

# Long-Range Electron Spin-Spin Interactions from Unparticle Exchange

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Unparticles as suggested by Georgi are identities that are not constrained by dispersion relations but are governed by their scaling dimension  $d$ . Their coupling to particles can result in macroscopic interactions between matter that are generally an inverse nonintegral power of distance. This is totally different from known macroscopic forces. We use the precisely measured long-ranged spin-spin interaction of electrons to constrain unparticle couplings to the electron. For  $1 < d < 1.5$  the axial vector unparticle coupling is excluded, and for  $1 < d < 1.3$  the pseudoscalar and vector couplings are also ruled out. These bounds and the ones for other ranges of  $d$  exceed or are complementary to those obtained previously from exotic positronium decays.

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Gravity and electromagnetism are the only known fundamental interactions extending to a macroscopic distance. Because of its basic importance, it has long been a tradition to search for extra long-ranged interactions (for a recent work, see [1]). Most experiments, especially those seeking deviations from the gravitational inverse square law, are sensitive only to spin-independent interactions. A microscopic spin-dependent interaction, which must be feeble to evade direct detection in particle physics experiments if it exists at all, would be simply averaged out for macroscopic bodies. To circumvent the decoherence effects, one has to utilize spin-polarized samples [2]. Although these are relatively new developments, they already yield interesting and unique information beyond spin-independent experiments (for experimental and theoretical reviews, see, e.g., [3,4], respectively).

In the language of quantum field theory (QFT), long-ranged interactions are mediated by massless force quanta, the photon for electromagnetism and the graviton for gravity. In the nonrelativistic (NR) limit, the interaction potential always starts in the form of  $r^{-1}$ , where  $r$  is the separation of the interacting particles. This is a joint result of the two facts in QFT that all particles, including force quanta, are constrained by dispersion relations quadratic in momentum and that we live in a three space. When the spins of interacting particles enter or when the small effects from relativistic corrections or simultaneous multiple exchange of quanta are considered, higher integral powers of  $r^{-1}$  are also present.

It is common in theories beyond the standard model that there exist hypothetical particles which have a mass tiny in the sense that its Compton wavelength could be macroscopic. These particles could then exert a force at a macroscopic distance. In the sense of interactions, there is no surprise: they always follow an inverse integral power of distance up to an exponential factor. These cover novel theories such as compactified extra dimensions where the size of extra dimensions provides an effective mass in four-dimensional theories.

So, what else can we imagine of a macroscopic force? The next simplest or least strange would be a nonintegral power law. What kind of force quantum could mediate such a force? It cannot be a particle excitation, as we discussed above. This may partly explain why such a force has not yet been analyzed. We must confess at this point that we are so used to the concept of particle that it is hard to move a step away from it. Nevertheless, very recently Georgi has made an interesting suggestion for an identity that is not a particle, dubbed unparticle [5]. He proposed a scenario showing how such an identity could appear as a low-energy degree of freedom from a scale invariant fundamental theory at high energy, such as the one studied in [6]. The unparticles must interact with ordinary matter, however feebly, to be physically relevant. These interactions can be well described in effective field theory (EFT) though it is generally difficult to calculate from a fundamental theory.

An unparticle is an identity that does not enjoy mass as one of its intrinsic properties. Namely, it is not constrained by a dispersion relation as for a particle of mass  $m$  and momentum  $p$ ,  $p^2 = m^2$ . Instead, its kinematic property is defined by its scaling dimension,  $d$ , which is generally a nonintegral number. Scale invariance essentially determines its state density and via unitarity its propagating property, up to a normalization factor [5,7,8]. If the normalization is fixed by analogy to the phase space of a system of massless particles, the unparticle with a nonintegral  $d$  could be virtualized as a  $d$  number of invisible massless particles [5].

