Extraordinary Acoustic Transmission through a 1D Grating with Very Narrow Apertures

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Recently, there has been an increased interest in studying extraordinary optical transmission (EOT) through subwavelength aperture arrays perforated in a metallic film. In this Letter, we report that the transmission of an incident acoustic wave through a one-dimensional acoustic grating can also be drastically enhanced. This extraordinary acoustic transmission (EAT) has been investigated both theoretically and experimentally, showing that the coupling between the diffractive wave and the wave-guide mode plays an important role in EAT. This phenomenon can have potential applications in acoustics and also might provide a better understanding of EOT in optical subwavelength systems.

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Recently, the extraordinary transmission of light through artificially micro-structured metal surface has attracted much attention [1–3]. It has been demonstrated that much more light can transmit through one or two dimensional metallic gratings [1–11] than the prediction by the conventional aperture theory [12]. Much effort devoted to the research on such an issue stems from both theoretical and practical interests. In practice, the extraordinary transmission can lead to a wide range of future applications such as subwavelength photolithography [13,14]. In principle, the physical origin of the effect has sparked many discussions [15–17]. Some models have been proposed to explain the underlying physics, such as surface plasmon polariton (SPP) [1–7], waveguide mode [8], cavity resonances [9], and dynamical diffraction [10]. More recently, a two-wave model has been proposed to analyze the properties of the waves scattered by nanoslit apertures. Two types of surface waves, SPP and a free-space surface creep wave with radiative and evanescent character [17], both can contribute to the transmission modulation [18,19]. The detailed physical mechanism for the transmission enhancement still needs further investigation [19,20]. Among these models, the SPP has been widely accepted. SPP is surface electromagnetic waves with collective electron oscillations, due to coupling between light and surface charges, capable of propagating along the metal surface with the amplitudes decaying into both sides. For other classical waves, however, such surface collective oscillations do not exist. Therefore, the investigation of enhanced transmission in these classical waves can be helpful for understanding the possible mechanisms, in addition to SPP, in extraordinary optical transmission (EOT).

The transmission through acoustic gratings has been studied for several decades [21]. In the subwavelength regime, the acoustic transmission coefficient of power is tiny and can be expressed as [22]:

$$t_P \approx \left(\frac{2\pi a}{\lambda}\right)^2 \tag{1}$$

where a is the width of the aperture, and λ is the wavelength of the incident acoustic wave, respectively. According to Eq. (1), the acoustic transmission will drop rapidly as λ increases. Rayleigh once pointed out that the acoustic resonant phenomena should appear at some special frequencies when acoustic wave impinges normally on a steel surface perforated by arrays of subwavelength slits with proper depths. The intensity of acoustic waves in these slits can be dramatically enhanced [23], which might result in extraordinary acoustic transmission (EAT). The surmises have been preliminarily studied by multiple-scattering numerical simulations [24], but without any experimental evidence and analytically physical explanation. In this Letter, we experimentally demonstrated the EAT phenomena through 1D acoustic gratings with subwavelength apertures. We also exploited a complete analytical model to investigate the physical origin of EAT, which is attributed to the strong coupling between the diffractive surface wave and waveguide modes.

A rectangular acoustic grating with very narrow apertures as shown in Fig. 1(a) has been considered. The grating consisted of 120 4 mm-wide square steel rods with the period of $d = 4.5$ mm; thus the aperture size is 0.5 mm and the thickness of the grating is 4 mm. The transmission spectra were measured by an analysis package of the 3560C Brüel & Kjær Pulse Sound and Vibration Analyzers. The zero-order transmission for normal incidence is shown in Fig. 1(a) as a function of λ. The Rayleigh minimum, corresponding to Wood's anomaly [23], is observed at the wavelength ($\lambda = d$), when Bragg diffraction is established. In contrast, the resonance-assisted enhanced transmission is also observed. Within the range of the scanned wavelength, there are two transmission peaks for the grating. For the peak at $\lambda = 2.02d$, the experimental transmission efficiency attains 92% (8.28 if normalized by the filling ratio of the aperture). Compared with the result predicted by Eq. (1) for a single aperture (about 0.11), the acoustic transmission has a 75-fold enhancement. Considering a 0.5 mm-wide, 4 mm-thick aperture in a steel

