Suppression of Turbulence by Self-Generated and Imposed Mean Flows

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The first direct experimental evidence of the suppression of quasi-two-dimensional turbulence by mean flows is presented. The flow either is induced externally or appears in the process of spectral condensation due to an inverse cascade in bounded turbulence. The observed suppression of large scales is consistent with an expected reduction in the correlation time of turbulent eddies due to shearing. At high flow velocities, sweeping of the forcing-scale vortices reduces the energy input, leading to a reduction in the turbulence level.

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An inverse turbulent cascade is a remarkable phenomenon of an energy transfer towards large scales which (somewhat counterintuitively) can be thought of as a process of turbulent self-organization [1]. Inverse cascades take place in two-dimensional (2D) and rotating flows, in magnetized plasma, for waves on the fluid surfaces, etc. [2,3]. In a bounded domain, an inverse cascade may create a mode coherent across the whole system [4–11]. How such a mode influences the turbulence that feeds it is a question of both fundamental and practical importance. For example, experiments in turbulent magnetized plasma reveal a correlation between the onset of strong mean flows and the reduction in the levels of turbulent fluctuations. This is often observed near transport barriers in plasma where turbulent diffusion is greatly reduced, leading to a better plasma confinement [12]. Turbulence suppression by flows has become an important direction in magnetized plasma research, since it offers a very effective method of turbulence control [13,14]. The suppression of turbulence by a coherent large-scale structure (condensate) has also been observed in a numerical simulation of optical turbulence and Bose-Einstein condensation [7]. Yet, despite its wide recognition in plasma physics and attempts to extend its application to other fields (see, e.g., [15]), the phenomenon of turbulence suppression by flows is not that familiar in hydrodynamics [16].

It was suggested that mean flows may affect turbulence via a shear flow suppression [17–19]. When a turbulent eddy is placed in a stable laminar flow whose velocity varies perpendicular to the flow direction, it becomes stretched and distorted. The shear suppression can be viewed as a reduction in the eddy's lifetime when the shearing rate ω_s exceeds the inverse eddy lifetime $\omega_s \tau_e > 1$.

In this Letter, we present experimental results which show that mean flows do suppress turbulence in quasi-2D fluid flows. Turbulence suppression is observed due to the self-generated mean flow during spectral condensation of turbulence and also in the presence of externally imposed large-scale flow. We present the first experimental evidence of the two mechanisms of turbulence suppression: shear decorrelation and a new mechanism due to *sweeping* of the force-generated vortices by the mean flow.

The experimental setup is similar to that described in Refs. [8,10]. A turbulent flow is generated in stratified thin layers of fluids. A heavier nonconducting fluid (Fluorinert FC-77, specific gravity of SG $= 1.8$) is placed at the bottom of the container. Then a lighter 4 mm thick conducting fluid, a NaCl water solution (SG $= 1.03$), is placed on top. A spatially varying magnetic field B normal to the fluid surface, produced by a 24×24 matrix of permanent magnets (10 mm spacing), interacts with the electric current flowing through the top layer. This results in 576 $J \times B$-driven vortices which interact to produce the turbulent flow. To visualize the flow, imaging particles (polyamid, 50 μm diameter, specific gravity of 1.03) are suspended in the top layer and are illuminated by a (1 mm) laser sheet aligned parallel to the free surface of the fluid. Laser light scattered by particles is filmed from above using a video camera at 25 frames per second. A cross-correlation-based particle image velocimetry technique is used to obtain the velocity fields from the sequence of video frames.

Test particles travel in the horizontal x-y plane over distances of more than 100 mm without leaving the 1 mm thick laser sheet that illuminates them (without drifting in the vertical z direction). Thus, the ratio of the vertical velocity component to the horizontal ones is small: $V_z/V_{x,y} < 0.01$, which confirms that the flow is quasi-2D.

