

Driving Light Pulses with Light in Two-Level Media

R. Khomeriki^{1,2} and J. Leon¹

¹Laboratoire de Physique Théorique et Astroparticules CNRS-IN2P3-UMR5207, Université Montpellier 2, 34095 Montpellier, France

²Physics Department, Tbilisi State University, 0128 Tbilisi, Georgia

(Received 28 July 2007; published 30 October 2007)

A two-level medium, described by the Maxwell-Bloch system, is engraved by establishing a standing cavity wave with a linearly polarized electromagnetic field that drives the medium on both ends. A light pulse, polarized along the other direction, then scatters the medium and couples to the cavity standing wave by means of the population inversion density variations. We demonstrate that control of the applied amplitudes of the grating field allows one to stop the light pulse and to make it move backward (eventually to drive it freely). A simplified limit model of the Maxwell-Bloch system with *variable boundary driving* is obtained as a discrete nonlinear Schrödinger equation with *tunable external potential*. It reproduces qualitatively the dynamics of the driven light pulse.

DOI: 10.1103/PhysRevLett.99.183601

PACS numbers: 42.50.Gy, 42.65.Re

Introduction.—Manipulation of light with light has become one of the hottest research spots in quantum optics this past decade. A widely studied field of research makes use of electromagnetically induced transparency in three-level systems, which allows one to slow down, and eventually stop, a light pulse [1–3]. Another interesting research option uses *resonantly absorbing Bragg reflectors* (RABR) which consist in a periodic array of dielectric films separated by layers of a two-level medium [4–6], allowing a light pulse not only to be stopped and trapped [7] but also to be released by scattering with another control pulse, thus creating a *gap soliton memory* [8].

The fundamental process underlying such novel light pulse dynamics is the cooperative action of nonlinearity and periodicity. Still, a serious drawback when making use of RABR is the built-in periodic structure that restricts both the freedom of pulse parameter and of pulse dynamics.

We propose to prepare a two-level system (TLS) by establishing a standing electromagnetic wave in a given polarization direction, and then to scatter a light pulse, orthogonally polarized. The incident pulse then feels the *electromagnetic induced grating* through the coupling mediated by the population density, as described by the governing Maxwell-Bloch (MB) system [9,10]. The freedom in the choice of the standing wave parameters (in particular the boundary amplitudes) allows us to demonstrate by numerical simulations as in Fig. 1 that the incident light pulse not only can be stopped but also can be *released back* to the incoming end. Engraving a medium with a cavity standing wave is a method previously used to create two-dimensional waveguide arrays in strongly anisotropic photonic crystals [11].

In a TLS of transition frequency Ω , the MB system is considered in the isotropic case for a plane polarized electromagnetic field propagating in direction z . The time is scaled to the inverse transition frequency Ω^{-1} , the space z to the length $\Omega c/\eta$ (η is the optical index of the medium), the population inversion to the density of active

dipoles N_0 , the energy to the average $W_0 = N_0 \hbar \Omega/2$, the electric field to $\sqrt{W_0/\epsilon}$, and the polarization to $\sqrt{\epsilon W_0}$. The resulting dimensionless MB system then reads

$$\begin{aligned} \mathbf{E}_{tt} - \mathbf{E}_{zz} + \mathbf{P}_{tt} &= -\gamma \mathbf{E}_t, \\ \mathbf{P}_{tt} + \mathbf{P} + \alpha N \mathbf{E} &= -\gamma_2 \mathbf{P}_t, \\ N_t - \mathbf{E} \cdot \mathbf{P}_t &= -\gamma_1(1 + N), \end{aligned} \quad (1)$$

where \mathbf{E} and \mathbf{P} denote vectors in the transverse plane, e.g., $\mathbf{E} = (E_x(z, t), E_y(z, t))$. The coupling eventually results in a unique dimensionless fundamental constant $\alpha = 2\mu_0 c^2/(3\eta^2)|\mu_{12}|^2 N_0/(\hbar \Omega)(\eta^2 + 2)^2/9$ (where the dipole moment μ_{12} is averaged over the orientations [10]) and by the normalization of the population inversion den-

