Charting the Landscape of Supercritical String Theory

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Special solutions of string theory in supercritical dimensions can interpolate in time between theories with different numbers of spacetime dimensions and different amounts of world sheet supersymmetry. These solutions connect supercritical string theories to the more familiar string duality web in ten dimensions and provide a precise link between supersymmetric and purely bosonic string theories. Dimension quenching and c duality appear to be natural concepts in string theory, giving rise to large networks of interconnected theories.

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Introduction.—Among the major advancements in string theory has been the realization that several seemingly distinct perturbative versions of the theory are linked by duality. Overwhelming evidence has accumulated that these theories collectively emerge from a more fundamental framework known as M theory. The duality web connects type I, type IIA, type IIB, and heterotic $SO(32)$ and $E_8 \times E_8$ superstring theories. This structure is often depicted as a pointed diagram with M theory residing in the middle, bounded by links among six vertices representing individual superstring theories and 11-dimensional supergravity. This picture describes only the duality network connecting *supersymmetric* string theories in ten spacetime dimensions. We show that a number of consistent string theories possessing either noncritical target space dimensions or exhibiting no spacetime supersymmetry (or both) can be connected to the duality web.

Perturbative string theory is described by a 2D field theory coupled to 2D gravity. The gravity theory on the string world sheet possesses a potentially anomalous Weyl symmetry. The Weyl anomaly can be made to vanish by formulating the theory in a specific, critical number of spacetime dimensions ($D_c = 10$ for the superstring, and $D_c = 26$ for the bosonic string). The condition $D = D_c$, however, is not the only way to cancel the Weyl anomaly: Consistent string theories exist in $D \neq D_c$ in the presence of a linear-dilaton background (see, e.g., [1–3]).

By first instance, there are an infinite number of noncritical string theories that appear to be disconnected, both from each other and from the ten-dimensional supersymmetric duality web. Also disconnected from the duality web is the set of bosonic string theories in critical and noncritical dimensions. Understanding the relationships among these seemingly disconnected theories has been a long-standing problem.

In this Letter, we address this problem by formulating precise connections among the aforementioned theories. We present a set of time-dependent classical solutions that interpolate in time among string theories with different numbers of spacetime dimensions, as well as different degrees of supersymmetry on the world sheet and in spacetime. The dynamics of our models describe domain walls moving at the speed of light, separating two distinct string theories. In each case, the phase on one side of the wall has lower potential energy. The configuration as a whole is the late-time limit of an *expanding bubble* of a lower-energy vacuum. Transitions to the new vacuum can be interpreted as a renormalization group (RG) flow in the world sheet theory, dressed with an exponential of the light-cone coordinate X^+ to render the deformation strictly scale-invariant. That is, if we think of treating the factor $\exp(\beta X^+)$ as a nondynamical mass scale, the theory admits a true RG flow. Letting $\exp(\beta X^+)$ be dynamical restores the conformal invariance of the entire theory: The perturbation remains marginal, and the theory has exactly vanishing beta function.

In some cases, the dynamics interpolate between string theories in different numbers of spacetime dimensions: In passing across the domain wall of the expanding bubble, the number of spatial dimensions changes dynamically. To simplify the exposition, we collectively refer to processes in this category as *dimension quenching*. In the examples we study, the stable end point of successive stages of dimension quenching is string theory in either the critical dimension (for type II theories) or in two dimensions (for bosonic and type 0 theories).

We also describe transitions that connect type 0 superstring theory dynamically with purely bosonic string theory. These solutions constitute precise transitions among simple string theories, unique in that they are realized *cosmologically* rather than as adiabatic motion along moduli space. (As with dimension-quenching transitions, time evolution in the target space acts as an RG flow on the world sheet.) For this reason, and because they involve a transfer of central charge contribution from various sources, we dub these transitions c dualities.

The intent of this Letter is to report the salient properties of these solutions and to present a global atlas of connections between various consistent string theories that arise from these processes. The resulting networks, while not exhaustive, demonstrate the rich interconnectedness of string theory. Detailed expositions of dimension quenching and c duality can be found in Refs. [4–6].

Dimension quenching.—The first set of solutions we study describes a reduction in the background spatial dimensions as a function of light-cone time X^+, triggered by a nonzero tachyon expectation value. The initial matter central charge is equal to the number of spacetime dimensions D. There is a timelike dilaton dependence $\Phi = -qX^0$, which compensates the matter central charge excess if we set $q = \sqrt{(D-26)/6\alpha'}$ for the bosonic string or $q = \sqrt{(D-10)/4\alpha'}$ for the superstring.