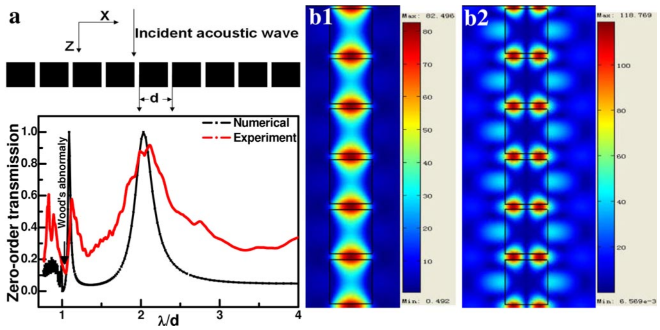

FIG. 1 (color online). (a) Numerical (black curve) and experimental (gray curve, red online) zero-order transmission spectra for a rectangular grating with the size of 4 mm, the period of 4.5 mm. The Wood's anomaly is indicated by an arrow. The upper panel is the sketch of the gratings. (b) The spatial intensity distribution of the pressure field in the grating at the wavelength of $2.02d$ (b1) and $1.09d$ (b2).

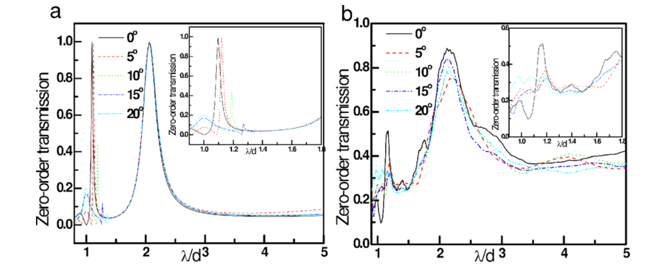

FIG. 2 (color online). (a) Theoretical and (b) experimental angular dependences of zero-order transmission spectra for the rectangular grating. Insets are the zoomed-in figures.

plate, the normalized transmission is about 0.6, so it still gets enhanced about 15 times through the grating.

By implementing the finite element simulation with Comsol Multiphysics 3.3, EAT has been numerically reproduced, concerning the spectrum shape and the position of transmission minima and maxima [See the black curve in Fig. 1(a)]. Notice that these two resonant peaks in the experiment are broader than those in the calculation, which may result from the dissipative loss, diffraction of finite size of the sample, and other experimental errors. The spatial intensity distributions of the pressure fields in the grating at the wavelength of $2.02d$ and $1.09d$, corresponding to two resonant transmission peaks, have been calculated to show in Figs. 1(b1) and 1(b2), respectively. The intensity of the fields inside the grating is about $80 \sim 119$ times larger than the original incidence (note that the fields are not zero inside the steel rods; indeed, for the real material, a weak penetration of the wave into rods can be expected). From the analysis of the spatial intensity distribution of the pressure field, the two resonant peaks are believed to originate from the Fabry-Perot resonance inside narrow apertures. These two peaks correspond to the fundamental and the second order resonant modes. From Figs. 1(b1) and 1(b2), it can be clearly seen that the surface diffractive waves are also excited on both sides of the grating.

The incident angular dependence of transmission spectra has been also studied. Figure 2 exhibits the zero-order transmission spectra of acoustic waves with different incident angles $\theta = 0°, 5°, 10°, 15°, 20°$, in which the measurement [Fig. 2(b)] is consistent with theoretical calculation [Fig. 2(a)]. The two resonant peaks show different angular dependence: the peak at the longer wavelength shows insensitive angular dependence, but the peak close to Wood's anomaly is very sensitive to incident angles. As shown in the inset of Fig. 2(b), the increase of the incident angle leads to the redshift of this peak with its amplitude rapidly fading. This angular dependence of zero-order transmission suggests that diffractive waves along the periodic direction must be responsible for the EAT. The wave vector of the transverse diffractive waves can be expressed as $k_{xm} = \frac{2\pi}{\lambda}\sin\theta \pm mG_x$ (here $G_x = \frac{2\pi}{d}$ is the grating momentum, and m is the order of the diffraction). The change of the incident angle θ may influence the diffractive surface waves and thus the resonant peak. The different behaviors of two transmission peaks with the incident angle will be discussed in details subsequently.

