\phi = 0$ on Σ. Then we can project this equation down to Σ using Eq. (1) to get

$$\tilde{\nabla}_{\mu}(\lambda\tilde{\nabla}^{\mu}\phi) = \lambda V'(\phi). \quad (3)$$

For $V(\phi)$ convex [i.e., $V''(\phi) \geq 0$], we multiply both sides of this equation by $V'(\phi)$ and integrate over the spacelike region Σ between the two horizons to get

$$\int_{\partial\Sigma}\lambda V'(\phi)n^{\mu}\tilde{\nabla}_{\mu}\phi + \int_{\Sigma}\lambda[V''(\phi)\tilde{\nabla}^{\mu}\phi\tilde{\nabla}_{\mu}\phi + V'^{2}(\phi)] = 0. \quad (4)$$

Here $\partial\Sigma$ is composed of the two spheres located at the two horizons, and n^{μ} is the Σ-ward pointing spacelike unit normal to these two spheres. Since $\tilde{\nabla}^{\alpha}\phi\tilde{\nabla}_{\alpha}\phi$ appears in the stress-energy tensor $T_{\mu\nu}$, it must be bounded at the two horizons. We may then apply Schwarz inequality, which in this case says that

$$|n^{\alpha}\tilde{\nabla}_{\alpha}\phi|^2 \leq (n^{\mu}n_{\mu})(\tilde{\nabla}^{\alpha}\phi\tilde{\nabla}_{\alpha}\phi) = (\tilde{\nabla}^{\alpha}\phi\tilde{\nabla}_{\alpha}\phi). \quad (5)$$

For generic $V(\phi)$, the boundedness of $T_{\mu\nu}$ on $\partial\Sigma$ implies that ϕ must also be bounded there. Since $\lambda(r) = 0$ on $\partial\Sigma$, it follows that the integral on $\partial\Sigma$ vanishes. Since Σ is spacelike $\tilde{\nabla}^{\alpha}\phi\tilde{\nabla}_{\alpha}\phi$ is non-negative, as is $V''(\phi)$ due to convexity, so Eq. (4) says that ϕ is a constant at its minimum everywhere on Σ, which is the no-hair result. For a massless ϕ, we can multiply the field equation by ϕ and insist that ϕ be measurable at the horizons, and the no-hair result follows. Note that we did not need to use the gravitational equations of motion.

The proof may not apply for a nonconvex potential $V(\phi)$. A real scalar field moving in the double-well potential $V(\phi) = \frac{\alpha}{4}(\phi^2 - v^2)^2$ can have a nontrivial static solution in Σ (it may be an unstable solution; see [11]). An interesting and not so obvious case is that of the conformal scalar, for which the interaction is $V(\phi) = \frac{1}{12}R\phi^2$. Then the part of the action containing ϕ is invariant under local conformal transformations, as are the scalars T^{μ}_{μ} and $T_{\mu\nu}T^{\mu\nu}$. Then in principle one can make a transformation to make ϕ or $\tilde{\nabla}_{\mu}\phi$ diverge at $\partial\Sigma$ without causing a curvature singularity. Then the $\partial\Sigma$ integral can be nonzero, which allows a nontrivial configuration of ϕ on Σ. Indeed solutions with conformal scalar hair with $\Lambda > 0$ are known [12]. The proof also will not apply to scalars with a kinetic term of the wrong sign, as in phantom models of dark energy [13]. Of course, in such models a static black-hole may not form in the first place, and a statement of no-hair theorems may not be possible.

For the massive vector field, the proof proceeds in a similar manner. The matter Lagrangian is

$$L = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}m^2A_{\mu}A^{\mu}. \quad (6)$$

Let us define the electric potential $\phi = \lambda^{-1}\zeta_{\mu}A^{\mu}$ and electric field $e^{\mu} = \lambda^{-1}\zeta_{\nu}F^{\mu\nu}$. A little algebra shows that

$$\tilde{\nabla}_{\mu}(\lambda\phi) = \lambda e_{\mu} + \mathcal{L}_{\zeta}a_{\mu}, \qquad \tilde{\nabla}_{\mu}e^{\mu} = \lambda^{-1}\zeta_{\alpha}\nabla_{\mu}F^{\mu\alpha}, \quad (7)$$

