Small Shear Viscosity of a Quark-Gluon Plasma Implies Strong Jet Quenching

Abhijit Majumder,[1] Berndt Müller,[1] and Xin-Nian Wang[2]
[1]*Department of Physics, Duke University, Durham, North Carolina 27708, USA*
[2]*Nuclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
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We derive an expression relating the transport parameter $\hat{q}$ and the shear viscosity η of a weakly coupled quark-gluon plasma. A deviation from this relation can be regarded as a quantitative measure of "strong coupling" of the medium. The ratio $T^3/\hat{q}$, where T is the temperature, is a more broadly valid measure of the coupling strength of the medium than η/s, where s denotes the entropy density. Different estimates of $\hat{q}$ derived from existing Relativistic Heavy Ion Collider data are shown to imply radically different structures of the produced matter.

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The highly excited, strongly interacting matter formed in collisions of large nuclei at the Relativistic Heavy Ion Collider (RHIC) exhibits two unusual properties: final hadrons at large transverse momentum p_T are strongly suppressed in central collisions [1] and the collective flow of the matter is well described by hydrodynamics with a negligible shear viscosity [2]. The suppression of hadrons at large p_T, generally referred to as jet quenching, is understood to be caused by parton energy loss induced by multiple collisions of the leading parton with color charges in the near-thermal medium [3–5]. It is still under debate which fraction of the energy loss is due to radiative and which to elastic collisions [6].

The RHIC data have been interpreted to imply that the quark-gluon plasma produced in the nuclear collision is a strongly coupled medium, which may not even contain quasiparticles whose interactions can be treated in some effective perturbation theory [7]. While the absolute values of the shear viscosity-to-entropy ratio η/s determined from model analysis of RHIC data tend to rule out a class of weak coupling approaches based on the hard-thermal loop (HTL) approximation, it may still be consistent with the general picture based on weakly coupled quasiparticles with partonic (quark and gluon) quantum numbers. The present Letter outlines a method for quantitatively testing this hypothesis.

The interaction of hard jets with strongly interacting matter may always be treated in a perturbative expansion involving short lived partonic excitations as the QCD coupling is weak at short distances. Thermal excitations in a QCD medium, on the other hand, may or may not be partonic quasiparticles. Here, we take the term "weakly coupled" to mean that the properties of the medium can be described perturbatively on the basis of an appropriate partonic quasiparticle picture. This notion does not preclude the possibility that the quasiparticles themselves may emerge nonperturbatively from the fundamental quanta of QCD. "Strong coupling" indicates the absence of such a partonic quasiparticle description. The central assertion advanced here is that there exists a general relation between the transport parameter $\hat{q}$ governing multiple scattering of an energetic parton and the shear viscosity η which holds for any weakly coupled partonic plasma where the interaction between the quasiparticles, which is responsible for the generation of a viscosity, has the same structure and strength as the interaction between the leading jet parton and the medium.

In any partonic quasiparticle framework, the transport parameter governing the radiative energy loss of a propagating parton in SU(3)-color representation R is [4]

$$\hat{q}_R \equiv \rho \int dq_\perp^2 q_\perp^2 \frac{d\sigma_R}{dq_\perp^2}. \quad (1)$$

The shear viscosity of a fluid is defined as the coefficient of the contribution to its stress tensor, which is proportional to the divergence-free part of the velocity gradient. In the framework of kinetic theory based on a quasiparticle picture, the shear viscosity η is determined by the mean-free path $\lambda_f(p)$ of a constituent particle of momentum p in the medium:

$$\eta \sim C\rho\langle p\rangle\lambda_f, \quad (2)$$

with $C \approx 1/3$ [8,9]. A heuristic connection between η and $\hat{q}$ can be established by the observation that the mean-free path is related to the average transport cross section of a quasiparticle in the medium: $\lambda_f = (\rho\sigma_{\rm tr})^{-1}$. When soft scattering dominates, as in the case of perturbative QCD, the transport cross section is related to the differential cross section by the relation:

$$\sigma_{\rm tr} \approx \frac{4}{\hat{s}} \int dq_\perp^2 q_\perp^2 \frac{d\sigma}{dq_\perp^2} \equiv \frac{4\hat{q}}{\hat{s}\rho}, \quad (3)$$

where $\sqrt{\hat{s}}$ is the center-of-mass energy. For a thermal ensemble of massless particles, $\langle p\rangle \approx 3T$ and $\langle\hat{s}\rangle \approx 18T^2$, and thus:

$$\eta \approx 13.5C\frac{T^3\rho}{\hat{q}}. \quad (4)$$

Using the relation $s \approx 3.6\rho$ for the entropy density of a gas of free massless bosons, the following heuristic equation relating the ratio of the shear viscosity to the entropy density with the transport coefficient may be derived:

$$\frac{\eta}{s} \approx 3.75C\frac{T^3}{\hat{q}}. \quad (5)$$

This relation shows that a large value of $\hat{q}$ implies a small value for the ratio η/s, which is thought to be bounded by the quantum limit $\eta/s \geq (4\pi)^{-1}$ [10].

When perturbation theory can be applied to the medium, the jet quenching parameter can be expressed in terms of the light-cone gluon correlation function [4,5]

$$\hat{q}_R = \frac{4\pi C_R\alpha_s}{N_c^2-1}\int dy^-\langle F^{ai+}(0)F_i^{a+}(y^-)\rangle e^{i\xi p^+y^-}. \quad (6)$$

In Eq. (6), $\langle\mathcal{O}\rangle = (2\pi)^{-3}\int d^3p/2p^+f(p)\langle p|\mathcal{O}|p\rangle$ denotes the ensemble average of an operator $\mathcal{O}$ in the medium composed of states $|p\rangle$ with occupation probability $f(p)$, $\xi = \langle k_T^2\rangle/2E\langle p^+\rangle$, $\langle k_T^2\rangle$ is the average transverse momentum carried by the gluons in $|p\rangle$, and the index i sums over the two transverse directions. $\rho = \int d^3pf(p)/(2\pi)^3$ denotes the density of scattering centers, mainly gluons, in the matter. $d\sigma_R/dq_\perp^2$ is the differential cross section for a parton on a scattering center. A nonperturbative definition of $\hat{q}_R$ in terms of a Wilson loop along the light cone has been proposed in Ref. [11].

In principle, $\hat{q}_R(E)$ depends weakly on the energy E and virtuality of the jet-inducing parton. In the limit of a thick (large opacity) medium of thickness L, the virtuality of the parton is determined by the saturation scale Q_s of the medium [11], and the transport parameter for parton energy loss takes on the universal form $\hat{q} = Q_s^2/L$. We will neglect a possible logarithmic energy dependence of $\hat{q}$ here. For a massless parton in a weakly coupled gluon plasma:

$$\hat{q}_R = \frac{8\zeta(3)}{\pi}N_cC_R\alpha_s^2T^3\ln\frac{1}{\alpha_s}, \quad (7)$$

in the leading log approximation [12]. In the remainder of this Letter, we will omit the subscript in the notation for the transport parameter $\hat{q} \equiv \hat{q}_A$ for a gluon jet.