The zero-order transmission as a function of the thickness of the grating has been shown in Fig. 3. More resonant modes appear as the thickness of the grating increases. In both the calculation [Figs. 3(a) and 3(c) (black curve)] and the measurement [Figs. 3(b) and 3(c) (gray curve, red online)], as the thickness increases from 2 to 12 mm, the fundamental frequency of resonance decreases monotonously. However, when the grating is thin, the frequency of the fundamental resonance is close to and affected by

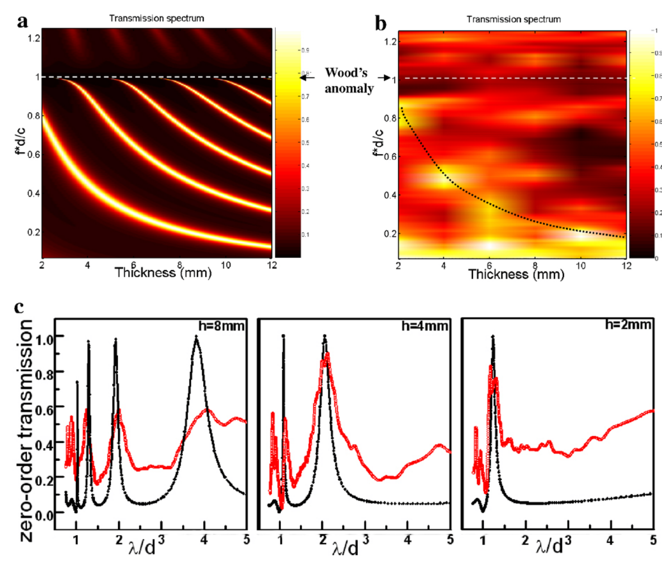

FIG. 3 (color online). (a) Theoretical and (b) experimental zero-order transmission spectra with different grating thickness. The Wood's anomaly is denoted by white dotted line and the black dotted line denotes the track of the fundamental resonant mode. (c) Theoretical (black curve) and experimental (gray curve, red online) zero-order transmission spectra with grating thickness $h = 8, 4, 2$ mm.

the Wood's anomaly, thereby deviating further from the Fabry-Perot resonant mode. If the thickness is large, the fundamental frequency of the resonant mode tends to be saturated, which also deviates from the Fabry-Perot resonant mode. Here, we stress that the acoustic transmission spectrum has a strong dependence on the period but is much less sensitive to the width of the aperture. When the width of aperture becomes larger, in spite of the decrease of the intensity of the transmission, the positions of the transmission maximum and minimum are almost invariable.

To further understand the EAT phenomenon, an original rigorous analysis model has been developed. First, the pressure field inside the grating is assumed as the superposition of two counter propagating waves, forward and backward zero-order acoustic modes, in the air aperture surrounded by steel walls. The monomodal approximation is then used in our analytical model [25]. Second, the composition of the diffractive waves above and below the grating should be considered to match the boundary condition on each side of the grating. So the pressure field P_w in the grating can be expressed as the rectangular waveguide mode:

$$P_w = \cos(\beta x)[A\exp(-jqz) + B\exp(jqz)], \qquad (-a/2 \le x \le a/2), \quad (2)$$

where A and B are the unknown amplitudes of the forward and backward waves; $q = \sqrt{k^2 - \beta^2}$ is the propagation constant of waveguide mode. The eigenwave vector of the aperture waveguide mode β can be determined by the dispersion equation as:

$$\beta\tan(\beta a) = jk\frac{\rho_0 c_0}{j\rho_{st}(d-a)\omega + \rho_0 c_0}, \quad (3)$$

where ρ_0, $\rho_{\rm st}$ are the mass densities of air and steel, respectively; c_0 is the acoustic velocity in air, and ω, k are the frequency and wave vector in air of the incident wave. Within the scope of this model, the zero-order transmittance is

$$t_0 = \left|\frac{qfs_0}{k_{z0}}\cdot\frac{4g_0[\mathrm{sinc}(\frac{\beta a}{2})]}{\Gamma_1^2 e^{jqh}}\cdot\frac{1}{1-\gamma^2}\right|^2, \quad (4)$$

where $\Gamma_1 = \sum_m \frac{g_m s_m q f}{k_{zm}} + [\mathrm{sinc}(\frac{\beta a}{2})]$, $\Gamma_2 = \sum_m \frac{g_m s_m q f}{k_{zm}} - [\mathrm{sinc}(\frac{\beta a}{2})]$, and $\gamma = \frac{\Gamma_2}{\Gamma_1}e^{-jqh}$ is defined as the resonant factor. Herein, $g_m = [\mathrm{sinc}(\frac{k_{xm}a}{2})]$, $s_m = \frac{1}{a}\times\int_{-a/2}^{a/2}\cos(\beta x)e^{jk_{xm}x}dx$, $k_{xm} = \frac{2\pi}{\lambda}\sin\theta - m\frac{2\pi}{d}$, $k_{zm} = (k^2 - k_{xm}^2)^{1/2}$, and f is the filling ratio of the aperture. More details of this model are elucidated in the Supplementary materials [26].