A self-generated coherent flow can develop spontaneously during spectral condensation of the bounded 2D turbulence [1]. It is related to the ability of 2D turbulence to self-organize and support an inverse energy cascade [1]. In a large domain, an inverse cascade proceeds up to the integral scale $\lambda_E \approx \epsilon^{1/2}\mu^{-3/2}$, where μ is the linear damping and ϵ is the energy dissipation rate. When λ_E is larger than the size of the boundary L, energy accumulates at the box scale, and self-generation of a large-scale single vortex occurs. This phenomenon has been confirmed in numerical simulations [4,6,9,11] and has been observed in experiments [5,8,10]. For given μ and ϵ, the easiest way to achieve spectral condensation is to reduce the size of the boundary to satisfy $\lambda_E \geq L$. In this experiment, square boundaries of different sizes of $L = 90$–120 mm were used. The spectral condensation leads to the onset of the self-generated mean flow, which interacts with the background turbulence.

In the other experiment described below, the boundary box substantially exceeded the integral scale, by about a factor of 3 ($L \approx 300$ mm). We refer to this configuration as ''unbounded'' turbulence. In this case, the mean flow was generated externally using a large permanent magnet.

First, we consider the effect of the self-generated flow on the bounded ($L = 110$ mm) turbulence. The time evolution of the total kinetic energy of the 2D turbulent flow is shown in Fig. 1(a). The inverse energy cascade leads to the development of larger eddies and to the growth of the kinetic energy of the system. By about 10 s, the kinetic energy reaches 80% of its maximum value. By this time, several large-scale coherent vortices develop in the flow, as seen in Fig. 1(b). These vortices persist for 4–5 turnover times (~10 s) before they start merging. After this transient stage, large vortices merge to form a single coherent vortex, which then persists in a steady state [Fig. 1(c)]. This

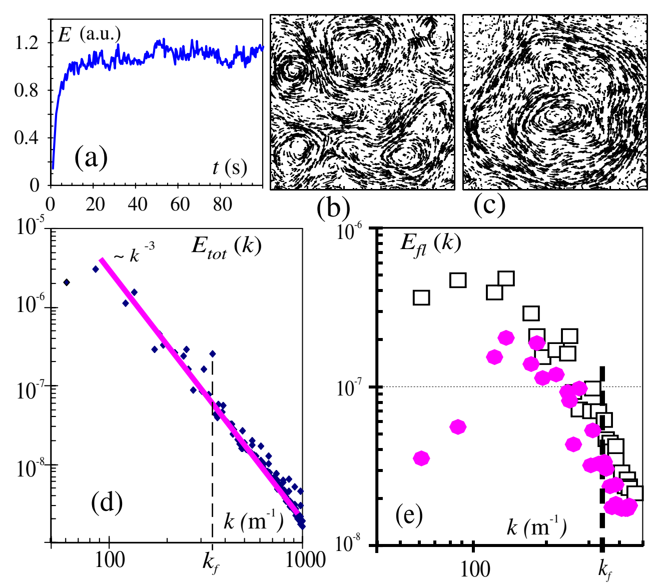

FIG. 1 (color online). (a) Time evolution of the total kinetic energy and instantaneous velocity fields measured at (b) $t = 13$ s and (c) $t = 71$ s. (d) Spectrum of the total spectral energy of the flow at $t = (9$–$17)$ s. (e) Spectra of the turbulent velocity fluctuations before, $t = (9$–$17)$ s (open squares, $N = 200$), and after the formation of a single large vortex, $t = (61$–$79)$ s (solid circles, $N = 400$).

stable vortex imposes mean flow, which affects 2D turbulence.

