Anderson Localization of Bogolyubov Quasiparticles in Interacting Bose-Einstein Condensates

P. Lugan, D. Clément, P. Bouyer, A. Aspect, and L. Sanchez-Palencia

Laboratoire Charles Fabry de l'Institut d'Optique, CNRS and Univ. Paris-Sud, Campus Polytechnique,
RD 128, F-91127 Palaiseau cedex, France

We study the Anderson localization of Bogolyubov quasiparticles in an interacting Bose-Einstein condensate (with a healing length ξ) subjected to a random potential (with a finite correlation length σ_R). We derive analytically the Lyapunov exponent as a function of the quasiparticle momentum k, and we study the localization maximum $k_{\max}$. For 1D speckle potentials, we find that $k_{\max} \propto 1/\xi$ when $\xi \gg \sigma_R$ while $k_{\max} \propto 1/\sigma_R$ when $\xi \ll \sigma_R$, and that the localization is strongest when $\xi \sim \sigma_R$. Numerical calculations support our analysis, and our estimates indicate that the localization of the Bogolyubov quasiparticles is accessible in experiments with ultracold atoms.

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An important issue in mesoscopic physics concerns the effects of disorder in systems where both quantum interference and particle-particle interactions play crucial roles. Multiple scattering of noninteracting quantum particles from a random potential leads to strong Anderson localization (AL) [1], characterized by an exponential decay of the quantum states over a typical distance, the localization length. AL occurs for arbitrarily weak disorder in 1D and 2D, and for strong-enough disorder in 3D [2]. The problem is more involved in the presence of interactions. Strong disorder in repulsively interacting Bose gases induces novel insulating quantum states, such as the Bose [3] and Lifshits [4] glasses. For moderate disorder and interactions, the system forms a Bose-Einstein condensate (BEC) [4–6], where the disorder induces a depletion of the condensed and superfluid fractions [7] and the shift and damping of sound waves [8].

These studies have direct applications to experiments on liquid ^{4}He in porous media [9], in particular as regards the understanding of the absence of superfluidity. Moreover, the realization of disordered gaseous BECs [10–14] has renewed the issue due to an unprecedented control of the experimental parameters. Using optical speckle fields [15], for instance, one can control the amplitude and design the correlation function of the random potential almost at will [14], opening possibilities for experimental studies of AL [16,17]. Earlier studies related to localization in the context of ultracold atoms include dynamical localization in δ-kicked rotors [18] and spatial diffusion of laser-cooled atoms in speckle potentials [19].

Transport processes in repulsively interacting BECs can exhibit AL [17,20]. However, for BECs at equilibrium, interaction-induced delocalizing effects dominate disorder-induced localization, except for very weak interactions [5,6]. The ground state of an interacting BEC at equilibrium is thus extended. Beyond, one may wonder how the many-body (collective) excitations of the BEC behave in weak disorder. In dilute BECs, these excitations correspond to quasiparticles (particle-hole pairs) described by the Bogolyubov theory [21]. In this case, the interplay

of interactions and disorder is subtle, and strong arguments indicate that the Bogolyubov quasiparticles (BQP) experience a random potential *screened* by the BEC density [5]. This problem has been addressed in the idealized case of uncorrelated disorder (random potentials with a delta correlation function) in Ref. [22].

In this Letter, we present a general quantitative treatment of the localization of the BQPs in an interacting BEC with healing length ξ in a weak random potential with arbitrary correlation length σ_R. For weak disorder, we introduce a transformation that maps rigorously the many-body Bogolyubov equations onto the Schrödinger equation for a noninteracting particle in a screened random potential, which we derive analytically. We calculate the Lyapunov exponent Γ_k (inverse localization length) as a function of the BQP wave number k for 1D speckle potentials. For a given ratio ξ/σ_R, we determine the wave number $k_{\max}$ for which Γ_k is maximum: We find $k_{\max} \sim 1/\xi$ for $\xi \gg \sigma_R$, and $k_{\max} \sim 1/\sigma_R$ for $\xi \ll \sigma_R$. The absolute maximum appears for $\xi \sim \sigma_R$ so that the finite-range correlations of the disorder need to be taken into account. Numerical calculations support our analysis. Finally, possibilities to observe the AL of BQPs in BECs placed in speckle potentials are discussed.