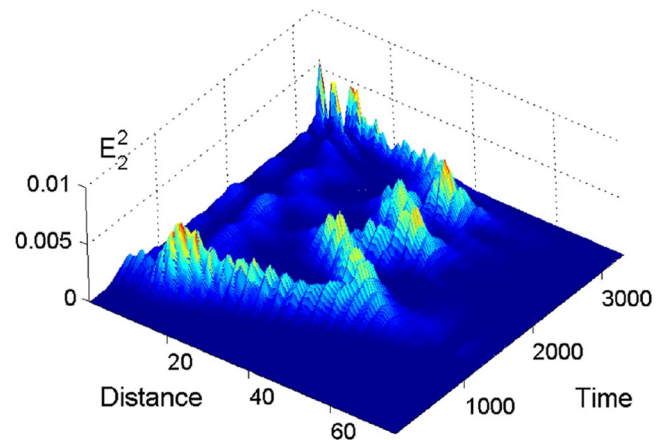


FIG. 1 (color online). A typical result of a numerical simulation of (1) under boundary values (3) and (4) with parameters $\mathcal{E}_1^{(0)} = \mathcal{E}_1^{(L)} = 0.4$, $\omega_1 = 1.5$, $\mathcal{E}_2^{\max} = 0.04$, $\omega_2 = 1.31$, $t_0 = 1000$, and $\mu = 1/150$. The plot shows the evolution of the energy density along the polarization direction y , trapped by the grating induced by the stationary wave. The pulse is eventually released back at time 4500 by increasing the amplitude on the incident side $\mathcal{E}_1^{(0)}$ by 25%.

sity: $N = -1$ when all active atoms are in the fundamental state, and $N = 1$ in the excited state. The dimensionless dissipation coefficients are $\gamma = (c/\eta)(\mathcal{A}/\Omega)$ resulting from the electric field attenuation \mathcal{A} , then $\gamma_1 = 1/(\Omega T_1)$ and $\gamma_2 = 2/(\Omega T_2)$ resulting, respectively, from the population inversion dephasing time T_1 and the polarization dephasing time T_2 .

In the strong coupling case $\alpha \sim 1$ (dense media), a multiscale analysis has shown that the model equation for the two directions of polarization results to be a system of coupled nonlinear Schrödinger equations [12]. Then reduced to a unique polarization, it allows for slow light soliton formation out of evanescent incident light [13], namely, under irradiation in the forbidden band gap. This gap results from the linear dispersion relation of the MB system, for a carrier $\exp[i(\omega t - kz)]$ on a medium at rest, namely,

$$\omega^2(\omega^2 - \omega_0^2) = k^2(\omega^2 - 1), \quad \omega_0^2 = 1 + \alpha. \quad (2)$$

The upper edge of the stop gap at frequency ω_0 corresponds to $k = 0$ and $d\omega/dk = 0$.

Numerical simulations.—We first proceed with numerical simulations of the MB Eqs. (1) submitted to the following boundary-value problem

$$\mathbf{E}(0, t) = \begin{pmatrix} \mathcal{E}_1^{(0)} \cos(\omega_1 t) \\ \mathcal{E}_2(t) \cos(\omega_2 t) \end{pmatrix}, \quad (3)$$

$$\mathbf{E}(L, t) = \begin{pmatrix} \mathcal{E}_1^{(L)} \cos(\omega_1 t) \\ 0 \end{pmatrix}, \quad (4)$$

where $\mathcal{E}_2(t)$ is the slowly varying low-amplitude pulse envelope

$$\mathcal{E}_2(t) = \frac{\mathcal{E}_2^{\max}}{\cosh[\mu(t - t_0)]}. \quad (5)$$

The carrier frequencies are chosen close to the gap edge ω_0 , inside the passing band for the grating field, in the gap for the incident pulse, namely,

$$\omega_1 > \omega_0, \quad \omega_2 < \omega_0, \quad |\omega_j - \omega_0| \sim \mathcal{O}(\epsilon^2) \quad (6)$$

for $j = 1, 2$, where ϵ is our small control parameter. We set from now on $\alpha = 1$.

We have chosen damping coefficients such as to reach a fast stabilization of the lattice created by the stationary driving: in the x direction we have set $\gamma = \gamma_2 = 0.01$ and $\gamma_1 = 0.001$, while in the y direction they have been set to zero in order to see clearly the process. Under such boundary data, with the parameter values indicated in the caption, we obtain the result displayed in Fig. 1. More precisely, the applied grating field (polarized along x) is settled smoothly and, at time $t = 1000$ the incident pulse enters the medium. It is naturally trapped and seen to oscillate about the center $z = 30$. At time $t = 4500$ the left-hand-side amplitude $\mathcal{E}_1^{(0)}$ is increased from 0.4 to 0.5, which produces the reverse

motion of the stored pulse. Actually, one may play with the driving amplitudes $\mathcal{E}_1^{(0)}$ and $\mathcal{E}_1^{(L)}$ to drive the pulse back and forth.