We compare the turbulence spectra during the transient stage, at $t = (9$–$17)$ s, and after the single vortex formation, at $t = (61$–$79)$ s. The analysis time in the transient stage is limited to 8 s, during which the flow is quasisteady. The wave number spectra are averaged over $N = 200$ realizations (400 in the steady condensate regime) of the ''instantaneous'' velocity fields (computed every 40 ms using two consecutive video frames):

$$E_{\mathrm{tot}}(k) = 1/N \sum_{n=1}^{N} F(V)F^*(V), \qquad (1)$$

where F denotes Fourier transform and F^* is its complex conjugate. This is a total spectrum which includes both mean and fluctuating velocity. Before the large vortex formation, this spectrum shows a power-law scaling of $E(k) \propto k^{-3}$ both above and below the forcing wave number $k_f = 350\ \mathrm{m}^{-1}$ [Fig. 1(d)]. Such a scaling, which was already observed in the experiments in the spectral condensate regime [10] and in numerical simulations [11,20–22], apparently contradicts the $E(k) \propto k^{-5/3}$ spectrum expected for the inverse energy cascade inertial range [1]. It was suggested in Ref. [11] that a k^{-3} power law is due to the presence of large-scale persistent vortices rather than due to the turbulent cascade. To eliminate that effect, we subtract from the instantaneous velocity the mean $\langle V \rangle = 1/N \sum_{n=1}^{N} V(x, y)$ obtained by averaging over N instantaneous fields $V(x, y)$. The resulting spectra

$$E_{fl}(k) = 1/N \sum_{n=1}^{N} F(V - \langle V \rangle)F^*(V - \langle V \rangle), \qquad (2)$$

computed for two time intervals before and after the generation of the single vortex, are shown in Fig. 1(e). Such a subtraction, proposed in Ref. [11], leads to a spectrum less steep than k^{-3}, somewhat close to $k^{-5/3}$.

After the formation of the single vortex, turbulence levels are significantly reduced for the wave numbers in the range of $k < 160\ \mathrm{m}^{-1}$. The explanation of this will be given below. The level of turbulent fluctuations changes less between $k \approx 160\ \mathrm{m}^{-1}$ and the injection scale $k_f \approx 350\ \mathrm{m}^{-1}$. That interval is too short to distinguish between $E_{fl}(k) \propto \epsilon^{2/3} k^{-5/3}$ and $E_{fl}(k) \propto \epsilon/\tau_s k$ that one may expect, assuming that the scale-independent energy transfer is of the order of the shear time τ_s. One can see that, in the forward cascade range ($k \geq k_f$), fluctuations are also reduced. This reduction at small k is significant (up to a factor of 10) and reproducible.

Now we discuss the results in unbounded 2D turbulence with and without an externally imposed large-scale flow. A large magnet ($40 \times 40\ \mathrm{mm}^2$) placed 2 mm above the free surface imposes a large-scale vortex flow, which slowly decays (for approximately 60 s) after the magnet is removed. Instantaneous velocity fields before and after the generation of this mean flow are shown in Figs. 2(a) and

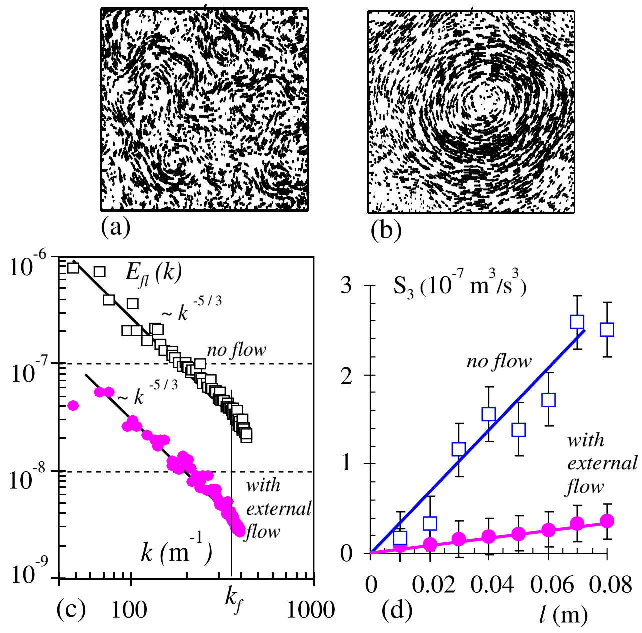

FIG. 2 (color online). Instantaneous velocity fields of unbounded turbulence (a) and in the presence of an externally generated large-scale azimuthal flow. The size of the box corresponds to the analyzed fraction of the fluid cell 0.18×0.18 m^2. Spectra of turbulence are computed with mean flow subtracted: before the large flow is imposed (open squares, $N = 400$) and in the presence of the mean flow (solid circles, $N = 200$). Third-order structure functions without (open squares) and with (solid circles) externally imposed mean flow (d).