We consider a d-dimensional Bose gas in a potential $V(\mathbf{r})$ with weak repulsive short-range atom-atom interactions, characterized by the coupling constant g. Its physics is governed by the many-body Hamiltonian

$$\hat{H} = \int d\mathbf{r}\{(\hbar^2/2m)[(\nabla\hat{\theta})^2\hat{n} + (\nabla\sqrt{\hat{n}})^2] + V(\mathbf{r})\hat{n} + (g/2)\hat{n}^2 - \mu\hat{n}\} \tag{1}$$

where m is the atomic mass, μ is the chemical potential, and $\hat{\theta}$ and $\hat{n}$ are the phase and density operators, which obey the commutation relation $[\hat{n}(\mathbf{r}), \hat{\theta}(\mathbf{r}')] = i\delta(\mathbf{r} - \mathbf{r}')$. According to the Bogolyubov-Popov theory [21,23,24], for small phase gradients ($\hbar^2|\nabla\theta|^2/2m \ll \mu$) and small density fluctuations ($\delta\hat{n} \ll n_c$, where $n_c = \langle\hat{n}\rangle$ and $\delta\hat{n} = \hat{n} - n_c$), Hamiltonian (1) can be diagonalized up to second

order as $\hat{H} = E_0 + \sum_\nu \epsilon_\nu \hat{b}_\nu^\dagger \hat{b}_\nu$, where $\hat{b}_\nu$ is the annihilation operator of the excitation (BQP) of energy ϵ_ν. The many-body ground state of the Bose gas is a BEC with a uniform phase and a density governed by the Gross-Pitaevskii equation (GPE):

$$\mu = -\hbar^2 \nabla^2 (\sqrt{n_c})/2m\sqrt{n_c} + V(\mathbf{r}) + gn_c(\mathbf{r}). \quad (2)$$

Expanding the density and phase in the basis of the excitations, $\hat{\theta}(\mathbf{r}) = [-i/2\sqrt{n_c(\mathbf{r})}]\sum_\nu[f_\nu^+(\mathbf{r})\hat{b}_\nu - \text{H.c.}]$ and $\delta\hat{n}(\mathbf{r}) = \sqrt{n_c(\mathbf{r})}\sum_\nu[f_\nu^-(\mathbf{r})\hat{b}_\nu + \text{H.c.}]$, the Hamiltonian reduces to the above diagonal form provided that $f_\nu^\pm$ obey the Bogolyubov–de Gennes equations (BdGE) [25]:

$$[-(\hbar^2/2m)\nabla^2 + V + gn_c - \mu]f_\nu^+ = \epsilon_\nu f_\nu^- \quad (3)$$

$$[-(\hbar^2/2m)\nabla^2 + V + 3gn_c - \mu]f_\nu^- = \epsilon_\nu f_\nu^+, \quad (4)$$

with the normalization $\int d\mathbf{r}\,[f_\nu^+ f_{\nu'}^{-*} + f_\nu^- f_{\nu'}^{+*}] = 2\delta_{\nu,\nu'}$. Equations (2)–(4) form a complete set to calculate the ground state (BEC) and excitations (BQPs) of the Bose gas, from which one can compute all properties of finite temperature or time-dependent BECs.

Here, we analyze the properties of the BQPs in the presence of weak disorder. According to Eqs. (3) and (4), they are determined by the interplay of the disorder V and the BEC density background n_c. Let $V(\mathbf{r})$ be a weak random potential ($\tilde{V} \ll \mu$, see below) with a vanishing average and a finite-range correlation function, $C(\mathbf{r}) = V_R^2 c(\mathbf{r}/\sigma_R)$, where $V_R = \sqrt{\langle V^2 \rangle}$ is the standard deviation, and σ_R the correlation length of V. As shown in Refs. [4–6], the BEC density profile is extended for strong-enough repulsive interactions (i.e., for $\xi \ll L$, where $\xi = \hbar/\sqrt{4m\mu}$ is the healing length and L the size of the BEC). More precisely, up to first order in V_R/μ, the GPE (2) yields