The on-shell condition for a linearized tachyon perturbation $\mathcal{T}(X)$ in the linear-dilaton background is $\partial^2\mathcal{T} - 2V^\mu\partial_\mu\mathcal{T} + \frac{4}{\alpha'}\mathcal{T} = 0$, where V^μ is the dilaton gradient. There are special choices of $\mathcal{T}(X)$ for which the world sheet theory is well-defined and conformal to all orders in perturbation theory (and nonperturbatively) in α'. Defining the light-cone frame $X^\pm \equiv (X^0 \pm X^1)/\sqrt{2}$, one sees that the exponential $\exp(\beta X^+)$ is nonsingular in the vicinity of another copy of itself. The conformal properties of vertex operators involving $\exp(\beta X^+)$ are therefore completely well-behaved. If we choose $\beta = 2\sqrt{2}/q$, this operator has weight $(1, 1)$ and zero anomalous dimension. This makes it possible to construct Lagrangian perturbations that deform the free theory while preserving conformal invariance exactly.

For the bosonic string, the tachyon couples to the free world sheet theory according to the interaction Lagrangian $\mathcal{L}_{\text{int}} = -\frac{1}{2\pi}\mathcal{T}$. To illustrate the dynamics of these solutions, we start by considering a tachyon profile of the form $\mathcal{T} = \mu_0^2 e^{\beta X^+} - \mu_k^2\cos(kX_2)e^{\beta_k X^+}$, where $\beta_k = (\sqrt{2}/q)\times(2/\alpha' - k^2/2)$ and μ_0 and μ_k are real parameters. One obtains a marginal perturbation that is Gaussian in X_2 in the long-wavelength limit $k \to 0$. The tachyon becomes large for large X^+, and the theory acquires a mass term for the coordinate X_2, which increases exponentially. In fact, the world sheet dynamics are exactly solvable at the *quantum level*, despite the fact that the underlying 2D theory is fully interacting. It can therefore be shown that the theory remains exactly conformal at all times.

The solution describes a domain wall moving to the left at the speed of light. When string trajectories carrying energy in the X_2 field encounter the domain wall, the X_2 field becomes frozen into a state of nonzero excitation and is pushed out along the wall to $X_1 \to -\infty$ at late times. Trajectories carrying *no* energy in the modes of the X_2 field, however, are allowed to propagate through the domain wall and into the bubble interior. (One might think of the interface as a *domain filter*.) The amount of dynamical matter on the world sheet therefore decreases dynamically as a function of X^+. The X_2 dimension becomes quenched for large X^+, and the region remaining at late times inside the tachyon condensate is a $(D-1)$-dimensional theory.

The contribution to the central charge from the dilaton sector of the theory is determined by the dilaton gradient and the string-frame metric. Both of these objects shift by finite amounts during the dimension-quenching transition. From the world sheet point of view, this is a one-loop renormalization of effective couplings arising when the X_2 field is integrated out. Quantum corrections to the classical theory terminate at one-loop order, so it is possible to compute this renormalization exactly. From the spacetime perspective, the effect is a backreaction of the tachyon onto the metric and dilaton. The result is that the dilaton contribution to the central charge increases by one unit, compensating the loss of the X_2 degree of freedom.

In mapping out the various possibilities, it is helpful to sort the allowed transitions into the following categories: stable transitions, in which no perturbation of the solution can destroy or alter the final state qualitatively; natural transitions, in which no perturbation can destabilize the solution without breaking additional symmetry; tuned transitions, in which the initial conditions of an unstable mode must be fine-tuned to preserve the qualitative nature of the final state. Under this classification scheme, transitions among bosonic string theories in different numbers of dimensions are tuned transitions. In particular, the parameters of the solution must be tuned to a particular value to set the bosonic string tachyon to zero in the far future of the lower-dimensional final state.

Transitions between type 0 string theories in even numbers of dimensions are natural by virtue of a chiral world sheet R parity. The action of this R parity is implemented by an operator g_L, which acts with a $+1$ on G and a -1 on $\tilde{G}$. However, type 0 string theories in *odd* dimensions preserve no such symmetry (see, e.g., [5] for a discussion of this point). Transitions between even- and odd-dimensional type 0 theories are therefore tuned transitions.