Interpretation.—The process described above is now understood, within a multiscale analysis of MB equations, in terms of a discrete nonlinear Schrödinger model with variable coefficients related to the variation of the boundary grating field amplitudes. Thanks to (6), a solution of (1) under boundary values (3) and (4) can be sought under the form

$$\mathbf{E}(z, t) = \begin{pmatrix} \epsilon E_1(\zeta, \tau_1) \\ \epsilon^2 E_2(\zeta, \tau_1, \tau_2) \end{pmatrix} e^{i\omega_0 t} + \text{c.c.} \quad (7)$$

with the slow variables $\zeta = \epsilon z$ and $\tau_n = \epsilon^{2n} t$. The second slow time τ_2 is meant to capture the nonlinear dynamics of the low-amplitude (ϵ^2) long duration (ϵ^{-2}) incident pulse. Note that from assumption (6), the frequency shift from ω_0 is contained in the slow time variations with $\tau_1 = \epsilon^2 t$.

Inserting then (7) in the MB system (1) we eventually obtain (with vanishing damping)

$$i \frac{\partial E_1}{\partial \tau_1} - \frac{\omega_0^2 - 1}{2\omega_0^3} \frac{\partial^2 E_1}{\partial \zeta^2} = \frac{\omega_0^2 + 3}{4\omega_0} |E_1|^2 E_1, \quad (8)$$

$$i \frac{\partial E_2}{\partial \tau_1} - \frac{\omega_0^2 - 1}{2\omega_0^3} \frac{\partial^2 E_2}{\partial \zeta^2} - \frac{\omega_0^2 + 1}{2\omega_0} |E_1|^2 E_2 + \frac{\omega_0^2 - 1}{4\omega_0} E_1^2 E_2^* = \epsilon^2 \left(-i \frac{\partial E_2}{\partial \tau_2} + \frac{\omega_0^2 + 3}{4\omega_0} |E_2|^2 E_2 \right), \quad (9)$$

where *star* means complex conjugation. The boundary values (3) and (4) for the x -component E_1 imply from Eq. (8) that $E_1(\zeta, \tau_1)$ is a periodic stationary solution of the nonlinear Schrödinger equation with frequency $\nu_1 = (\omega_1 - \omega_0)\epsilon^{-2}$. It acts then as an external periodic potential in the evolution (9).

In order to take into account variations $|E_1|(\zeta)$ resulting from the variations of the boundary driving (3) we set [remember $\nu_1 \tau_1 = (\omega_1 - \omega_0)t$]

$$E_1 = |E_1|(\zeta) e^{i\nu_1 \tau_1}, \quad |E_1|^2(\zeta) = V_0(\zeta) + \epsilon^2 V(\zeta), \quad (10)$$

where $V_0(\zeta)$ is purely periodic while $V(\zeta)$ describes the *aperiodic inhomogeneities* of $|E_1|(\zeta)$ induced by the boundary values.

We then seek a solution of (9) on a suitable orthonormal basis of Wannier functions $\varphi_j(\zeta)$ which are localized with respect to the site j [14], within the *one-band approximation* [15], where j actually indexes the minima of the periodic potential $V_0(\zeta)$. A solution of (9) is sought as

$$E_2 = \sum_j e^{i\nu_1 \tau_1} \mathcal{F}_j(\tau_1, \tau_2) \varphi_j(\zeta), \quad (11)$$

and the equation for the coefficients \mathcal{F}_j is worked out by inserting (11) in (9) and by projecting on a chosen φ_j . In

the *tight-binding approximation* [16] we eventually obtain

$$i \frac{\partial \mathcal{F}_j}{\partial \tau_1} - (\Omega_0 + \nu_1) \mathcal{F}_j + \Lambda_0 \mathcal{F}_j^* = \epsilon^2 \left[-i \frac{\partial \mathcal{F}_j}{\partial \tau_2} + \Omega_j \mathcal{F}_j - \Lambda_j \mathcal{F}_j^* + Q(\mathcal{F}_{j-1} + \mathcal{F}_{j+1}) + U |\mathcal{F}_j|^2 \mathcal{F}_j \right], \quad (12)$$

where the coefficients are given from the Wannier basis by (integrals run on $\zeta \in \mathbb{R}$)