so that the equation of motion for e^{μ} is

$$\tilde{\nabla}_{\mu}e^{\mu} - m^2\phi = 0. \quad (8)$$

Multiplying both sides by $\lambda\phi$ and integrating, we find

$$\int_{\partial\Sigma}\lambda\phi e^{\mu}n_{\mu} + \int_{\Sigma}[\lambda(e_{\mu}e^{\mu} + m^2\phi^2) + e^{\mu}\mathcal{L}_{\zeta}A_{\mu}] = 0, \quad (9)$$

where n^{μ} is the Σ-ward unit normal to $\partial\Sigma$, as before. Since ϕ^2 and $e_{\mu}e^{\mu}$ both appear in $T_{\mu\nu}$, ϕ must be finite, and by Schwarz inequality $e^{\mu}n_{\mu}$ is finite, so the $\partial\Sigma$ integral vanishes. The Lie derivative vanishes by staticity, so the vanishing Σ integral contains positive definite quantities. It follows that $\phi = 0 = e_{\mu}$ on Σ.

The equation of motion for the magnetic field is

$$\tilde{\nabla}_{\mu}(\lambda f^{\mu\nu}) - m^2\lambda a^{\nu} = 0, \quad (10)$$

where a_{μ} and $f_{\mu\nu}$ are the Σ-projections of A_{μ} and $F_{\mu\nu}$. Multiplying both sides by a_{ν} and integrating, we find

$$\int_{\partial\Sigma}\lambda a_{\nu}f^{\mu\nu}n_{\mu} + \int_{\Sigma}\lambda\left(\frac{1}{2}(f^{\mu\nu})^2 + m^2(a^{\mu})^2\right) = 0. \quad (11)$$

Since a^{μ} and $f_{\mu\nu}$ appear in $T_{\mu\nu}$, these must be regular, which ensures that the $\partial\Sigma$ integral vanishes. The second integral is over a sum of squares, so $a_{\mu} = 0 = f_{\mu\nu}$ on Σ, which is the desired no-hair result.

For the massless vector field the Lagrangian has a local gauge symmetry, which nullifies the boundedness argument. A gauge transformation can always change a bounded function ϕ to one that becomes unbounded on the horizon. Thus we cannot set the $\partial\Sigma$ integration to zero, so e_μ need not vanish on Σ either. In fact Reissner-Nördstrom solutions with a positive cosmological constant are known.

There are two gauge-invariant Lagrangians which describe a massive Abelian gauge field. The no-hair conjecture fails for both of these cases in the presence of a positive Λ, as we describe now.

The first mechanism we consider is described by the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{m}{4}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}F_{\rho\sigma}, \tag{12}$$

where $B_{\mu\nu}$ is an antisymmetric tensor potential and $H_{\mu\nu\rho} = (\nabla_\mu B_{\nu\rho} + \text{cyclic})$ is its field strength. This system describes equally well either a massive vector or a massive antisymmetric tensor. A static, spherical, asymptotically flat black hole can carry a charge of the B field, with both $F_{\mu\nu}$ and $H_{\mu\nu\rho}$ vanishing everywhere outside the black-hole horizon [14]. It is easy to see that a similar solution exists for $\Lambda > 0$ as well.

Let $f^{\mu\nu}$ and h^μ be the Σ-projections of $F^{\mu\nu}$ and $H^\mu \equiv \frac{1}{6}\epsilon^{\mu\nu\lambda\rho}H_{\nu\lambda\rho}$, respectively. Then the "magnetic equations" can be written as

$$\tilde{\nabla}_\nu(\lambda f^{\mu\nu}) = \lambda m h^\mu, \qquad \tilde{\nabla}_{[\alpha}h_{\beta]} = -m f_{\alpha\beta}. \tag{13}$$

If we also define $e^\mu = \lambda^{-1}\zeta_\nu F^{\mu\nu}$ and $\psi = \lambda^{-1}\zeta_\rho H^\rho$, we find the "electric equations"

$$\tilde{\nabla}_\mu e^\mu = -m\psi, \qquad \tilde{\nabla}_\mu(\lambda\psi) = -\lambda m e_\mu, \tag{14}$$

where we have used $\mathcal{L}_\zeta H^\mu = 0$.