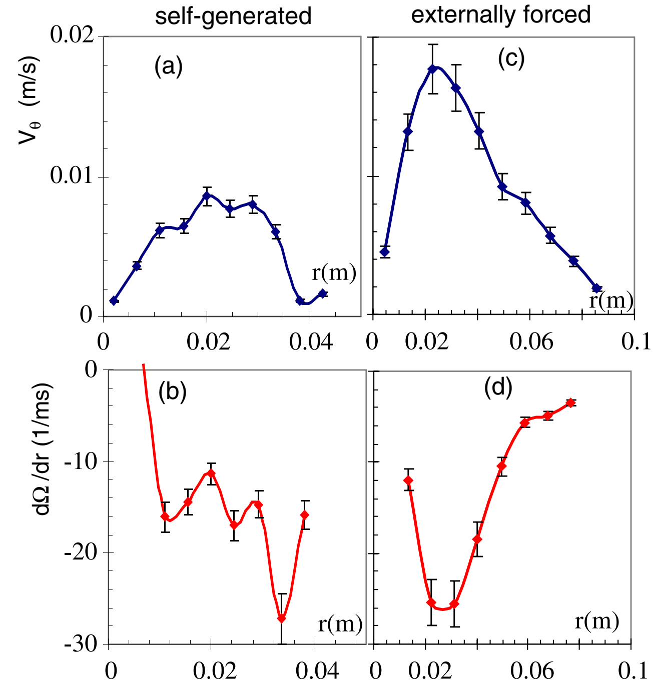

FIG. 3 (color online). Mean azimuthal velocity of the flow after the large-scale vortex generation (a),(c) and the derivative of its angular velocity (b),(d) during spectral condensation of the bounded turbulence (a),(b) and in the case of externally forced unbounded flow (c),(d).

2(b). Energy spectra shown in Fig. 2(c) are computed after subtracting the mean flow, using Eq. (2). Both with and without the large vortex, spectra are close to the $k^{-5/3}$ scaling. The mean flow reduces the spectral power of the turbulent fluctuations everywhere within the inverse cascade range by a factor of 8.

In the presence of the self-generated large vortex, the observed reduction in the spectral power of turbulent eddies is consistent with the mechanism of the shear turbulence suppression. We estimate the shear suppression criterion $s = \omega_s \tau_e > 1$ as follows. The turnover time of an eddy of the scale l is $\tau_e \approx l/\langle|\delta V(l)|\rangle = l/S_1(l)$, which is estimated from the mean velocity difference across l: $\delta V(l) = V(r_0 + l) - V(r_0)$. The angular brackets denote the averaging over all possible positions r_0 within the boundary box (or within the computation box in the unbounded case), and $S_1 = \langle \delta V \rangle$ is the first-order structure function averaged over 100 velocity fields.