The expression for the shear viscosity depends on the mechanism that limits momentum transport in the medium. We start with the shear viscosity due to parton collisions in the quark-gluon plasma, following Arnold *et al.* [13]. In the Chapman-Enskog approach to linearized transport theory, one parametrizes the deviation $f_1(p)$ of the parton distribution from equilibrium due to shear by a function $\bar{\Delta}(p)$ in the form (using the notation of [14])

$$f_1(p) = -\frac{\bar{\Delta}(p)}{ET^2}p_ip_j(\nabla u)_{ij}, \quad (8)$$

where $(\nabla u)_{ij}$ denotes the traceless velocity gradient. One can then derive an integral equation for $\bar{\Delta}(p)$ from the linearized Boltzmann equation. An analytic estimate can be obtained by restricting $\bar{\Delta}$ to the functional form $\bar{\Delta}(p) = Ap/T$. The shear viscosity is expressed in terms of $\bar{\Delta}(p)$ as

$$\eta_C = -\frac{1}{15T}\int\frac{d^3p}{(2\pi)^3}\frac{p^4}{E^2}\bar{\Delta}(p)\frac{\partial f_0}{\partial E}, \quad (9)$$

where $f_0(p)$ denotes the equilibrium distribution of partons. Assuming that collisions in the medium are dominated by soft scattering, the integral over the differential scattering cross section can be extracted from the collision integral, yielding a factor $\hat{q}/\rho$, where $\hat{q}$ is defined by Eq. (1) for a gluon jet. After a lengthy calculation one finds that in this limit the shear viscosity for a thermal gluon plasma can be expressed in the form:

$$\eta_C \approx C'\frac{T^3\rho}{\hat{q}}. \quad (10)$$

In order to extract the constant, we can make use of the known result for the shear viscosity of a pure gluon gas in the leading logarithmic approximation [13]:

$$\eta_{\rm LL} = \frac{3.81}{\pi^2}\frac{N_c^2-1}{N_c^2}\frac{T^3}{\alpha_s^2\ln(1/\alpha_s)}. \quad (11)$$

Inserting the jet quenching parameter in the weak coupling limit (7) and using the expression for the density of a thermal gas of free gluons, we obtain $C' \approx 4.85$. Comparing with Eq. (4), we find that this corresponds to $C \approx 0.36$.

We next consider the case of anomalous shear viscosity [14], which is generated by dynamically created turbulent color fields in a rapidly expanding quark-gluon plasma. Unstable collective plasma modes [15] result in the growth of long wavelength modes of the glue fields, which ultimately saturate due to their nonlinear self-interaction [16,17]. The turbulent quark-gluon plasma is characterized by a nonvanishing expectation value of the gluon correlator $\langle F^{\sigma+}(0)F_\sigma^+(y^-)\rangle$ of the soft color fields.

The presence of localized domains of color fields induces an anomalous contribution to the shear viscosity of the matter [14]. This contribution is derived by considering the effect of random color fields on the propagation of quasithermal partons, which is described by a Fokker-Planck equation of the form:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\mathbf{\nabla}_r - \nabla_pD^R(\mathbf{p},t)\nabla_p\right]f(\mathbf{p},\mathbf{r},t) = 0 \quad (12)$$

with the average parton phase space distribution f and the diffusion tensor

$$D_{ij}^R = \int_{-\infty}^t dt'\langle F_i[\bar{r}(t'),t']F_j[\bar{r}(t),t]\rangle. \quad (13)$$

Here $\mathbf{F} = gQ_a^R(\mathbf{E}^a + \mathbf{v}\times\mathbf{B}^a)$ is the color Lorentz force generated by the turbulent color fields and Q^R is color charge of the parton in the given representation. The anomalous shear viscosity implied by Eq. (12) can be evaluated, e.g., for randomly distributed color fields with a correlation time τ_m along the light cone [14]:

$$\eta_A = \frac{16\zeta(6)(N_c^2 - 1)^2 T^6}{\pi^2 N_c g^2 \langle E^2 + B^2 \rangle \tau_m}. \quad (14)$$

The diffusion coefficient D_{ij} in Eq. (13) is closely related to the transport parameter for radiative energy loss, Eq. (6). Applying the definition of $\hat{q}_A$ as rate of growth of the transverse momentum fluctuations of a fast gluon to an ensemble of turbulent color fields, one finds (for partons in the adjoint color representation):

$$\hat{q} = \frac{8\pi\alpha_s N_c}{3(N_c^2 - 1)} \langle E^2 + B^2 \rangle \tau_m. \quad (15)$$

