Storage of Spin Squeezing in a Two-Component Bose-Einstein Condensate

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A simple scheme for storage of spin squeezing in a two-component Bose-Einstein condensate is investigated by considering rapidly turning-off the external field at a time that maximal spin squeezing occurs. We show that strong reduction of spin fluctuation can be maintained in a nearly fixed direction. We explain the underlying physics using the phase model and present analytical expressions of the maximal-squeezing time and the corresponding squeezing parameter.

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Spin squeezing has attracted much attention for decades not only because of fundamental physical interests [1–5], but also for its possible application in atomic clocks for reducing quantum noise [2] and quantum information [6–10]. Formally, the spin squeezing is quantified via a parameter $\xi = (\Delta\hat{J}_{\mathbf{n}})_{\min}/\sqrt{j/2}$, where $(\Delta\hat{J}_{\mathbf{n}})_{\min}$ represents the smallest variance of a spin component $\hat{J}_{\mathbf{n}} = \hat{J}\cdot\mathbf{n}$ normal to the mean spin $\langle\hat{J}\rangle$. For the coherent spin state (CSS), the variance $(\Delta\hat{J}_{\mathbf{n}})_{\min}$ is $\sqrt{j/2}$, i.e., $\xi = 1$. If the mean spin is in the x direction, the spin component with the reduced variance is in the (y, z) plane. A state is called spin squeezed state (SSS) if its variance of the spin component is smaller than that of the CSS, i.e., $\xi < 1$.

Kitagawa and Ueda have investigated the spin squeezing generated by the so-called one-axis twisting (OAT) model with Hamiltonian: $\hat{H}_{\mathrm{OAT}} = 2\kappa\hat{J}_z^2$ [1]. Possible realization of the OAT-type spin squeezing in a two-component Bose-Einstein condensate (TBEC) has been proposed in Refs. [6,11]. Sørensen *et al.* also argued that macroscopic quantum entanglement can be characterized by using the OAT-type spin squeezing in the TBEC [6]. Besides the TBEC, Takeuchi *et al.* recently proposed another realization of the OAT-type spin squeezing by using the interactions between atoms and off-resonant light (paramagnetic Faraday rotation) [12]. To coherently control spin squeezing, Law *et al.* introduced an additional Josephson-like (or Raman) coupling to the OAT model: $\hat{H}_R = 2\kappa\hat{J}_z^2 + \Omega_R\hat{J}_x$ [13]. Such a model has also been used to prepare arbitrary Dicke states [14].

In addition to the generation of the SSS itself, it is desirable to maintain not only the squeezing but also its direction for a long time [7]. Jaksch *et al.* have shown that the OAT-type SSS can be stored for an arbitrarily long time by removing the self-interaction [15]. However, it might not be easy to handle in experiment since the precisely designed additional pulses are crucially required. In this Letter, we propose a simple mechanism to obtain long-lasting spin squeezing in the TBEC. Our scheme is quite easy to realize in experiment since it can be achieved by turning-off the Josephson coupling once the TBEC reaches its maximal spin squeezing. More importantly the storage is achieved even though the inherent self-interaction among atoms persists.

We consider a two-component weakly interacting BEC [16,17] consisting of N atoms in different atomic hyperfine states $|a\rangle$ and $|b\rangle$ coupled by a time-varying microwave field with Rabi frequency Ω_{rf}. Based on the two-mode approximation [18–23], the total Hamiltonian can be described by ($\hbar = 1$):

$$\hat{H}(t) = 2\kappa\hat{J}_z^2 + \Omega(t)\hat{J}_x, \tag{1}$$

where $\kappa = (\kappa_{aa} + \kappa_{bb} - 2\kappa_{ab})/4$, and $\kappa_{\alpha\beta} = g_{\alpha\beta}\int d^3\mathbf{r}|\phi_\alpha(\mathbf{r})\phi_\beta(\mathbf{r})|^2$, with $g_{\alpha\beta} = 4\pi a_{\alpha\beta}/m$ (α, $\beta = a, b$) being the s-wave scattering strengths between atoms. The normalized condensate-mode functions ϕ_α satisfy coupled Gross-Pitaevskii equations [24]. Here we focus on the case that the external field is turned off rapidly at a time t_M so that the time-dependent Josephson-like coupling can be written as $\Omega(t) = \Omega_R\Theta(t_M - t)$, where $\Omega_R = \Omega_{\mathrm{rf}}\int d^3\mathbf{r}\phi_a^*(\mathbf{r})\phi_b(\mathbf{r})$ and $\Theta(t)$ is the step function.