To estimate the shearing rate of the large-scale mean flows, both self-generated [Fig. 1(c)] and externally forced [Fig. 2(b)], the polar coordinate system with its origin in the center of the vortex is used. The azimuthal component of the velocity V_θ dominates the flow after the vortex is formed. Its radial distribution is shown in Figs. 3(a) and 3(c). In the case of the self-generated flow, radial coordinates $r = 0$ and $r = 0.05$ m correspond to the vortex center and to the square boundary, respectively. In the case of the externally driven flow, $r = 0.09$ m corresponds to the size of the imaged area of unbounded turbulent flow. It is seen that the amplitude of velocity of the externally forced flow is a factor of 2 higher than in the self-organized case. The shearing rate is determined as follows:

$$\omega_s = l d\Omega/dr, \tag{3}$$

where l is the radial extent of the eddy. The derivative of the radially localized angular velocity $\Omega = V_\theta/r$ is determined as $d\Omega/dr = (1/r)(dV_\theta/dr) - (V_\theta/r^2)$, which is zero for the solid-body mean flow rotation and nonzero for the sheared flow. Figures 3(b) and 3(d) show $d\Omega/dr$ for the self-generated and the externally driven shear flows, respectively. Since both ω_s and τ_e grow with l, the shear affects larger scales first.

For the case of the self-generated flow $d\Omega/dr \approx 15$ (m s)$^{-1}$, $S_1 \approx 8 \times 10^{-3}$ m/s, and the shearing parameter $s \approx 2 \times 10^3 l^2$. The criterion for the shear suppression $s > 1$ is satisfied for the scales $l > 0.022$ m. This gives an estimate of the affected wave number range $k = \pi/l \leq 145$ m^{-1}, which is in agreement with the observation of the turbulence suppression in the wave number range of $k \leq 160$ m^{-1} seen in Fig. 1(e).

For the externally forced mean flow $d\Omega/dr \approx 22$ (m s)$^{-1}$, $S_1 \approx 2 \times 10^{-3}$ m/s, and the shearing parameter $s \approx 1.1 \times 10^4 l^2$. The suppression criterion is satisfied for the scales $l > 0.0095$ m, which extends very close to the forcing scale $l_f \approx 9$ mm ($k_f = 350$ m^{-1}). Again, this is in agreement with our observation that the spectral energy is reduced everywhere within the inverse energy cascade inertial range [Fig. 2(c)].

The externally driven flow must be strong enough to affect the energy flux through the $k < k_f$ inertial range. To test this, we computed the third-order structure function $S_3(l) = \langle \delta V(l)^3 \rangle$ to estimate the energy flux ϵ from the Kolmogorov law $S_3(l) = -(3/2)\epsilon l$. Similarly to S_1, S_3 is computed by averaging over the boundary box and then by averaging S_3 in time over 100 subsequent velocity fields. It should be noted that $\delta V(l)$ represents here the longitudinal velocity increment $\delta V_{\|}(l)$, defined in Kolmogorov's theory [2]. Also, S_3 is computed allowing positive and negative values of the velocity increments $\delta V_{\|}(l)$. Such computations require very large statistical averaging to obtain converged results. We obtained satisfactory convergence for the steady-state unbounded turbulence and for the turbulence in the presence of the slowly decaying externally driven flow. The result is illustrated in Fig. 2(d). Both before and after the mean flow is imposed, S_3 is a linear function of the scale l. As a result, the energy flux $\epsilon = -(2/3)S_3/l$ is constant to within 15% for all scales in the energy inertial range. This flux ϵ is reduced in the presence of the flow by 1 order of magnitude compared to the case without the flow.