The lack of a dispersion relation and the existence of a generally nonintegral scaling dimension make unparticles sharply different from particles. It is the purpose of this Letter to demonstrate that an unparticle can mediate a long-range force between particles of a nonintegral inverse power of distance depending on  $d$ . We stress that this is an excluding characteristic of unparticles that cannot be mimicked by any other theory of particle physics, and is thus of fundamental importance. An experimental indication of

such a potential would definitely point to unparticle physics and help discover a scale invariant fundamental theory at high energy. Inversely, by employing experimental constraints on extra macroscopic forces, this sets bounds on the energy scale of unparticle physics. These bounds could be more stringent than those from precision QED tests [9], because a feeble interaction between single particles can be coherently amplified by a macroscopic mass if the force is long-ranged.

Additional surprises have been unveiled previously. Because of lack of a dispersion relation, a kinematically forbidden one-to-one particle transition of different masses becomes possible for a one particle to one unparticle transition [9]. For a nonintegral  $d$ , the propagator gets a nontrivial phase in the timelike region. This produces unusual interference patterns in some processes [7], and serves as a “strong phase” to help discern  $CP$  violating effects [10]. The studies so far have focused on unparticle effects at colliders [7,8,11–14], precision QED tests [8,9,15], flavor-changing neutral current processes [10,15–22], interactions with Higgs bosons [23], in gauge boson scattering [24] and in astrophysics [25]. Some theoretical issues are addressed in Refs. [23,26].

We shall restrict ourselves to the system of electrons although we are aware that there are constraints involving nucleons. The reason is theoretical; for nucleons we have to study unparticle interactions with quarks and gluons to make a direct connection to theory, which are then converted with unavoidable uncertainties to interactions of nucleons. This implies in turn that we should focus on the spin-dependent part of the electron interaction since the spin-independent interaction of macroscopic samples is dominated by that of nucleons. The leading interactions in EFT of the electron ( $\psi$ ) and unparticles are

$$\mathcal{L}_{\text{int}} = C_S \bar{\psi} \psi \mathcal{U}_S + C_P \bar{\psi} i \gamma_5 \psi \mathcal{U}_P + C_V \bar{\psi} \gamma_\mu \psi \mathcal{U}_V^\mu + C_A \bar{\psi} \gamma_\mu \gamma_5 \psi \mathcal{U}_A^\mu, \quad (1)$$

which will induce long-ranged interactions between electrons. Here  $\mathcal{U}_{S,P,V,A}$  stand for the fields of scalar, pseudo-scalar, vector, and axial vector unparticles, respectively. For simplicity, we assign to them the same scaling dimension  $d$ . The couplings can be parametrized by  $C_{S,P,V,A} = \pm c_{S,P,V,A} \Lambda_{S,P,V,A}^{1-d}$ , where  $\Lambda_i$  and  $c_i$  are unknown energy scales and dimensionless positive numbers, respectively. One could set  $\Lambda_i \sim 1$  TeV, say, and constrain  $c_i$ , but we find it simpler to put  $c_i = 1$  and work with  $\Lambda_i$ . The two can easily be converted into each other indeed.

The propagator for a spin-0 unparticle is [7,8]

$$\frac{A_d}{2 \sin(\pi d)} \frac{i}{(-p^2 - i\epsilon)^{2-d}}, \quad (2)$$

$$A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d}} \frac{\Gamma(d + \frac{1}{2})}{\Gamma(d-1)\Gamma(2d)}, \quad (3)$$

with  $p$  being the momentum. For a vector or an axial vector unparticle, we attach a tensor projector for its spin. For the vector one, it is immaterial whether to include the  $p_\mu p_\nu$  term since it vanishes due to current conservation. For the axial one, however, there is no similar conservation law. For definiteness, we shall simply work with  $-g_{\mu\nu}$ . Note that theoretical considerations prefer a narrow range for  $d \in (1, 2)$  [7].