Let us consider the type 0 case for which the dimension of spacetime decreases by $\Delta D = 1$. The tachyon $\mathcal{T}$ couples to the world sheet as a superpotential: $\Delta\mathcal{L} = \frac{i}{2\pi}\times\int d\theta_+ d\theta_-{:}\,\mathcal{T}(X){:}$ (colons represent normal ordering), with

$$:\mathcal{T}: = \exp\beta X^+\left(\frac{\mu}{2\alpha'}{:}X_2^2{:} + \frac{\mu}{\alpha' q\sqrt{2}}X^+ + \mu'\right).$$

The quantity μ' is a regulator-dependent coefficient that must be tuned to make the effective superpotential vanish in the limit $X^+ \to \infty$. Unlike the even-dimensional type 0 theories, which come in two varieties (0A and 0B), there is only one kind of odd-dimensional type 0 theory. Starting from odd-dimensional type 0, the sign of μ determines whether one reaches type 0A or type 0B string theory in $D-1$ dimensions as a final state. After defining the odd-dimensional GSO (Gliozzi-Scherk-Olive) projection with a particular phase $[(-1)^{F_W} = \pm i]$ in the Ramond/Ramond sectors, altering the sign of μ changes the ground state of the ψ_2 and $\tilde{\psi}_2$ fermions and thereby reverses the effective GSO projection for the $D-1$ remaining Majorana fermions. These tuned transitions are represented by the diagonal lines in Fig. 1 and can be defined to connect type 0 linear-dilaton backgrounds in any number of dimensions, with any value of ΔD, as long as the final state is at least two-dimensional.

The natural transitions described above can be converted to stable transitions if one orbifolds by a discrete R symmetry, such as the R parity g_L described above (see Fig. 1). One can thereby start in a supercritical string theory and condense the tachyon without fine-tuning initial conditions to recover a stable, supersymmetric ten-dimensional background in the distant future. In the limit $X^+ \to \infty$, the massive string modes of the 10D theory have a vanishing expectation value, the dilaton is lightlike and rolling to weak coupling, and the background preserves half of the supersymmetries of type II string dynamics. Starting from an initial state with $D > 10$, the final state has lightlike linear dilaton with no fluxes, gradients, or curvature of the string-frame metric. The final state is therefore a particular half–Bogomol'nyi-Prasad-Sommerfeld background of critical superstring theory in the limit $X^+ \to \infty$, where all massive background fields have decayed to zero. This establishes that supercritical string theories are connected by dimension quenching to the standard duality web of critical superstring theory.

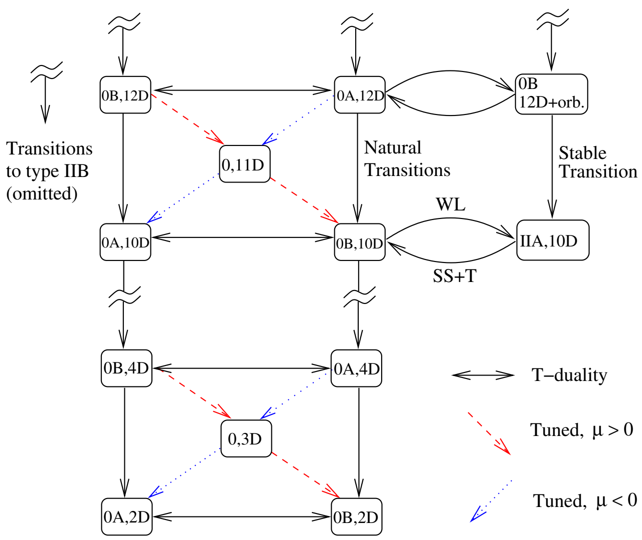

FIG. 1 (color online). The dimension-quenching transitions in type 0 string theory define a semi-infinite lattice of connected theories. Diagonal lines are tuned transitions that reduce the spacetime dimensionality by one. Right-pointing downward (dashed) arrows represent transitions with $\mu > 0$, and left-pointing downward (dotted) arrows represent transitions with $\mu < 0$. Vertical lines are natural transitions reducing the spacetime dimensionality by two. The straight horizontal lines indicate either T duality or orbifolding by a left-moving spacetime fermion number (the 2D version is a thermal T duality). Natural dimension-quenching transitions of orbifolded type 0 string theories terminate with type II string theory in the critical dimension ($D = 10$). These hierarchies are connected laterally (curved arrows) to the type 0 series by Scherk-Schwarz compactification and T duality (SS + T) or by a discrete Wilson line (WL) construction [8]. (Orbifolded type 0B transitions to stable type IIA are shown on the right; analogous connections among orbifolded type 0A and critical type IIB theories have been omitted.)