The reduction in the energy flux can be attributed to two phenomena in this case. First, it is the shearing of the forcing-scale vortices discussed above. However, the force-connected vortices (with $k \approx k_f$) must be more resistant to shearing than the inertial scale eddies at $k < k_f$. Second, the force-fed vortices are *swept* by the mean flow relative to the magnets, which must also reduce the energy input. One can define a dimensionless sweeping parameter $\mathrm{sw} = \omega_{\mathrm{sw}} \tau_e$, where the sweeping rate is given by $\omega_{\mathrm{sw}} = V_\theta / l$. Since $\mathrm{sw} = (V_\theta / l)(l/S_1) \sim V_\theta (\epsilon l)^{-1/3}$, sweeping acts more efficiently on the smaller scales (while shearing is more effective on larger scales). At the forcing scale l_f, this parameter is $\mathrm{sw} \approx 0.75$ for the self-generated flow, and it is $\mathrm{sw} \approx 7$ with the externally forced mean flow. Thus, the sweeping can be responsible for the reduction in the energy flux through the inverse cascade range in the presence of an externally induced flow. The dominant role of sweeping in this case is also supported by the fact that the spectrum of the inverse cascade remains $k^{-5/3}$, just shifted down as shown in Fig. 2(c). Such modifications to the spectrum are also consistent with a tenfold decrease in the energy flux ϵ, since $E(k) = C_k \epsilon^{2/3} k^{-5/3}$. Let us stress the qualitative difference between Fig. 1(e) (strong decrease at small k) and Fig. 2(c) (uniform decrease for all k) which shows that there are two mechanisms of suppression. Sweeping may also be responsible for the reduction in the enstrophy flux through the forward cascade ($k > k_f$) in the presence of the self-generated flow [see Fig. 1(e)]. In this case, we could not obtain statistically converged computations of S_3 during spectral condensation to compare ϵ before and after the formation of the large vortex. New experiments with substantially higher spatial resolution of the velocity field (currently under way) will address this issue.

We have shown that turbulence in quasi-2D flow is significantly reduced in the presence of a large coherent vortex. In the case of a self-generated vortex, larger scales are affected more than the smaller ones. This qualitatively agrees with the description of the shear turbulence suppression mechanism as a reduction in the eddy lifetime [17]. In the presence of the externally imposed flow, two effects may be responsible for the observed strong reduction in the turbulence level. The vortex sweeping by the mean flow seems to play an important role here. In this case, the shape of the spectrum is not modified, but the (inverse) spectral energy flux is substantially reduced.

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[1] R. H. Kraichnan, Phys. Fluids **10**, 1417 (1967).
[2] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, England, 1995).
[3] V. Zakharov, V. Lvov, and G. Falkovich, *Kolmogorov Spectra of Turbulence* (Springer, Berlin, 1992).
[4] M. Hossain, W. H. Matthaeu, and D. Montgomery, J. Plasma Phys. **30**, 479 (1983).
[5] J. Sommeria, J. Fluid Mech. **170**, 139 (1986).
[6] L. M. Smith and V. Yakhot, Phys. Rev. Lett. **71**, 352 (1993).
[7] A. Dyachenko and G. Falkovich, Phys. Rev. E **54**, 5095 (1996).
[8] J. Paret and P. Tabeling, Phys. Fluids **10**, 3126 (1998).
[9] D. Molenaar, H. J. H. Clercx, and G. J. F. van Heijst, Physica (Amsterdam) **196D**, 329 (2004).
[10] M. G. Shats, H. Xia, and H. Punzmann, Phys. Rev. E **71**, 046409 (2005).
[11] M. Chertkov *et al.*, Phys. Rev. Lett. **99**, 084501 (2007).
[12] F. Wagner *et al.*, Phys. Rev. Lett. **49**, 1408 (1982).
[13] T. S. Hahm and K. H. Burrell, Phys. Plasmas **2**, 1648 (1995).
[14] K. H. Burrell, Phys. Plasmas **6**, 4418 (1999).
[15] E.-J. Kim, Astron. Astrophys. **441**, 763 (2005).
[16] P. W. Terry, Phys. Plasmas **7**, 1653 (2000).
[17] H. Biglari, P. H. Diamond, and P. W. Terry, Phys. Fluids B **2**, 1 (1990).
[18] K. C. Shaing, E. J. Crume, and W. A. Houlberg, Phys. Fluids B **2**, 1492 (1990).
[19] M. Leconte *et al.*, Phys. Plasmas **13**, 112301 (2006).
[20] V. Borue, Phys. Rev. Lett. **72**, 1475 (1994).
[21] L. M. Smith and F. Waleffe, Phys. Fluids **11**, 1608 (1999).
[22] C. V. Tran and J. C. Bowman, Phys. Rev. E **69**, 036303 (2004).