The state vector at arbitrary time t can be expanded in terms of eigenstates of $\hat{J}_z$: $|\psi(t)\rangle = \sum_m c_m(t)|j, m\rangle$, where $-j \le m \le j$ and $j = N/2$. The equations of motion for the amplitudes $c_m(t)$ are obtained by solving the time-dependent Schrödinger equation. We consider an initial CSS $|j, -j\rangle_x = e^{-i\pi J_y/2}|j, -j\rangle$, then the initial amplitudes

$$c_m(0) = \frac{(-1)^{j+m}}{2^j}\binom{2j}{j+m}^{1/2}.$$

Our initial CSS satisfies $c_{-m}(0) = c_m(0)$ for even N, and $c_{-m}(0) = -c_m(0)$ for odd N, from which we will prove that the mean spin appears always in the x direction. In addition, we will consider only positive κ case by assuming $a_{aa} + a_{bb} > 2a_{ab}$. However, our results keep valid for negative κ if $|j, j\rangle_x$ is exploited as an initial state.

Now let us briefly explain the basic principle of our scheme. The initial CSS is prepared by applying a short $\pi/2$ pulse to a single-component BEC with all the atoms being in the internal ground state $|a\rangle$ [6,23]. After that, the external Josephson coupling is immediately switched on, so the dynamics of the TBEC is governed by the Hamiltonian (1) with $\Omega(t) = \Omega_R$. The squeezing parame-

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ter ξ rapidly decreases to the first local minimum ξ_M at the maximal-squeezing time denoted as t_M, and then exhibits collapsed oscillations [25] as shown by the dashed curve in Fig. 1. If the coupling is optimally chosen, the Josephson interaction results in further enhancement of the spin squeezing compared with that of the OAT [13]. The optimal coupling, defined as Ω_R at which the first local minimum ξ_M is optimized, depends on the number of atoms, e.g., $\xi_M = 8.7076 \times 10^{-2}$ at $\Omega_R = 10.8\kappa$ for $N = 10^3$. We find that if we turn off the Josephson field at the time t_M, the maximal squeezing ξ_M can be stored in a fixed direction (i.e., $\theta_{\min} = 0$). The basic features of our scheme are exhibited by the solid lines of Fig. 1. It is nontrivial to explain such a result since neither does $\hat{J}_{\mathbf{n}}$ commute with the Hamiltonian $2\kappa\hat{J}_z^2$ nor is the SSS at t_M an eigenstate of $\hat{J}_z^2$.

To understand the above observation, we investigate probability distribution of the spin state, $|c_m|^2 = |\langle j, m|\psi(t)\rangle|^2$. As shown in the insets of Fig. 2, we find that compared with the initial CSS, the maximal SSS at t_M has a very sharp probability distribution centered at the lowest spin projection, i.e., $m = 0$ (for even N) or $m = \pm 1/2$ (for odd N). Such a sharp probability distribution of the SSS can be explained qualitatively by considering the familiar phase model [26]. By replacing $\hat{J}_z \to p_{\hat{\phi}} = -i\partial_{\hat{\phi}}$ and $\hat{J}_x \to (N\cos\hat{\phi})/2$, with $\hat{\phi}$ being the macroscopic phase difference between two condensate components, one obtains the following Hamiltonian

$$H_{\hat{\phi}} = -2\kappa\frac{\partial^2}{\partial\hat{\phi}^2} + \frac{\Omega_R N}{2}\cos\hat{\phi}. \tag{2}$$

The phase model allows us to regard the spin system as the fictitious particle with effective mass $(4\kappa)^{-1}$ subjected to a pendulum potential. The motion of the particle can be described as the rotation in phase space (ϕ, p_ϕ) in terms of a Wigner distribution function.