To obtain the potential between electrons, it is sufficient to work out the  $t$ -channel electron scattering amplitude. We shall keep terms up to  $O(m^{-2})$  in the NR expansion where  $m$  is the electron mass, while higher order terms are suppressed at a macroscopic distance. For this, we expand the kinetic term in the Schrödinger equation to the same relative order, as well as the propagator and spinor bilinears [27]. Ignoring terms involving averaged velocities of the electrons in the center of mass frame that are of no interest here, we obtain the potential:

$$U_i^{--}(\mathbf{r}) = U_{\text{spin}}^{--}(\mathbf{r}) + U_{\text{non}}^{--}(\mathbf{r}), \quad (4)$$

where, extracting the common factors  $A_d r^{1-2d}/(4\pi^2)$ ,

$$\begin{aligned} U_{\text{spin}}^{--}(\mathbf{r}) &= -C_A^2 \Sigma_s \Gamma(2(d-1)) + \frac{\Gamma(2d)}{4m^2 r^2} \left[ (d-2) C_A^2 \Sigma_s \right. \\ &\quad \left. - C_P^2 \frac{\Sigma_s - (2d+1)\Sigma_a}{2(d-1)} \right. \\ &\quad \left. + (C_A^2 - C_V^2) \frac{(1-2d)\Sigma_s + (2d+1)\Sigma_a}{2(d-1)} \right], \\ U_{\text{non}}^{--}(\mathbf{r}) &= (C_V^2 - C_S^2) \Gamma(2(d-1)) \\ &\quad + \frac{\Gamma(2d)}{4m^2 r^2} [(2-d)C_V^2 - (3-d)C_S^2], \end{aligned} \quad (5)$$

and  $\Sigma_s = \sigma_1 \cdot \sigma_2$ ,  $\Sigma_a = \sigma_1 \cdot \hat{\mathbf{r}} \sigma_2 \cdot \hat{\mathbf{r}}$ ,  $\hat{\mathbf{r}} = \mathbf{r}/r$  with the subscripts 1, 2 referring to the  $e^- e^-$  pair. The standard result for an exchange of particles is recovered in the limit  $d \rightarrow 1$ , up to contact terms proportional to  $\delta^3(\mathbf{r})$ . The latter cannot be obtained from the general result because a simpleminded computation would give incorrectly  $\nabla^2 r^{-1} \rightarrow 0$ , although this is safe for  $d \neq 1$ .

Before we embark on the long-ranged interactions, we calculate the hyperfine splitting (hfs) between the ortho- and parapositronium ground state. There are two contributions to the  $e^- e^+$  potential, one from the  $t$ -channel exchange and the other from the  $s$ -channel annihilation. The former,  $U_i^{-+}$ , is obtained from  $U_i^{--}$  by  $C_V^2 \rightarrow -C_V^2$ . The latter gives in the NR limit:

$$\begin{aligned} U_s^{-+}(\mathbf{r}) &= A_d/[4 \sin(\pi d)](-4m^2 c^2 - i\epsilon)^{d-2} \delta^3(\mathbf{r})[(3C_V^2 \\ &\quad + C_P^2 + C_A^2) + (C_V^2 - C_P^2 - C_A^2)\sigma_- \cdot \sigma_+], \end{aligned} \quad (6)$$

which is generally complex. Here the indices  $\pm$  refer to the  $e^- e^+$  pair. Since the above is higher order than the  $t$  channel for  $d < 2$ , we ignore it from now on.

Some comments are in order. Our main aim is to work out long-ranged interactions of electrons. For this, the naive NR expansion is suitable: higher terms will yield less important terms. But for short-ranged bound state problems there is no guarantee that higher terms make sense as they become more singular than lower ones. This happens already in QED: the expansion works well until terms of  $(mc)^{-2}$  (with  $c$  being the velocity of light) because radiation enters only at  $O(c^{-3})$  [27]. We thus retain only the leading term  $\sim r^{1-2d}$  in  $U_i^{-+}$ . For  $d \in (1, 1.5)$ , it behaves well; for  $d \in (1.5, 2)$ , it still yields a meaningful result for the level shifts as long as it is treated as a perturbation, although a negative potential singular than  $r^{-2}$  results in the phenomenon of falling to center. This is again similar to the QED case.