c duality.—One surprising outcome arises when considering type 0 string theory in a flat, timelike linear-dilaton background. We consider a world sheet theory deformed by the interaction Lagrangian (the tachyon again couples as a superpotential) $\mathcal{L}_{\text{int.}} = -\frac{\alpha'}{8\pi} G^{\mu\nu}\partial_\mu \mathcal{T} \partial_\nu \mathcal{T} + \frac{i\alpha'}{4\pi}\partial_\mu\partial_\nu \mathcal{T} \tilde{\psi}^\mu \psi^\nu$, where ψ and $\tilde{\psi}$ are, respectively, right- and left-moving world sheet fermions. In addition, the supersymmetry algebra is modified by the following F term: $F^- = \{Q_-, \psi^\mu\} = -\{Q_+, \tilde{\psi}^\mu\} = -(\sqrt{\alpha'}/8)G^{\mu\nu}\partial_\nu \mathcal{T}$. When the tachyon is lightlike, the world sheet potential vanishes, but the Yukawa coupling, the F terms, and the interaction terms all grow as $X^+ \to \infty$. Despite this fact, the fermion mass matrix remains nilpotent, and the physical frequencies of the fermion modes do not grow. With no world sheet potential, no states are expelled from the interior of the domain bubble, and the number of spacetime dimensions does not change. The presence of the F term, however, indicates that the world sheet supersymmetry is spontaneously broken as $X^+ \to \infty$.

Although the world sheet theory is strongly interacting in its original variables at large X^+, we can describe the physical content precisely by invoking a series of canonical transformations. The starting point is to exchange the lightcone fermions $\psi^\pm$ for a bc ghost system with weights $3/2$ and $-1/2$, denoted by b_1 and c_1, respectively (and similarly for left-movers): $\psi^+ = 2c_1' - M^{-1}\tilde{b}_1 + 2\beta(\partial_+ X^+)c_1$; $\tilde{\psi}^+ = -2\tilde{c}_1' + M^{-1}b_1 + 2\beta(\partial_- X^+)\tilde{c}_1$; $\psi^- = M\tilde{c}_1$; $\tilde{\psi}^- = -Mc_1$. Here we have defined $M \equiv \mu\exp(\beta X^+)$. In this new set of variables, the Lagrangian consists of a free theory plus a perturbation proportional to $\exp(-\beta X^+)$, which becomes vanishingly small deep inside the region of a large tachyon condensate.

A subsequent series of canonical transformations can be invoked that render the stress tensor and supercurrent in the following form [6] (T_{mat} is just the remaining matter contribution to the stress tensor):

$$T = -\frac{3i}{2}\partial_+ c_1 b_1 - \frac{i}{2} c_1 \partial_+ b_1 + \frac{i}{2}\partial_+(c_1 \partial_+^2 c_1) + T_{\text{mat}},$$

$$G = b_1 + i\partial_+ c_1 b_1 c_1 - c_1 T_{\text{mat}} - \frac{5}{2}\partial_+^2 c_1.$$

At this stage, the supersymmetry is completely nonlinearly realized. In this form, the D-dimensional theory is a free world sheet theory with a bc ghost system, D free scalars, and $D - 2$ free fermions (with appropriate left- or right-moving counterparts). The central charge receives a contribution of 26 from the matter-dilaton system (the dilaton gradient is again renormalized) and -11 from the bc ghosts. The theory therefore exhibits the correct central charge for the world sheet of an Ramond-Neveu-Schwarz superstring in the conformal gauge.

In fact, the system at late times fits into a construction due to Berkovits and Vafa [7], in which the bosonic string is embedded in the solution space of the superstring (i.e., it is a formal rewriting of the bosonic string as a superstring). This is just a manifestation of the fact that to any theory