Multiplying the first of Eq. (13) by h_μ, and the first of Eq. (14) by $\lambda\psi$, and integrating, we obtain

$$\int_{\partial\Sigma}\lambda f^{\mu\nu}h_\mu n_\nu + \int_\Sigma m\lambda\left(\frac{1}{2}f^{\mu\nu}f_{\mu\nu} + h^\mu h_\mu\right) = 0, \tag{15}$$

$$\int_{\partial\Sigma}\lambda\psi e_\mu n^\mu - \int_\Sigma m\lambda(e_\mu e^\mu + \psi^2) = 0. \tag{16}$$

The surface integrals contribute nothing. It follows that all components of the field strengths $H_{\mu\nu\lambda}$ and $F_{\mu\nu}$ vanish on Σ. The solution is then the de Sitter–Schwarzschild black hole, with an arbitrary charge q corresponding to the B field, whose nonvanishing component is

$$B_{\theta\phi} = \frac{q}{4\pi r^2}. \tag{17}$$

This charge should be measurable via a stringy Bohm-Aharonov effect, just as for asymptotically flat space-times [15]. We should mention here that the free Abelian two form will leave the same kind of charge on the black hole, the proof of $H_{\mu\nu\rho} = 0$ on Σ proceeds in a similar fashion for that theory.

The other case is that of the Abelian Higgs model. In the absence of cosmological constant, a static spherically symmetric black hole does not carry electric (or magnetic) charge if the gauge field becomes massive via spontaneous symmetry breaking. However, as we shall see now, the presence of a positive cosmological constant allows a charged black hole in the false vacuum. The matter Lagrangian for the Abelian Higgs model is

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}q^2\rho^2\left(A_\mu + \frac{1}{qv}\nabla_\mu\eta\right)\left(A^\mu + \frac{1}{qv}\nabla^\mu\eta\right) - \frac{1}{2}\nabla_\mu\rho\nabla^\mu\rho + \frac{\alpha}{4}(\rho^2 - v^2)^2. \tag{18}$$

The equations for the magnetic and the electric fields on Σ read

$$\tilde{\nabla}_\nu(\lambda f^{\mu\nu}) + \lambda q^2\rho^2\left(a^\mu + \frac{1}{qv}\tilde{\nabla}^\mu\eta\right) = 0, \tag{19}$$

$$\tilde{\nabla}_\mu e^\mu - q^2\rho^2\left(\phi + \frac{1}{qv}\mathcal{L}_\zeta\eta\right) = 0, \tag{20}$$

where the definitions for e^μ and $f_{\mu\nu}$ are as in Eqs. (7) and (10). Applying the now familiar techniques to Eq. (19), we get

$$\int_{\partial\Sigma}\lambda\left(a_\mu + \frac{1}{qv}\tilde{\nabla}_\mu\eta\right)f^{\mu\nu}n_\nu - \int_\Sigma\lambda\left[\frac{1}{2}f^{\mu\nu}f_{\mu\nu} + q^2\rho^2\left(a_\mu + \frac{1}{qv}\tilde{\nabla}_\mu\eta\right) \times\left(a^\mu + \frac{1}{qv}\tilde{\nabla}^\mu\eta\right)\right] = 0. \tag{21}$$

The Σ integral can be nonvanishing only if the $\partial\Sigma$ integral is also nonvanishing, which means that the norm of either $f_{\mu\nu}$ or $(a_\mu + \tilde{\nabla}_\mu\eta)$ must diverge at the horizon. However, since we have assumed spherical symmetry, a nonvanishing $f_{\mu\nu}$ is essentially that of the magnetic monopole. But then $(a_\mu + \tilde{\nabla}_\mu\eta)$ cannot be both spherically symmetric and divergent at the horizon. So $f_{\mu\nu} = 0$ on Σ.

For the electric field we use Eq. (20) to find

$$\int_{\partial\Sigma}\lambda\left(\phi + \frac{1}{\lambda qv}\dot{\eta}\right)e^\mu n_\mu + \int_\Sigma\left[\lambda e^\mu e_\mu + \lambda q^2\rho^2\left(\phi + \frac{1}{\lambda qv}\dot{\eta}\right)^2\right] = 0, \tag{22}$$

where $\dot{\eta} = \mathcal{L}_\zeta\eta$, and we have used $\mathcal{L}_\zeta(a_\mu + \frac{1}{qv}\tilde{\nabla}_\mu\eta) = 0$ because of staticity. Since $e_\mu e^\mu$ appears in $T_{\mu\nu}$, we can use Schwarz inequality to say that $e^\mu n_\mu$ is finite on $\partial\Sigma$. So the Σ integral can be nonzero only if $(\phi + \frac{1}{qv}\lambda^{-1}\dot{\eta})$ diverges on at least one horizon. In this case ρ must vanish on that horizon.