Combining this expression with Eq. (14) we obtain a relation between the contributions of turbulent color fields to the transport parameter in gluon energy loss and the anomalous shear viscosity of gluons:

$$\eta_A = \frac{32\zeta(6)(N_c^2 - 1)}{3\pi^2} \frac{T^6}{\hat{q}} \approx 4.51 \frac{T^3 \rho}{\hat{q}}. \quad (16)$$

The numerical coefficient here is very close to the one obtained for the collisional viscosity. Alternatively, we can calculate the dimensionless ratio of the shear viscosity and the entropy density. For a free thermal gluon gas, the entropy density is given by

$$s = 2(N_c^2 - 1) \frac{2\pi^2}{45} T^3, \quad (17)$$

which implies the following result for the shear viscosity-to-entropy ratio:

$$\frac{\eta_A}{s} = \frac{8\pi^2}{63} \frac{T^3}{\hat{q}} \approx 1.25 \frac{T^3}{\hat{q}}, \quad (18)$$

again in agreement with (5) for $C \approx 1/3$. This confirms our assertion that Eq. (5) holds if the transport properties of a quark-gluon plasma are described by kinetic theory based on partonic quasiparticles.

In order to explore the opposite limit, we first consider a solvable strongly coupled theory, $N = 4$ supersymmetric Yang-Mills (SYM) theory. Analytical results for η/s and $\hat{q}$ have been derived for this theory in the limit of large 't Hooft coupling $\lambda = g^2 N_c$. The strong coupling expression for the ratio of shear viscosity-to-entropy density is well established [18,19]:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + \frac{135\zeta(3)}{(8\lambda)^{3/2}} + \cdots\right]. \quad (19)$$

There is not yet universal agreement about the correct generalization of the jet quenching parameter. Here we follow Liu *et al.* [11] who defined $\hat{q}$ by means of an adjoint Wilson loop along the light cone and found the result

$$\hat{q} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)} \sqrt{\lambda} T^3 \approx 7.53 \sqrt{\lambda} T^3. \quad (20)$$

Inserting this expression into the right-hand side of (5) for $C \approx 1/3$, we obtain

$$1.25 \frac{T^3}{\hat{q}} \approx \frac{0.166}{\sqrt{\lambda}} \ll \frac{\eta}{s} \quad (21)$$

for $\lambda \gg 4.35$. Obviously, the left-hand side of (21) is a better measure of the strength of the coupling in the strong coupling limit than the right-hand side.

The strong coupling limit of QCD exhibits quark confinement. As is well known, this limit of QCD is also amenable to rigorous calculations in the chiral limit and at low temperature. In this limit, the shear viscosity-to-entropy density ratio of a pion gas is given by [20]:

$$\frac{\eta}{s} \approx \frac{15 f_\pi^4}{16\pi T^4}. \quad (22)$$

The ratio grows rapidly with decreasing temperature, because the pion-pion interaction weakens at low momentum. On the other hand, the jet quenching parameter for a gluon jet propagating through a pion gas has the form

$$\hat{q}_A^{(\pi)} \approx \frac{4\pi^2 N_c \alpha_s}{N_c^2 - 1} \rho_\pi [x G_\pi(x)]_{x\to 0}, \quad (23)$$

where $G_\pi(x)$ is the gluon distribution function of the pion. Since the pion density in the chiral limit is proportional to T^3, this implies that the left-hand side of (21) is approximately independent of T:

$$1.25 \frac{T^3}{\hat{q}_A^{(\pi)}} \approx \frac{0.235}{\alpha_s [x G_\pi(x)]_{x\to 0}}. \quad (24)$$

Again, $T^3/\hat{q}$ is a much better measure of the underlying QCD coupling strength in this domain than η/s. We note that $T^3/\hat{q} \ll \eta/s$ in both cases of strong coupling. It is not surprising that the heuristic relation (4) does not hold in the strong coupling limit: in a strongly coupled system, thermal modes are not approximately described as elementary quasiparticles [21] probed by jet quenching. On the other hand, highly energetic excitations can retain a quasiparticle nature, and their interactions with the medium continue to track the strength of the coupling. The violation of the relation (4) may thus be considered as a general criterion for "strongly coupling" in a gauge theory.