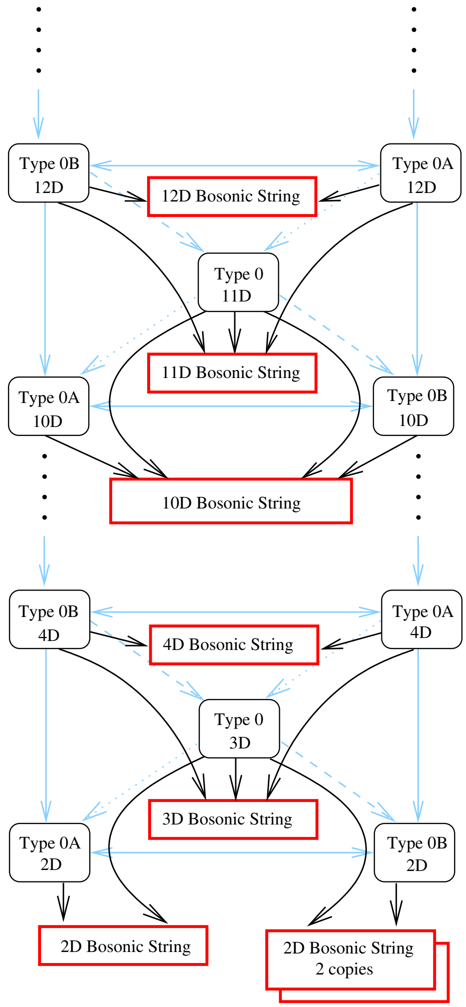

FIG. 2 (color online). Transitions to bosonic string theory in two dimensions and higher can occur via c duality, starting from points in the type 0 hierarchy. The transitions connect type 0 theories in D dimensions to bosonic string theory in D (straight, solid arrows) or $D - 1$ (curved, solid arrows) noncompact dimensions, with compact current-algebra factors. The curved solid arrows are the detuned versions of tuned dimension-changing transitions with $\Delta D = 1$. The transitions to bosonic string theory in this figure are understood to be superimposed on the corresponding structure in the type 0 series depicted in Fig. 1.

one may add any amount of nonlinearly realized local symmetry as a redundancy of description. From the perspective of the bosonic theory, the role of the local supersymmetry is to restrict the b_1 and c_1 fields to their ground states, up to gauge transformation.

We emphasize that, in formulating the transition to bosonic string theory, we have not integrated out any fields, and no information contained in the original type 0 phase has been lost. The late-time description is achieved entirely through canonical variable redefinitions. In one set of variables, the theory is weakly coupled in the initial (UV) phase, while in another set, the theory is weakly coupled in the late-time (IR) limit. No information is lost in moving to IR variables, in which the theory reduces to bosonic string theory at large X^+: The change of variables constitutes a precise duality. The nature of the transition is cosmological, in that time evolution in the target space drives RG flow on the world sheet, and there is again a transfer of central charge from the UV fermions to the dilaton. [We reiterate that the RG flow comes dressed with a factor of $\exp(\beta X^+)$, which preserves the exact marginality of the world sheet perturbation.] We therefore refer to this transition as c duality. Bosonic string theory in various dimensions can be reached via c duality from type 0 string theory (we require a tachyonic phase in the bulk to drive the transition). The basic transitions induced by the tachyon profile $\mathcal{T} = \mu \exp(\beta X^+)$ connect D-dimensional type 0 string theory to the bosonic string in D noncompact dimensions with an $SO(D-2)_L \times SO(D-2)_R$ current algebra.

A related set of transitions can be obtained by "detuning" the tuned dimension-changing type 0 transitions. For instance, we can modify the transition from type 0 in D dimensions to type 0 in $D - 1$ dimensions by deforming μ' away from its tuned value. This is equivalent to deforming the $(D-1)$-dimensional final state with a tachyon vacuum expectation value of the form $\mathcal{T} = (\mu' - \mu'_{\rm tuned}) \exp(\beta X^+)$. This induces a further transition to the bosonic string in $D - 1$ noncompact dimensions with an $SO(D-3)_L \times SO(D-3)_R$ current algebra. Various allowed c duality transitions are depicted in Fig. 2.

Conclusions.—The solutions described in this Letter interpolate between theories that were previously thought to be completely distinct; they connect string theories in different numbers of spacetime dimensions and with different amounts of world sheet and spacetime supersymmetry. The duality web of critical superstring theory is therefore naturally embedded in a much wider class of string theories. We expect that the mechanisms described in this Letter will play a substantial role in mapping out the full, nonperturbative phase structure of quantum gravity with and without supersymmetry.

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[1] A. H. Chamseddine, Nucl. Phys. **B368**, 98 (1992).
[2] R. C. Myers, Phys. Lett. B **199**, 371 (1987).
[3] A. M. Polyakov, Phys. Lett. **103B**, 207 (1981).
[4] S. Hellerman and I. Swanson, arXiv:hep-th/0611317.
[5] S. Hellerman and I. Swanson, arXiv:hep-th/0612051.
[6] S. Hellerman and I. Swanson, arXiv:hep-th/0612116.
[7] N. Berkovits and C. Vafa, Mod. Phys. Lett. A **9**, 653 (1994).
[8] S. Hellerman and J. Walcher, arXiv:hep-th/0604191.