After these considerations, the only term relevant for hfs is the leading  $C_A^2$  term. Using  $\langle r^{1-2d} \rangle = 2^{2d-2} a^{1-2d} \Gamma[2(2-d)]$  for the positronium ground state with  $a = 2/(m\alpha)$ , we obtain the relative shift:

$$E(1^3S_1) - E(1^1S_0) = -m\alpha^{2d-1} \left( \frac{C_A}{m^{1-d}} \right)^2 \frac{A_d}{2\pi^2} \Gamma(2(d-1)) \Gamma(2(2-d)), \quad (7)$$

which is negative for  $d \in (1, 2)$ . Note that the  $s$ -channel contribution is lower by a factor of  $\alpha^{2(2-d)}$ . The most recent QED calculations [28,29] yield the value  $+203.391\,69(41, 16)$  GHz, to be compared with the measured ones,  $+203.387\,5(16)$  GHz [30] and  $+203.389\,10(74)$  GHz [31]. Since it is hard to imagine higher order QED corrections can further reduce the discrepancy, we suppose the gap is filled by the unparticle. Using the most precise experimental value, we obtain

$$\Lambda_A \geq 21 \text{ TeV} \quad \text{for } d = 1.5. \quad (8)$$

The bound decreases as  $d$  increases.

Now we come to the macroscopic force mediated by unparticles. As explained earlier, our main interest is in the spin-spin force between electrons. To our knowledge, there are four precise yet reliable experiments so far. Two of them used a torsion pendulum [3,32]. They got a similar bound on anomalous electron's spin-spin interaction that is less stringent by a factor of 20 or 40 than those by experiments of induced paramagnetization [33,34]. In [33], a pair of spin-polarized bodies made of  $\text{Dy}_6\text{Fe}_{23}$  were used. With all magnetic fields shielded and in the presence of an anomalous spin-spin interaction, they would induce magnetization in a paramagnetic salt sample made of  $\text{TbF}_3$ . The anomalous interaction is parametrized by a standard magnetic dipole-dipole interaction with a global factor  $\alpha_s$  measuring the relative strength. They set the limit  $\alpha_s = (2.7 \pm 2.4) \times 10^{-14}$  [33]. In [34], another pair of spin-polarized bodies made of  $\text{HoFe}_3$  were added and aligned perpendicularly to the pair of  $\text{Dy}_6\text{Fe}_{23}$ . There are now two kinds of signals as the table holding the bodies rotates. The limit set from the new pair is  $\alpha_s = (-2.1 \pm 3.5) \times 10^{-14}$ .

They combined the two to reach the final limit:

$$\alpha_s = (1.2 \pm 2.0) \times 10^{-14}. \quad (9)$$

When employing the above limit, we should note the differences between our interaction and the one used in fitting. Ours is generally not of a standard dipole-dipole form in either the  $r$  dependence ( $r^{1-2d}$  or  $r^{-1-2d}$  instead of  $r^{-3}$ ) or the relative weight of  $\Sigma_{s,a}$  [not in a ratio of 1:(-3)]. An accurate Monte Carlo simulation based on our interaction is certainly welcome, but this is not possible without detailed knowledge of the samples and apparatus, especially their geometric properties. Fortunately, because of the special arrangement in those experiments, we can make reasonably good approximations. We note that the magnetization direction of the salt lies in a plane parallel to the plane of polarization of the spin-polarized bodies. Their dimensions are much smaller than the vertical separation between the salt and the bodies, and the bodies are close to the apparatus's axis where the salt is placed. Considering all of this, we expect that the spin-spin interaction between the masses scales with the vertical distance up to an order one geometric factor and that the  $\Sigma_a$  term is much smaller than  $\Sigma_s$  because  $\hat{\mathbf{r}}$  is very close to being perpendicular to the spins for most pairs of the electrons in the salt and the bodies. Isolating the  $\Sigma_s$  terms whose coefficients are constrained by  $-0.8 < \alpha_s \times 10^{14} < 3.2$ , we can set bounds on  $C_i$ 's.