$$n_c(\mathbf{r}) = [\mu - \tilde{V}(\mathbf{r})]/g \quad (5)$$

where $\tilde{V}(\mathbf{r}) = \int d\mathbf{r}' G_\xi(\mathbf{r} - \mathbf{r}')V(\mathbf{r}')$, and G_ξ, the Green function of the linearized GPE [4,6], reads $G_\xi(\mathbf{q}) = (2\pi)^{-d/2}/[1 + (|\mathbf{q}|\xi)^2]$ in Fourier space [26]. Then,

$$\tilde{V}(\mathbf{q}) = V(\mathbf{q})/[1 + (|\mathbf{q}|\xi)^2]. \quad (6)$$

Thus, ξ is a threshold in the response of the density n_c to the potential V, as $\tilde{V}(\mathbf{q}) \simeq V(\mathbf{q})$ for $|\mathbf{q}| \ll \xi^{-1}$, while $\tilde{V}(\mathbf{q}) \ll V(\mathbf{q})$ for $|\mathbf{q}| \gg \xi^{-1}$. The potential $\tilde{V}(\mathbf{r})$ is a smoothed potential [6]. If V is a homogeneous random potential, so is $\tilde{V}$, and according to Eq. (5), the BEC density profile n_c is random but extended [4,6].

Solving the BdGEs (3) and (4) is difficult in general because they are strongly coupled. Yet, we show that for a weak (possibly random) potential $V(\mathbf{r})$, this hurdle can be overcome by using appropriate linear combinations $g_\mathbf{k}^\pm$ of the $f_\mathbf{k}^\pm$ functions, namely $g_\mathbf{k}^\pm = \pm\rho_k^{\pm 1/2}f_\mathbf{k}^+ + \rho_k^{\mp 1/2}f_\mathbf{k}^-$, with $\rho_k = \sqrt{1 + 1/(k\xi)^2}$ and $k = |\mathbf{k}|$. For $V = 0$, the equations for $g_\mathbf{k}^\pm$ are uncoupled [see Eqs. (7) and (8)] and

we recover the usual plane-wave solutions of wave vector $\mathbf{k}$ and energy $\epsilon_k = \rho_k(\hbar^2 k^2/2m)$ [21]. For weak but finite V, inserting Eq. (5) into Eqs. (3) and (4), we find

$$\frac{\hbar^2 k^2}{2m}g_\mathbf{k}^+ = -\frac{\hbar^2}{2m}\nabla^2 g_\mathbf{k}^+ - \frac{2\rho_k \tilde{V}}{1 + \rho_k^2}g_\mathbf{k}^- + \left[V - \frac{3 + \rho_k^2}{1 + \rho_k^2}\tilde{V}\right]g_\mathbf{k}^+ \quad (7)$$

$$-\rho_k^2\frac{\hbar^2 k^2}{2m}g_\mathbf{k}^- = -\frac{\hbar^2}{2m}\nabla^2 g_\mathbf{k}^- - \frac{2\rho_k \tilde{V}}{1 + \rho_k^2}g_\mathbf{k}^+ + \left[V - \frac{1 + 3\rho_k^2}{1 + \rho_k^2}\tilde{V}\right]g_\mathbf{k}^-. \quad (8)$$