For the asymptotically flat black hole ($\Lambda = 0$), it can be shown that ρ cannot vanish on the horizon, and so the black hole cannot have electric charge [6]. Let us see what happens for our present choice of $\Lambda = 0$. The equation of motion for ρ projected down to Σ reads

$$\tilde{\nabla}_\mu \lambda \tilde{\nabla}^\mu \rho = -\lambda q^2 \rho \left(\phi + \frac{1}{qv} \lambda^{-1} \dot{\eta} \right)^2 + \lambda \alpha \rho (\rho^2 - v^2). \tag{23}$$

Let us assume for the moment that ρ vanishes on the black-hole horizon at $r = r_H$, and starts increasing with increasing r. Then ρ must increase monotonically from $\rho = 0$ at $r = r_H$ to one of: (i) $\rho = \rho_C < v$ at $r = r_C$; (ii) $\rho = v$ at $r = r_v \le r_C$; (iii) $\rho = \rho_{\max} < v$ at the turning point $r = r_{\max} < r_C$.

In all three cases, we multiply Eq. (23) by $(\rho - v)$ and integrate over a region Ω to get

$$\int_{\partial\Omega} \lambda(\rho - v) n^\mu \tilde{\nabla}_\mu \rho - \int_\Omega \lambda \left[\tilde{\nabla}_\mu \rho \tilde{\nabla}^\mu \rho - \rho(\rho - v) \left(\phi + \frac{1}{\lambda q v} \dot{\eta} \right)^2 + \alpha (\rho - v)^2 \rho (\rho + v) \right] = 0. \tag{24}$$

The region Ω and its boundary $\partial\Omega$ for the three cases are taken, respectively, to be (i) $\Omega = \Sigma$, $\partial\Omega = \partial\Sigma$; (ii) $\Omega = \Sigma|_{r<r_v}$, $\partial\Omega =$ spheres at r_H, r_v; (iii) $\Omega = \Sigma|_{r<r_{\max}}$ $\partial\Omega =$ spheres at r_H, $r_{\max}$.

In all three cases, the integral over $\partial\Omega$ vanishes, and all terms in the Ω integral are non-negative everywhere on Ω. So we have a contradiction and ρ cannot increase from zero as r increases from r_H. These arguments can be trivially modified to show that ρ cannot decrease from zero as r increases from r_H, nor can ρ increase or decrease from zero as r decreases from r_C. So in general, $\rho \neq 0$ at either horizon, so the electric field vanishes on Σ, and the black-hole does not carry an electric charge, which is the no-hair statement. There is however one exception. This is the solution for which $\rho = 0$ on all of Σ. Then Eqs. (20) and (22) are the same as those for the ordinary Maxwell-Einstein system. Then the black hole may carry an electric charge, and the space-time is described by the Reissner-Nördstrom–de Sitter solution.

We also note here that the assumption of spherical symmetry is not crucial for the proofs, except for the Higgs model. So axisymmetric black-holes are hairless for most field theories, while dipole or other axisymmetric hair cannot be ruled out for the Higgs model.

We have proved various no-hair theorems by restricting attention to the region between the two horizons for black-hole space-times with $\Lambda = 0$. Unlike usual investigations of black-hole space-times, we have managed to completely ignore the asymptotic behavior. This is the new paradigm referred to earlier, which we believe should be useful in further investigations of $\Lambda > 0$ space-times. Interestingly, the Abelian Higgs system allows a charged solution which has no counterpart in the asymptotically flat case. This suggests the intriguing possibility that, even for the $\Lambda = 0$ black holes with hair, there may be additional classes of solutions for $\Lambda = 0$, coming from nontrivial boundary conditions at the two horizons. For example, black holes pierced by a cosmic string [16], black holes with nontrivial external Yang-Mills and Higgs fields, or Skyrme black holes [17,18] may have more varied counterparts for $\Lambda > 0$. Black holes with discrete gauge hair (see [19] for a review), because of the underlying Higgs model, may be dressed differently for $\Lambda > 0$. There may also be new axisymmetric solutions in a Higgs background. Other kinds of quantum hair such as non-Abelian quantum hair [19,20] or spin-two hair [21], whose existence are related to the topology of the space-time, are likely to be present also for $\Lambda > 0$.

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