The largest contribution comes from the  $C_A^2 r^{1-2d}$  term with others suppressed by a tiny factor of  $(mr_0)^{-2}$ , where  $r_0$  is the characteristic distance in the experiment. Since the term is negative, we use the lower bound of  $\alpha_s$  to get

$$\left( \frac{\Lambda_A}{\text{TeV}} \right)^{2(d-1)} \geq 3.17 \times 10^4 \frac{\Gamma(d - \frac{1}{2})}{(2\pi)^{2d} \Gamma(d)} K^{2(d-2)}, \quad (10)$$

with  $K = 0.2 \times 10^{-16} \text{ cm}/r_0$ . The bound is shown in Table I for a typical  $r_0 = 25 \text{ cm}$ . For  $1.5 < d < 2$ ,  $\Lambda_A$  is very stringently bounded. [Equivalently, one could assume  $\Lambda_A \sim 1 \text{ TeV}$  and constrain  $c_A$ ; for instance, at  $d = 1.6$ , one has  $c_A < 10^{-8}$ .] For  $1 < d < 1.5$ , we have practically  $C_A \sim 0$  since  $\Lambda_A$  is close to or exceeds the Planck scale, so that we should consider the  $O(m^{-2})$  terms. Though  $C_{P,V}$  terms differ in sign and partly cancel, we cannot gain more by treating them together because their scaling dimensions

TABLE I. Bounds on  $\Lambda_{A,P,V}$  (in TeV) are shown as a function of  $d$ . Data from Ref. [34] are used with a typical distance  $r_0 = 25 \text{ cm}$ .  $\times$  stands for scales far in excess of the Planck scale and  $-$  for scales too low to be useful.

$d$	$\log_{10} \Lambda_A$	$\log_{10} \Lambda_P$	$\log_{10} \Lambda_V$	$d$	$\log_{10} \Lambda_A$
1.2	$\times$	$\times$	$\times$	1.6	13.7
1.3	$\times$	6.44	5.77	1.7	9.04
1.4	$\times$	0.126	-0.307	1.8	5.53
1.5	20.3	-	-	1.9	2.81

are generally different. We choose to consider them one by one and get separate bounds as follows:

$$\left(\frac{\Lambda_P}{\text{TeV}} \frac{1}{K}\right)^{2(d-1)} \geq 6.07 \times 10^{16} \frac{\Gamma(d + \frac{1}{2})}{(2\pi)^{2d}\Gamma(d)}, \quad (11)$$

$$\left(\frac{\Lambda_V}{\text{TeV}} \frac{1}{K}\right)^{2(d-1)} \geq 1.52 \times 10^{16} \frac{\Gamma(d + \frac{1}{2})(2d-1)}{(2\pi)^{2d}\Gamma(d)}. \quad (12)$$

The bounds are also shown in Table I.

Unparticles result in a long-ranged force between matter, which is generally an inverse nonintegral power of distance, most likely between Coulomb and dipole ones. This is unique to unparticles and cannot be disguised by particles in any other model conceived so far. An experimental indication of it would unambiguously point to unparticle physics and significantly modify our standard conception of particle physics. On the other hand, existing experiments on macroscopic electron's spin-spin interactions are already useful in assessing the relevance of unparticles in our world. The obtained pattern of constraints is complementary to that in positronium decays [9], and they together constitute the best constraints worked out hitherto. This highly restricts the relevance of unparticles in electron-involved processes studied in the literature. For  $1 < d < 1.5$ , the axial vector unparticle coupling is excluded. For  $1.5 \leq d < 2$ , the bound on it is much more stringent than in positronium decays. For  $1 < d < 1.3$ , the pseudoscalar and vector unparticle couplings are also ruled out. At  $d \sim 1.5$ , however, the bounds on them are less stringent than from positronium decays. Since we are restricted to spin-spin interactions, the scalar unparticle does not set in at the considered order, which, however, is constrained by positronium decays. Finally, we have studied the positronium hfs due to unparticles. Although this is best measured in positronium spectroscopy in absolute precision, it cannot compete with its decays or macroscopic experiments.

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