Equations (7) and (8), which are coupled at most by a term of the order of V [since $|\tilde{V}| \leq |V|$ and $2\rho_k/(1 + \rho_k^2) \leq 1$], allow for perturbative approaches. Note that the functions $g_\mathbf{k}^+$ and $g_\mathbf{k}^-$ have very different behaviors owing to the signs in the left-hand side terms in Eqs. (7) and (8). Equation (8) can be solved to the lowest order in $\tilde{V}_R/\mu$ with a Green kernel defined in Ref. [6]: we find $g_\mathbf{k}^-(\mathbf{r}) \simeq \frac{2/\epsilon_k}{1 + \rho_k^2} \times \int d\mathbf{r}' G_{\xi_k}(\mathbf{r} - \mathbf{r}')\tilde{V}(\mathbf{r}')g_\mathbf{k}^+(\mathbf{r}')$, where $\xi_k = \xi/\sqrt{1 + (k\xi)^2}$. Equation (7) cannot be solved using the same method because the perturbation series diverges. Nevertheless, from the solution for $g_\mathbf{k}^-$, we find that $|g_\mathbf{k}^-/g_\mathbf{k}^+| \lesssim \frac{2/\epsilon_k}{1 + \rho_k^2}|\tilde{V}| < |\tilde{V}|/\mu \ll 1$. The coupling term in Eq. (7) can thus be neglected to first order in $\tilde{V}_R/\mu$, and we are left with the closed equation

$$-(\hbar^2/2m)\nabla^2 g_\mathbf{k}^+ + \mathcal{V}_k(\mathbf{r})g_\mathbf{k}^+ \simeq (\hbar^2 k^2/2m)g_\mathbf{k}^+, \quad (9)$$

where

$$\mathcal{V}_k(\mathbf{r}) = V(\mathbf{r}) - \frac{1 + 4(k\xi)^2}{1 + 2(k\xi)^2}\tilde{V}(\mathbf{r}). \quad (10)$$

Equation (9) is formally equivalent to a Schrödinger equation for noninteracting bare particles with energy $\hbar^2 k^2/2m$, in a random potential $\mathcal{V}_k(\mathbf{r})$. This mapping allows us to find the localization properties of the BQPs using standard methods for bare particles in 1D, 2D, or 3D [27]. However, since $\mathcal{V}_k(\mathbf{r})$ depends on the wave vector $\mathbf{k}$ itself, the localization of the BQPs is dramatically different from that of bare particles as discussed below.

In the remainder of the Letter, we restrict ourselves to the 1D case, for simplicity, but also because AL is expected to be stronger in lower dimensions [2]. The Lyapunov exponent Γ_k is a self-averaging quantity in infinite 1D systems, which can be computed in the Born approximation using the phase formalism [27] (see also Ref. [17]). We get $\Gamma_k = (\sqrt{2\pi}/8)(2m/\hbar^2 k)^2 \mathcal{C}_k(2k)$, where $\mathcal{C}_k(q)$ is the Fourier transform of the correlation function of $\mathcal{V}_k(z)$, provided that $\Gamma_k \ll k$ [16,17,27]. Since $\mathcal{C}_k(q) \propto \langle|\mathcal{V}_k(q)|^2\rangle$, the component of $\mathcal{V}_k$ relevant for the calculation of Γ_k is $\mathcal{V}_k(2k)$. From Eqs. (6) and (10), we find

$$\mathcal{V}_k(2k) = \mathcal{S}(k\xi)V(2k); \qquad \mathcal{S}(k\xi) = \frac{2(k\xi)^2}{1 + 2(k\xi)^2}, \quad (11)$$

and the Lyapunov exponent of the BQP reads

$$\Gamma_k = [\mathcal{S}(k\xi)]^2 \gamma_k \qquad (12)$$

where $\gamma_k = (\sqrt{2\pi}/32)(V_R/\mu)^2(\sigma_R/k^2\xi^4)c(2k\sigma_R)$ is the Lyapunov exponent for a bare particle with the same wave number k [17,27].

Let us summarize the validity conditions of the perturbative approach presented here. It requires (i) the smoothing solution (5) to be valid (i.e., $\tilde{V}_R \ll \mu$), (ii) the coupling term proportional to g_k^- in Eq. (7) to be negligible (which is valid if $\tilde{V}_R \ll \mu$), and (iii) the phase formalism to be applicable. The latter requires $\Gamma_k \ll k$, i.e., $(V_R/\mu) \times (\sigma_R/\xi)^{1/2} \ll (k\xi)^{3/2}[1 + 1/2(k\xi)^2]$, which is valid for any k if $(V_R/\mu)(\sigma_R/\xi)^{1/2} \ll 1$.