The zero-order transmission predicted by the model was marked by the black curve in Fig. 4. The resonances of the zero-order transmission should appear when the imaginary part of the resonant factor γ is zero. These zeros occur when the "Fabry-Perot resonance" condition is satisfied:

$$\arg(\gamma) = qh - \arg\left(\frac{\Gamma_2}{\Gamma_1}\right) = m\pi, \quad (5)$$

where m is an integer, indicating the mth-order Fabry-Perot resonance. Figure 4(a) shows the principle phase of the resonant factor. The phase becomes zero at the wavelength of the fundamental resonant peak. At the second order resonant peak, π or $-\pi$ presents, meaning the switching from the first Brillouin zone to the second one. The first term in Eq. (5) is from the expression of the waveguide mode and is independent of the incident angle θ. The second term is an additional phase shift due to the composition of the diffractive waves along the grating surfaces. That is, the waveguide resonance has been dressed through the coupling of waveguide mode to the diffractive waves. As shown in Fig. 4(a), when incident angle $\theta = 0°$, the phase shift changes tardily if λ is relatively larger than the period, but rapidly if $\lambda \approx d$. This means the position of the resonant peak strongly depends on the additional phase shift. So the position of the fundamental resonant peak is mainly dependent on the waveguide mode and nearly independent of incident angles. Equation (4) indicates that zero transmission can occur when Γ_1 becomes infinite, which means $k = k_{xm}$, corresponding to Wood's anomaly.

With narrow apertures, the characteristics of the acoustic grating are quite different from the conventional one with larger apertures [23]. The scattering potential for the wide aperture is small so that the first-order Born's approximation is proper and the zero-order transmissivity can be expressed as [27]:

$$T \sim \sqrt{t_p}\left[\mathrm{sinc}\left(\frac{k_{x0}a}{2}\right)\mathrm{comb}\left(\frac{k_{x0}d}{2}\right)\right] = \sqrt{t_p}\left[g_0\mathrm{comb}\left(\frac{k_{x0}d}{2}\right)\right]. \quad (6)$$

However, for the narrow apertures with subwavelength dimensions, the multiple scattering and the dynamical diffraction effect have to be considered. So in our grating, many diffractive modes can be simultaneously excited [11,18,19]. Thus, the strongly excited diffractive waves should play a crucial role in the extraordinary transmission.

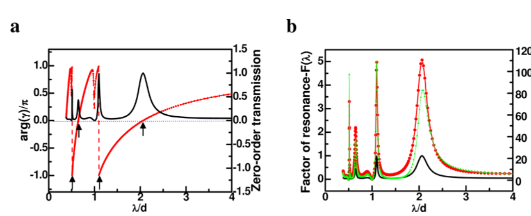

FIG. 4 (color online). (a) The dependence of the phase of the resonant factor γ (gray curve, red online) on the wavelength, and the zero-order transmission spectra (black curve) for the grating. (b) The dependence of the enhancement factor of diffraction $F(\lambda)$ on the wavelength (gray curve, red online), and the zero-order transmission spectra (black curve) for the grating. The light gray curve (green online) is the wavelength dependence of the normalized intensity of the pressure field inside the aperture.

Comparing Eq. (4) with Eq. (6), we can define $F(\lambda) = (\Gamma_1^2 e^{jqh} - \Gamma_2^2 e^{-jqh})^{-1}$ as the enhancement factor of diffractive waves. Figure 4(b) illustrates the wavelength dependence of the enhancement factor $F(\lambda)$. When $F(\lambda)$ is maximal, t_0 also reaches the maximum. The intensity enhancement $|F(\lambda)|^2$ for the fundamental resonant peak is about 25 times, consistent with the simulation and experimental results. Moreover, the normalized intensity of the pressure filed in the aperture was also calculated to show in Fig. 4(b). The maximum of the field intensity is also corresponding to the maximum of the zero-order transmission, and reaches 110 for the second order resonant mode and 84 for the fundamental resonant mode, which also agree well with the finite element simulations [Figs. 1(b1) and 1(b2)].