As shown in Fig. 2, starting from vertically elongated distribution, one obtains the distribution elongated horizontally at the time $t \simeq T/4$, where the period $T = 2\pi/\omega_{\text{eff}}$ and $\omega_{\text{eff}} = \sqrt{2\kappa\Omega_R N}$. Based upon the phase model, one can intuitively understand why the SSS at t_M has a sharp distribution. Moreover, we obtain analytical expression of the maximal-squeezing time

$$\kappa t_M \simeq \kappa\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{\kappa}{2\Omega_R N}}, \tag{3}$$

which is valid for large N ($\geq 10^3$). It should be noted that t_M given in Eq. (3) corresponds not necessarily to the time that ξ is minimized but rather to the time that the distribution is sharp in terms of p_ϕ. Only for Ω_R near or larger than the optimal coupling, it provides the maximal-squeezing time, which will be discussed in detail below. Compared with the exact numerical result, we find Eq. (3) gives accurate prediction of the maximal-squeezing time for wide range of Ω_R. For example, with $N = 10^3$ and $\Omega_R = 10.8\kappa$ (the optimal coupling), Eq. (3) predicts $\kappa t_M = 1.069 \times 10^{-2}$, which agrees quite well with 1.104×10^{-2} obtained numerically.

If the external field is turned off at t_M, the spin system is governed only by the self-interaction Hamiltonian $2\kappa J_z^2$. Therefore, the amplitudes $|c_m(t)|$'s are conserved while the relative phases among the spin projections are subjected to change. It is found when the SSS exhibits a very sharp distribution in m as shown in the inset of Fig. 2(b), the relative phases have negligible influence on the spin squeezing. To show this, we suppose the SSS at t_M takes the form

$$|\psi(t_M)\rangle = \frac{e^{i\varphi}\sin\alpha}{\sqrt{2}}(|j, 1\rangle + |j, -1\rangle) + \cos\alpha|j, 0\rangle \tag{4}$$

for even N, or

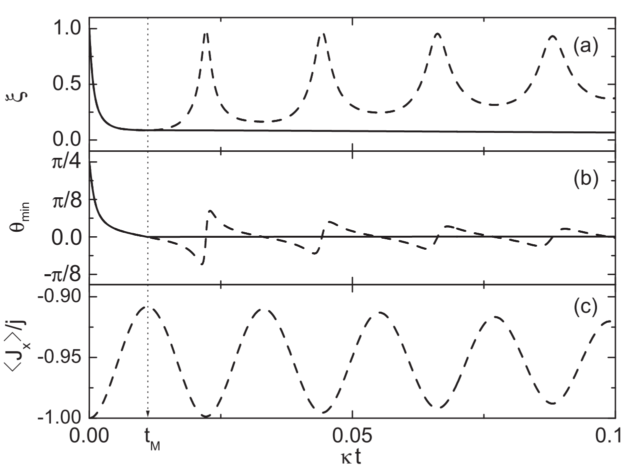

FIG. 1. Time evolution of (a) the squeezing parameter, (b) the squeezing angle, and (c) the mean spin $\langle J_x\rangle$ for the optimal coupling $\Omega_R = 10.8\kappa$ with $N = 10^3$. Dashed curves: the constant-coupling case; solid curves: the case for turning-off the coupling at the time t_M indicated by the vertical dotted line.

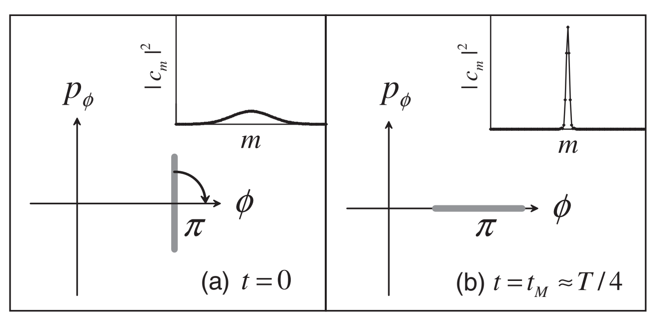

FIG. 2. Schematic picture of the probability distribution in phase space (ϕ, p_ϕ) for (a) the initial CSS and (b) the SSS at $t = T/4$. The insets: the corresponding distribution $|c_m|^2$ as a function of m (or p_ϕ) obtained numerically with the parameters used in Fig. 1.