Applying Eq. (12) to uncorrelated potentials [$C(z) = 2D\delta(z)$ with $\sigma_R \to 0$, $V_R \to \infty$ and $2D = V_R^2\sigma_R \int dxc(x) = cst$], one recovers the formula for Γ_k found in Ref. [22]. Our approach generalizes this result to potentials with finite-range correlations, which proves useful since uncorrelated random potentials are usually crude approximations of realistic disorder, for which σ_R can be significantly large. We show below that if $\xi \lesssim \sigma_R$, as, e.g., in the experiments of Refs. [10–14], the behavior of Γ_k versus k is dramatically affected by the finite-range correlations of the disorder.

Let us discuss the physical content of Eqs. (9) and (10). According to Eqs. (3) and (4), the properties of the BQPs are determined by both the bare random potential V and the BEC density n_c in a nontrivial way. Equation (9) makes their roles more transparent. As the occurrence of the smoothed potential $\tilde{V}(z)$ in Eq. (10) is reminiscent of the presence of the meanfield interaction gn_c in the BdGEs (3) and (4), it appears that the random potential $\mathcal{V}_k(z)$ results from the *screening* of the random potential $V(z)$ by the BEC density background [5]. More precisely, the expression (11) for the Fourier component $\mathcal{V}_k(2k)$, relevant for the Lyapunov exponent of a BQP, shows that the screening strength depends on the wave number k. In the *free-particle regime* ($k \gg 1/\xi$), we find that the Lyapunov exponent of a BQP equals that of a bare particle with the same wave number ($\Gamma_k \simeq \gamma_k$), as expected. In the *phonon regime* ($k \ll 1/\xi$), the disorder is strongly screened, and we find $\Gamma_k \ll \gamma_k$, as in models of elastic media [28]. Here, the localization of a BQP is strongly suppressed by the repulsive atom-atom interactions, as compared to a bare particle in the same bare potential. These findings agree with and generalize the results obtained from the transfer matrix method, which applies to potentials made of a 1D random series of δ-scatterers [22].

Our approach applies to any weak random potential with a finite correlation length. We now further examine the case of 1D speckle potentials used in quantum gases [10–14]. Inserting the corresponding reduced correlation func-

tions, $c(\kappa) = \sqrt{\pi/2}(1 - \kappa/2)\Theta(1 - \kappa/2)$ where Θ is the Heaviside function [17], into Eq. (12), we find

$$\Gamma_k = \frac{\pi}{8}\left(\frac{V_R}{\mu}\right)^2 \frac{\sigma_R k^2(1 - k\sigma_R)}{[1 + 2(k\xi)^2]^2}\Theta(1 - k\sigma_R) \qquad (13)$$

which is plotted in Fig. 1. To test our general approach on the basis of this example, we have performed numerical calculations using a direct integration of the BdGEs (3) and (4) in a finite but large box of size L. The Lyapunov exponents are extracted from the asymptotic behavior of $\log[r_k(z)/r_k(z_k)]/|z - z_k|$, where z_k is the localization center and $r_k(z)$ is the envelope of the function g_k^+, obtained numerically. The numerical data, averaged over 40 realizations of the disorder, are in excellent agreement with formula (13) as shown in Fig. 2. These results validate our approach. It should be noted, however, that our numerical calculations return BQP wave functions that can be strongly localized for very small momenta k. This will be discussed in more details in a future publication [29].