We can now ask whether the relation (4) applies to the matter produced at RHIC, according to the phenomenological analysis of jet quenching measurements. At present, the values of $\hat{q}$ deduced from the data by means of different approaches to jet quenching differ from each other by substantial factors [22]. Recent studies within the framework of the twist expansion, fitting to the experimental data on hadron suppression in the most central Au + Au collisions at RHIC [23,24], gave values $\hat{q}_0^{(\mathrm{ht})} = $ 1–2 $\mathrm{GeV}^2/\mathrm{fm}$ for the gluon quenching parameter at the initial time $\tau_0 \approx 1$ fm/c. A study [25] using a variant of the eikonal approach [4] resulted in a drastically different estimate $\hat{q}_0^{(\mathrm{eik})} = $ 10–30 $\mathrm{GeV}^2/\mathrm{fm}$ for the same quantity (scaled from the value given in [25] for a propagating quark).

Note that the above studies only considered radiative energy loss. Inclusion of collisional energy loss will affect the value of the extracted $\hat{q}_0$. Fortunately, the uncertainty in the value of $\hat{q}$ caused by the ambiguity of the energy loss mechanism can be eliminated in the future by measuring $\hat{q}_0$ directly via the transverse momentum broadening of jets.

One can estimate [26] the average initial entropy density from the measured charge hadron multiplicity of the most central Au + Au collisions as $s_0 = (33 \pm 3)\ \mathrm{fm}^{-3}$ at $\tau_0 = 1\ \mathrm{fm}/c$ corresponding to an initial temperature $T_0 = (337 \pm 10)$ MeV. Using Eq. (5) with $C = 1/3$ one then finds that the lower value $\hat{q}_0^{(\mathrm{ht})}$ implies $1.25T_0^3/\hat{q}_0 =$ 0.12–0.24, whereas the higher value $\hat{q}_0^{(\mathrm{eik})}$ implies $1.25T_0^3/\hat{q}_0 = 0.008$–$0.024$. This first result is close to the conjectured lower bound for η/s and lies well within the range of values for the shear viscosity-to-entropy density ratio ($\eta/s < 0.3$) which are compatible with the measured hadron spectra from Au + Au collisions at RHIC [27]. If ultimately confirmed, it would indicate that the quark-gluon plasma produced in Au + Au collisions is (marginally) weakly coupled. On the other hand, the second result lies far below the lower bound for η/s and thus implies that the quark-gluon plasma formed at RHIC is deep in the strong coupling regime and cannot be described as a quasiparticle plasma.

In summary, we have derived a general relation between the shear viscosity η and the parameter $\hat{q}$ for a "weakly coupled", i.e., quasiparticle dominated quark-gluon plasma. The relation associates a small ratio of shear viscosity-to-entropy density to a large value of the parameter $\hat{q}$ governing multiple scattering of a hard parton in the quark-gluon plasma.

The fact that η/s saturates in the limit of strong coupling of the SYM theory but $\hat{q}$ continues to increase, suggests that the ratio $T^3/\hat{q}$ may serve as a more broadly applicable measure of the coupling strength of a quark-gluon plasma. We thus conjecture that the following relations hold generally:

$$\frac{\eta}{s}\left\{\begin{matrix}\approx\\ \gg\end{matrix}\right\}1.25\frac{T^3}{\hat{q}}\left\{\begin{matrix}\text{for weak coupling,}\\ \text{for strong coupling.}\end{matrix}\right. \tag{25}$$

An unambiguous determination of both sides of (25) from experimental data would thus permit a model independent, quantitative assessment of the strongly coupled nature of the quark-gluon plasma produced in heavy ion collisions [28].