$$|\psi(t_M)\rangle = \frac{e^{i\varphi}\sin\alpha}{\sqrt{2}}(|j, 3/2\rangle - |j, -3/2\rangle) + \frac{\cos\alpha}{\sqrt{2}}(|j, 1/2\rangle - |j, -1/2\rangle) \quad (5)$$

for odd N, where α describes the amplitude and φ represents the relative phase difference induced by the self-interaction. Figure 3 presents the squeezing parameter ξ as a function of α and φ, where two distinct features are observed. First, ξ is minimized as $\alpha \to 0$ [2], which implies that the maximal squeezing occurs when the SSS is dominated by the lowest spin projection: i.e., $m = \pm 1/2$ for odd N, or $m = 0$ for even N. Second, ξ is rather *insensitive* to the relative phase difference φ. Remember that the self-interaction makes only φ vary but α is kept almost fixed. This explains our scheme for the storage of spin squeezing.

We check the above argument in two exactly solvable cases with $N = 2$ and $N = 3$. Taking the optimal coupling $\Omega_R = \kappa$ for $N = 2$ and $\Omega_R = 2\kappa$ for $N = 3$, we get the maximally squeezed states $|\psi(t_n)\rangle = -i(-1)^{n+1}e^{-i\kappa t_n}|j = 1, m = 0\rangle$ [2] and $|\psi(t_n)\rangle = \frac{i(-1)^n}{\sqrt{2}}e^{-3i\kappa t_n/2}(|3/2, 1/2\rangle - |3/2, -1/2\rangle)$, respectively, where $t_n = (2n + 1)\pi/S$ with an integer n, and $S = 2\sqrt{\Omega_R^2 + \kappa^2}$ for $N = 2$ and $S = 2\sqrt{\Omega_R^2 + 2\kappa\Omega_R + 4\kappa^2}$ for $N = 3$. Since the spin squeezed state $|\psi(t_n)\rangle$ is the ground state of $\hat{J}_z^2$, the rapid switching-off the external field at t_n obviously results in constant ξ with $\theta_{\min} = 0$. Even though for larger N the SSS at t_M no longer lies at the ground state of $\hat{J}_z^2$, almost constant ξ can also be achieved as long as the distribution is sharp enough as explained above.

It does not depend on Ω_R whether the storage itself can be achieved or not in our scheme. For Ω_R near or larger than the optimal coupling the maximal squeezing can be stored. Whereas for Ω_R smaller than the optimal coupling it is found that there exist two time scales: t_M at which turning-off the field leads to the storage, and namely τ_M at which the maximal squeezing occurs. Equation (3) still works very well to give t_M, but fails to predict the maximal-squeezing time. It is shown in Fig. 4(a) that for $\Omega_R = 5\kappa$ true maximal squeezing occurs at $\tau_M = 6.915 \times 10^{-3}\kappa^{-1}$, while the storage is achieved when turning-off the field at $t_M = 1.687 \times 10^{-2}\kappa^{-1}$ at which the probability distribution of the SSS appears to be sharp [see the inset of Fig. 4(b)]. As a result, nonmaximally squeezed variance is stored by turning-off the external field at the time t_M [see the red curves of Fig. 4(a)].

Interestingly it is found in Figs. 1(a) and 4 that at t_M the squeezing angle $\theta_{\min}$ is always equal to zero, which can be explained by considering the phase model again. The average $\langle\cos\phi\rangle$ over the distribution in Fig. 2 as time goes monotonically increases and turns to decrease at t_M since $\cos\phi$ is an even function on $\phi = \pi$. Consequently $\langle\cos\phi\rangle$ has its local maximum at t_M. Considering the relation $\hat{J}_x = (N\cos\hat{\phi})/2$, it is found that $d\langle\hat{J}_x\rangle/dt \propto d\langle\cos\hat{\phi}\rangle/dt = 0$ at t_M (see also $\langle J_x\rangle$ in Figs. 1 and 4). It leads us to $\theta_{\min} = 0$ due to $\theta_{\min} \propto \tan^{-1}(A^{-1}d\langle\hat{J}_x\rangle/dt)$ for $A \neq 0$ (see below). Consequently t_M provides not only the time when $\theta_{\min}$ is equal to zero, but also the time when the distribution becomes very sharp.