Of special interest are the maxima of Γ_k, which denote a maximum localization of the BQPs. It is straightforward to show that, for a fixed set of parameters ($V_R/\mu, \xi, \sigma_R$), Γ_k is nonmonotonic and has a single maximum, k_{max}, in the range $[0, 1/\sigma_R]$ (see Fig. 2). This contrasts with the case of bare particles, for which the Lyapunov exponent γ_k decreases monotonically as a function of k, provided that $c(2k\sigma_R)$ decreases versus k (which is valid for a broad class of random potentials [27]). The existence of a localization maximum with respect to the wave number k is thus specific to the BQPs and results from the strong screening of the disorder in the phonon regime. In general, the value of k_{max}, plotted in the inset of Fig. 2 versus the correlation length of the disorder, depends on both ξ and σ_R. For $\sigma_R \ll \xi$, we find $k_{max} \simeq \frac{1}{\sqrt{2}\xi}(1 - \frac{\sigma_R/\xi}{2\sqrt{2}})$, so that the localization is maximum near the crossover between the phonon and the free-particle regimes as for uncorrelated potentials [22]. For $\sigma_R \gg \xi$, however, we find $k_{max} \simeq 2/3\sigma_R$ so that k_{max} is no longer determined by the healing length but rather by the correlation length of the disorder, and lies deep in the phonon regime. For $k > 1/\sigma_R$, Γ_k vanishes. This defines an *effective mobility edge* due to long-range
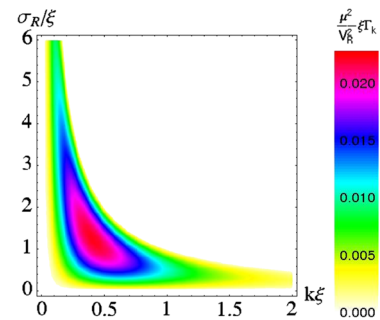

FIG. 1 (color online).   Density plot of the Lyapunov exponent of the BQPs for a 1D speckle potential.
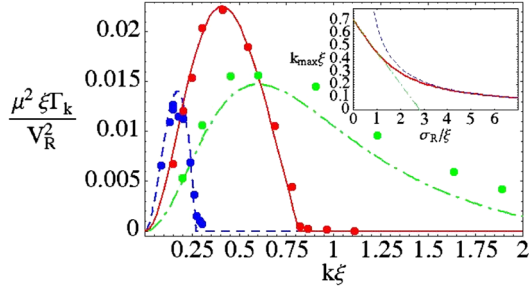
FIG. 2 (color online). Lyapunov exponent of the BQPs for a 1D speckle potential. The lines correspond to Eq. (13), and the points to numerical results for $\xi = 1.25 \times 10^{-5}L$, $V_R = 0.075\mu$, and $\sigma_R = \sqrt{3/2}\xi$ (solid line), $\sigma_R = 3.7\xi$ (dashed line), $\sigma_R = 0.4\xi$ (dash-dotted line). Inset: localization maximum versus the ratio of the correlation length of the disorder to the healing length of the BEC (the dash-dotted and dashed lines correspond to the limits $\sigma_R \ll \xi$ and $\sigma_R \gg \xi$ respectively; see text).

correlations in speckle potentials, as for bare particles [17,30].

Finally, let us determine the absolute localization maximum. The Lyapunov exponent Γ_k decreases monotonically versus ξ and V_R/μ. However, for fixed values of V_R/μ and ξ, Γ_k has a maximum at $\sigma_R = \sqrt{3/2}\xi$ and $k = \xi^{-1}/\sqrt{6}$ (see Fig. 1), and we find the corresponding localization length ($L_{max} = 1/\Gamma_{max}$):

$$L_{max}(\xi) = (512\sqrt{6}/9\pi)(\mu/V_R)^2\xi. \qquad (14)$$

At the localization maximum, we have $\sigma_R \sim \xi$ so that the disorder cannot be modeled by an uncorrelated potential, and the long-range correlations must be accounted for as in our approach. For $\sigma_R = 0.3 \ \mu m$ [14] and $V_R = 0.2 \ \mu$, we find $L_{max} \simeq 280 \ \mu m$, which can be smaller than the system size in disordered, ultracold gases [11,14,31].

In conclusion, we have presented a general treatment for the AL of BQPs in an interacting BEC subjected to a random potential with finite-range correlations. We have calculated the Lyapunov exponents for a 1D speckle potential, and we have shown that the localization is strongest when $\sigma_R \sim \xi$. We have found that the localization length can be smaller than the size of the BEC for experimentally accessible parameters. We expect that the AL of BQPs could be observed directly, for instance as a broadening of the resonance lines in Bragg spectroscopy, a well mastered technique in gaseous BECs [32].

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