In conclusion, similar EAT phenomena in acoustic subwavelength systems have been investigated. Based on rigorous analysis, the resonant enhancement of the zero-order transmission is believed to be attributed to the coupling between the compositions of diffractive waves excited on the surfaces and the Fabry-Perot resonant modes inside the apertures. This enhancement stems from dynamic diffractions [11] and is also valid for the optical counterpart. However, the discrepancies between EAT and EOT are obvious: (1) there is no SPP excited in acoustic systems; (2) the zero-order propagating waveguide mode exists in acoustic narrow apertures, but in optic subwavelength systems, the cutoff frequency can lead to the truncated evanescent waveguide modes. So in the visible and near infrared regimes, SPP plays a key role in EOT [17–19]. However, the contribution from the dynamic diffractions cannot be neglected. Although more studies are necessary, especially for 2D metallic gratings, we believe that our work provides a profound basis for such extraordinary transmission in subwavelength systems. This remarkable effect will bring a great impact on ultrasonic devices and applications such as thickness selected frequency acoustic filters, acoustic collimators, and compacted acoustic devices with subwavelength, etc.

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[1] T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff, Nature (London) **391**, 667 (1998).
[2] H. F. Ghaemi, T. Thio, D. E. Grupp, T. W. Ebbesen, and H. J. Lezec, Phys. Rev. B **58**, 6779 (1998).
[3] C. Genet and T. W. Ebbesen, Nature (London) **445**, 39 (2007).
[4] F. J. Garcia-Vidal, H. J. Lezec, T. W. Ebbesen, and L. Martin-Moreno, Phys. Rev. Lett. **90**, 213901 (2003).
[5] A. Barbara, P. Quemerais, E. Bustarret, and T. Lopez-Rios, Phys. Rev. B **66**, 161403(R) (2002).
[6] D. C. Skigin and R. A. Depine, Phys. Rev. Lett. **95**, 217402 (2005).
[7] K. G. Lee and Q. H. Park, Phys. Rev. Lett. **95**, 103902 (2005).
[8] Y. Kurokawa and H. T. Miyazaki, Phys. Rev. B **75**, 035411 (2007).
[9] Q. Cao and P. Lalane, Phys. Rev. Lett. **88**, 057403 (2002).
[10] M. M. J. Treacy, Appl. Phys. Lett. **75**, 606 (1999).
[11] W. Srituravanich, N. Fang, C. Sun, Q. Luo, and X. Zhang, Nano Lett. **4**, 1085 (2004).
[12] H. A. Bethe, Phys. Rev. **66**, 163 (1944).
[13] D. B. Shao and S. C. Che, Appl. Phys. Lett. **86**, 253 107 (2005).
[14] J. Bravo-abad, A. Degiron, F. Przybilla, C. Genet, F. J. Garcia-vidal, L. Martin-Moreno, and T. T. Ebbesen, Nature Phys. **2**, 120 (2006).
[15] J. A. Porto, F. J. Garcia-Vidal, and J. B. Pendry, Phys. Rev. Lett. **83**, 2845 (1999).
[16] L. Martin-Moreno, F. J. Garcia-Vidal, H. J. Lezec, K. M. Pellerin, T. Thio, J. B. Pendry, and T. W. Ebbesen, Phys. Rev. Lett. **86**, 1114 (2001).
[17] L. Aigouy, P. Lalane, J. P. Hugonin, G. Julié, V. Mathet, and M. Mortier, Phys. Rev. Lett. **98**, 153902 (2007).
[18] L. Chen, J. T. Robinson, and Michal Lipson, Opt. Express **14**, 12 629 (2006).
[19] P. Lalanne and J. P. Hugonin, Nature Phys. **2**, 551 (2006).
[20] G. Gay, O. Alloschery, B. Viaris De Lesegno, C. O'Dwyer, J. Weiner, and H. J. Lezec, Nature Phys. **2**, 262 (2006).
[21] P. Filippi, D. Habault, J.-P. Lefebvre, and A. Bergassoli, *Acoustics: Basic Physics, Theory and Methods* (Academic, London, 1999).
[22] L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of Acoustics* (John Wiley & Sons, Inc, New York, 1982).
[23] L. Rayleigh, Philos. Mag. **39**, 225 (1920).
[24] X. D. Zhang, Phys. Rev. B **71**, 241102 (2005).
[25] R. C. Mcphedran and D. Maystre, Applied Physics (Berlin) **14**, 1 (1977).
[26] See EPAPS Document No. E-PRLTAO-99-002743 for detailed elucidation of the analytical model and the schematic of the experiment setup. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[27] See, for example, M. Born and E. Wolf *Principles of Optics* (Pergamon, Oxford, UK, 1975).