Our explanation for the storage of spin squeezing should be applied with great care to the SSS with a rather broad probability distribution. In fact, typical OAT scheme [1] relies solely on the evolution of relative phases induced by the self-interaction, where the initial CSS shows a broad probability distribution. As shown by the red line of Fig. 4(b), for a large coupling $\Omega_R = 50\kappa$, the squeezing parameter decreases *slightly* after turning-off the external coupling. This is because the SSS at t_M exhibits a relatively broad probability distribution [see the inset of Fig. 4(b)]. As mentioned above, the squeezing angle is also maintained at zero as shown by the green curve in Fig. 4(b).

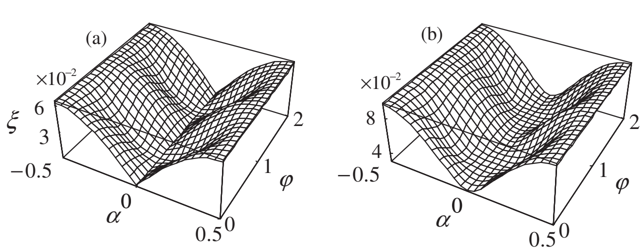

FIG. 3. The squeezing parameter ξ as a function of α and φ (in units of π) for (a) the even number case ($N = 1000$); (b) the odd number case ($N = 1001$), calculated by using Eqs. (4) and (5), respectively. Note that the scale of ξ's differ for (a) and (b) although the difference in N is tiny.

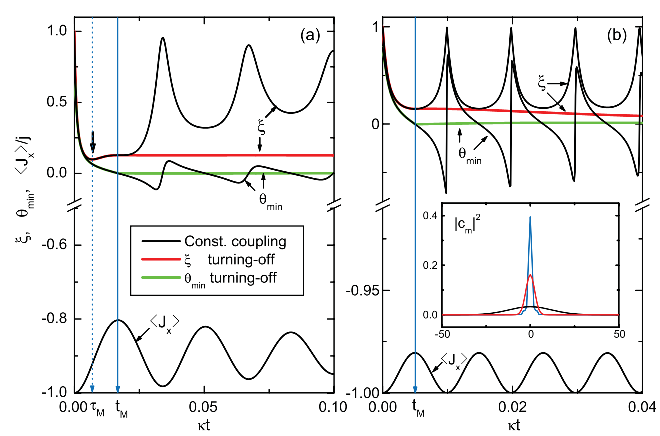

FIG. 4 (color). Time evolution of ξ, $\theta_{\min}$, and $\langle\hat{J}_x\rangle/j$ for $N = 10^3$ with (a) $\Omega_R = 5\kappa$ and (b) $\Omega_R = 50\kappa$. In both (a) and (b) the red and the green curves represent ξ and $\theta_{\min}$ for the case of turning-off the coupling at t_M, while the black curves denote them for constant-coupling case. t_M and τ_M are defined in the text. The inset: the probability distribution of the SSS at τ_M in (a) (the black curve), at t_M in (a) (the blue curve), and in (b) (the red curve).

Equation (3) still gives a fairly good estimation of $t_M = 4.967 \times 10^{-3}\kappa^{-1}$ compared with the numerical one $4.945 \times 10^{-3}\kappa^{-1}$.