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[1] K. Adcox *et al.*, Phys. Rev. Lett. **88**, 022301 (2001); C. Adler *et al.*, Phys. Rev. Lett. **89**, 202301 (2002).
[2] D. Teaney, Phys. Rev. C **68**, 034913 (2003).
[3] M. Gyulassy and X.-N. Wang, Nucl. Phys. **B420**, 583 (1994); B. G. Zhakharov, JETP Lett. **63**, 952 (1996); U. Wiedemann, Nucl. Phys. **B588**, 303 (2000); M. Gyulassy, P. Lévai, and I. Vitev, Nucl. Phys. **B594**, 371 (2001).
[4] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff, Nucl. Phys. **B483**, 291 (1997); **B484**, 265 (1997).
[5] X. F. Guo and X.-N. Wang, Phys. Rev. Lett. **85**, 3591 (2000); Nucl. Phys. **A696**, 788 (2001).
[6] See, e.g., S. Wicks, W. Horowitz, M. Djordjevic, and M. Gyulassy, Nucl. Phys. **A784**, 426 (2007).
[7] M. Gyulassy and L. McLerran, Nucl. Phys. **A750**, 30 (2005).
[8] P. Danielewicz and M. Gyulassy, Phys. Rev. D **31**, 53 (1985).
[9] S. R. deGroot, W. A. van Leeuwen, and Ch. G. van Weert, *Relativistic Kinetic Theory* (North-Holland, Amsterdam, 1980).
[10] P. K. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005).
[11] H. Liu, K. Rajagopal, and U. A. Wiedemann, Phys. Rev. Lett. **97**, 182301 (2006).
[12] R. Baier and D. Schiff, J. High Energy Phys. 09 (2006) 059.
[13] P. Arnold, G. D. Moore, and L. G. Yaffe, J. High Energy Phys. 11 (2000) 001.
[14] M. Asakawa, S. A. Bass, and B. Müller, Phys. Rev. Lett. **96**, 252301 (2006); Prog. Theor. Phys. **116**, 725 (2006).
[15] J. Randrup and S. Mrowczynski, Phys. Rev. C **68**, 034909 (2003); P. Romatschke and M. Strickland, Phys. Rev. D **68**, 036004 (2003); P. Arnold, J. Lenaghan, and G. D. Moore, J. High Energy Phys. 08 (2003) 002.
[16] A. Rebhan, P. Romatschke, and M. Strickland, J. High Energy Phys. 09 (2005) 041.
[17] P. Arnold and G. D. Moore, Phys. Rev. D **73**, 025006 (2006); **73**, 025013 (2006).
[18] G. Policastro, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001).
[19] A. Buchel, J. T. Liu, and A. O. Starinets, Nucl. Phys. **B707**, 56 (2005).
[20] J. W. Chen and E. Nakano, Phys. Lett. B **647**, 371 (2007).
[21] D. Teaney, Phys. Rev. D **74**, 045025 (2006).
[22] A. Majumder, J. Phys. G **34**, S377 (2007).
[23] H. Chang, J. F. Owens, E. Wang, and X.-N. Wang, Phys. Rev. Lett. **98**, 212301 (2007).
[24] A. Majumder, C. Nonaka, and S. A. Bass, arXiv:nucl-th/0703019.
[25] B. Sahlmueller, J. Phys. G **34**, S969 (2007).
[26] B. Müller and K. Rajagopal, Eur. Phys. J. C **43**, 15 (2005).
[27] R. Baier and P. Romatschke, Eur. Phys. J. C **51**, 677 (2007).
[28] J.-P. Blaizot, J. Phys. G **34**, S243 (2007).