$$\begin{aligned} \Omega_0 &= \frac{\omega_0^2 - 1}{2\omega_0^3} \int \varphi_j'' \varphi_j + \frac{\omega_0^2 + 1}{2\omega_0} \int V_0 \varphi_j^2, & \Lambda_0 &= \frac{\omega_0^2 - 1}{4\omega_0} \int V_0 \varphi_j^2, & \Omega_j &= \frac{\omega_0^2 + 1}{2\omega_0} \int V \varphi_j^2, \\ \Lambda_j &= \frac{\omega_0^2 - 1}{4\omega_0} \int V \varphi_j^2, & U &= \frac{\omega_0^2 + 3}{4\omega_0} \int \varphi_j^4, & \epsilon^2 Q &= \frac{\omega_0^2 - 1}{2\omega_0^3} \int \varphi_{j\pm 1}'' \varphi_j. \end{aligned}$$

Note that translational invariance guarantees that the above coefficients are j independent, except, of course, for Ω_j and Λ_j that bear the aperiodic inhomogeneity of the external potential $|E_1|(\zeta)$.

Equation (12) can now be solved first for the τ_1 dependence of \mathcal{F}_j as a linear system, which provides then a discrete nonlinear Schrödinger coupled system for the τ_2 -dependent amplitudes. This is done by seeking a solution under the form

$$\mathcal{F} = \mathcal{G}_j^1 e^{-i\Delta\tau_1} + \mathcal{G}_j^{-1} e^{i\Delta\tau_1} \quad (13)$$

for which the leading order of (12) furnishes the two coupled linear equations

$$\begin{aligned} -[\Omega_0 + \nu_1 - \Delta] \mathcal{G}_j^1 + \Lambda_0 (\mathcal{G}_j^{-1})^* &= 0, \\ -[\Omega_0 + \nu_1 + \Delta] (\mathcal{G}_j^{-1})^* + \Lambda_0 \mathcal{G}_j^1 &= 0. \end{aligned}$$

The dispersion relation and the relation between \mathcal{G}_j^1 and $(\mathcal{G}_j^{-1})^*$ automatically follows as

$$\begin{aligned} \Delta &= \sqrt{[\Omega_0 + \nu_1]^2 - \Lambda_0^2}, \\ (\mathcal{G}_j^{-1})^* &= \frac{\Lambda_0}{\Omega_0 + \nu_1 + \Delta} \mathcal{G}_j^1. \end{aligned}$$

At next order we readily get for \mathcal{G}_j^1 the discrete nonlinear Schrödinger equation, the parameters of which are greatly simplified if we note that in our numerical simulations (for sufficiently deep lattice) $\Omega_0 \gg \Lambda_0$ and $\Omega_j \gg \Lambda_j$. In such a case the equation reads

$$i \frac{\partial \mathcal{G}_j^1}{\partial \tau_2} - \Omega_j \mathcal{G}_j^1 = Q[\mathcal{G}_{j+1}^1 + \mathcal{G}_{j-1}^1] + U |\mathcal{G}_j^1|^2 \mathcal{G}_j^1. \quad (14)$$

The electric field envelope in the y direction of polarization reads

$$E_2 = \sum_j e^{-i\Omega_0 \tau_1} \mathcal{G}_j^1(\tau_2) \varphi_j(\zeta), \quad (15)$$

in terms of the solution of the chosen Wannier basis.

The dynamics of the pulse is thus interpreted out of the discrete nonlinear Schrödinger model (14) as the action of the *potential* Ω_j that translates the applied variations of the boundary driving in the x direction of polarization.

Application.—In order to illustrate the above interpretation, we proceed now with numerical simulations of (14) where $\tau_2 = t$ to read quantities in physical dimensions. The potential Ω_j models the variations of $|E_1|$ away from a purely periodic function as soon as $\mathcal{E}_1^{(0)} \neq \mathcal{E}_1^{(L)}$. We set

$$\Omega_j = \begin{cases} 0, & t < 650, \\ -0.07j, & t > 650, \end{cases} \quad (16)$$

and obtain Fig. 2, which shows the same qualitative behavior as Fig. 1.

Conclusion and comments.—We have shown that the Maxwell-Bloch system, the celebrated fundamental and general semiclassical model of interaction of radiation with matter, may serve as a tool to store and drive light pulses in an arbitrary way by conveniently using laser light to engrave the medium with a controllable standing wave pattern. The extreme genericity of the MB model, together with the freedom in the boundary values of the engraving field, constitutes a decisive advantage over other techniques to store and manipulate light pulses.