Before closing, we provide several mathematical derivations supporting the above argument. The linear combinations of the probability amplitudes $p_m^{(\pm)}(t) = c_m(t) \pm c_{-m}(t)$ obey two closed sets of first-order ordinary differential equations. For even N, the fact that all $p_m^{(-)}(0) = 0$ results in $p_m^{(-)}(t) = 0$, namely, $c_{-m}(t) = c_m(t)$. On the other hands, for odd N all $p_m^{(+)}(t)$ are zero, i.e., $c_{-m}(t) = -c_m(t)$. Since $c_{-m}(t) = \pm c_m(t)$, we obtain simple expressions: $\langle \hat{J}_y \rangle = \langle \hat{J}_z \rangle = 0$, and $\langle \hat{J}_x \rangle \neq 0$, i.e., the mean spin is along the x direction. Consequently, the spin component normal to the mean spin reads $\hat{J}_{\mathbf{n}} = \hat{J}_y \sin\theta + \hat{J}_z \cos\theta$. By minimizing the variance $(\Delta \hat{J}_{\mathbf{n}})^2$ with respect to θ, we obtain the squeezing angle $\theta_{\min} = \frac{1}{2}\tan^{-1}(B/A)$ and $(\Delta \hat{J}_{\mathbf{n}})^2_{\min} = \frac{1}{2}C - \frac{1}{2}\sqrt{A^2 + B^2}$, where $A = \langle \hat{J}_z^2 - \hat{J}_y^2 \rangle$, $B = \langle \hat{J}_z \hat{J}_y + \hat{J}_y \hat{J}_z \rangle$, and $C = \langle \hat{J}_z^2 + \hat{J}_y^2 \rangle$. From Heisenberg equations of motion of the spin $\hat{J}_\alpha$ for $\alpha = x, y, z$, one can obtain formal solutions for the constant-coupling case: $C = j(j+1) - \langle \hat{J}_x^2 \rangle$, $A = -C + j(1 - \Omega_R/\kappa) - (\Omega_R/\kappa)\langle \hat{J}_x \rangle$, and $B = -(2\kappa)^{-1} d\langle \hat{J}_x \rangle/dt$. Note that for $B = 0$ and $A \neq 0$, the spin squeezing takes place along z axis (i.e., $\theta_{\min} = 0$) with the corresponding squeezing parameter

$$\xi_M^2 = 1 - (\Omega_R/\kappa)[1 + \langle \hat{J}_x \rangle_M / j], \tag{6}$$

where $\langle \hat{J}_x \rangle_M$ is the maximum value of the mean spin. For an extremely strong coupling ($\Omega_R \gg \kappa N$), $\langle \hat{J}_x \rangle_M \to -j$ and $\xi_M \to 1$ so the squeezing becomes very weak. This is the reason why we discuss the spin squeezing in the small-coupling regime ($\kappa < \Omega_R \ll N\kappa$).

Finally, we estimate several important parameters for experimental realization. Following Ref. [6], we consider ^{23}Na atoms in the hyperfine states $|F = 1, M_F = \pm 1\rangle$ trapped in a spherically symmetric potential $V_a = V_b = m\omega^2 r^2/2$. The self-interaction strength can be solved by applying the Thomas-Fermi approximation, yielding

$$\kappa \simeq \frac{15^{2/5}\hbar\omega}{14} \frac{a_{\rm eff}}{a_0} \left(\frac{a_0}{N a_{aa}} \right)^{3/5}, \tag{7}$$

where $a_0 = \sqrt{\hbar/m\omega}$ is the harmonic oscillator length and $a_{\rm eff} = a_{aa} + a_{bb} - 2a_{ab}$ the effective scattering length. For ^{23}Na atoms, we take $a_{aa} = a_{bb} = 1.076 a_{ab}$ [27] and $a_{\rm eff} = 0.15 a_{aa} = 0.41$ nm [17], then the self-interaction strength $\kappa \simeq 4.87 \times 10^{-5}\hbar\omega$. For the case $N = 10^3$ and $\Omega_R = 10.8\kappa$, we have obtained $t_M = 1.1041 \times 10^{-2}/(\hbar^{-1}\kappa) = 226.9\omega^{-1}$, which corresponds to the maximal-squeezing time about 72.2 ms for $\omega = 2\pi \times 500$ Hz.

In summary, we have investigated the coherent control of spin squeezing of TBEC with the time-dependent Josephson coupling induced by a microwave electromagnetic field. By rapidly turning-off the external driving at the time when the maximal squeezing occurs, the spin squeezing parameter can be maintained with its direction fixed along the z axis. In such a scheme, the storage of the spin squeezing is achieved even though the inherent self-interaction among atoms in a BEC still exists. We find the analytical expression of the maximal-squeezing time by considering phase model. Our scheme for the storage of spin squeezing is quite robust for wide range of parameters. We hope our scheme will be realized in experiment near future.

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