The process is understood by deriving a discrete nonlinear Schrödinger model where the external tunable potential actually translates the effects of boundary driving

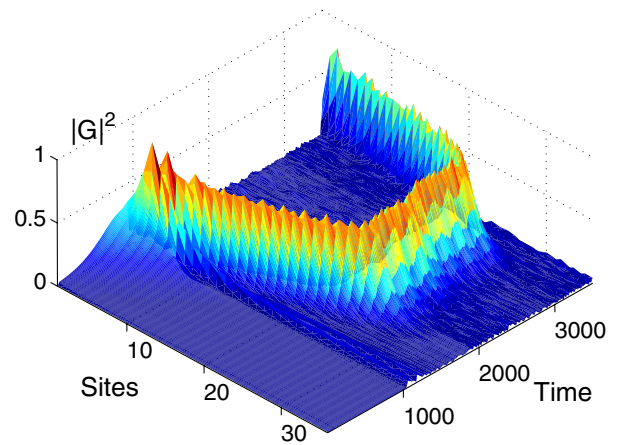


FIG. 2 (color online). Plot of the solution of (14) with the parameters $Q = 0.2$, $U = 0.4$ and an incident pulse $\mathcal{G}_0^1(t) = 0.305 e^{-0.418it} / \cosh[(t - 750)/250]$ which propagates, becomes trapped, and is then released back by the action of Ω_j in (16).

variations. Our purpose was simply to provide a *qualitative* interpretation.

A more detailed study, reported in future work, would require first to use exact solutions of (8) for the grating field (e.g., in terms of Jacobi elliptic functions), second to construct the corresponding most adequate Wannier basis, third to evaluate precisely the effect of boundary driving variations on the factor Ω_j , and last to study the full system (12) that couples \mathcal{F}_j to \mathcal{F}_j^* .

This work was done under Contract No. CNRS GDR-PhoNoMi2 (*Photonique Nonlinéaire et Milieux Microstructurés*). R. K. acknowledges the invitation and support of the *Laboratoire de Physique Théorique et Astroparticules* and USA CRDF Grant No. GEP2-2848-TB-06.

-
- [1] C. Liu, Z. Dutton, C. Behroozi, and L. V. Hau, *Nature* (London) **409**, 490 (2001).
 - [2] A. Andre and M. D. Ludkin, *Phys. Rev. Lett.* **89**, 143602 (2002).
 - [3] M. Bajcsy, A. S. Zibrov, and M. D. Ludkin, *Nature* (London) **426**, 638 (2003).
 - [4] A. E. Kozhokin and G. Kurizki, *Phys. Rev. Lett.* **74**, 5020 (1995).
 - [5] A. E. Kozhokin, G. Kurizki, and B. Malomed, *Phys. Rev. Lett.* **81**, 3647 (1998).
 - [6] N. Aközbeke and S. John, *Phys. Rev. E* **58**, 3876 (1998).
 - [7] W. N. Xiao, J. Y. Zhou, and J. P. Prineas, *Opt. Express* **11**, 3277 (2003).
 - [8] I. V. Mel'nikov and J. S. Aitchison, *Appl. Phys. Lett.* **87**, 201111 (2005).
 - [9] L. C. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover, New York, 1987).
 - [10] R. H. Pantell and H. E. Puthoff, *Fundamentals of Quantum Electronics* (Wiley, New York, 1969).
 - [11] T. Schwartz, G. Bartal, Sh. Fishman, and M. Segev, *Nature* (London) **446**, 52 (2007).
 - [12] F. Ginovart and J. Leon, *J. Phys. A* **27**, 3955 (1994).
 - [13] J. Leon, *Phys. Rev. A* **75**, 063811 (2007).
 - [14] W. Kohn, *Phys. Rev. B* **7**, 4388 (1973).
 - [15] G. L. Alfimov, P. G. Kevrekidis, V. V. Konotop, and M. Salerno, *Phys. Rev. E* **66**, 046608 (2002).
 - [16] A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, *Phys. Rev. Lett.* **79**, 4950 (1997); S. Raghavan, A. Smerzi, S. Fantoni, and S. R. Shenoy, *Phys. Rev. A* **59**, 620 (1999); A. Smerzi and A. Trombettoni, *Phys. Rev. A* **68**, 023613 (2003).