Isotopic Dependence of the Giant Monopole Resonance in the Even-A $^{112-124}$Sn Isotopes and the Asymmetry Term in Nuclear Incompressibility

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The strength distributions of the giant monopole resonance (GMR) have been measured in the even-A Sn isotopes ($A = 112$–124) with inelastic scattering of 400-MeV α particles in the angular range $0° - 8.5°$. We find that the experimentally observed GMR energies of the Sn isotopes are lower than the values predicted by theoretical calculations that reproduce the GMR energies in ^{208}Pb and ^{90}Zr very well. From the GMR data, a value of $K_\tau = -550 \pm 100$ MeV is obtained for the asymmetry term in the nuclear incompressibility.

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Incompressibility of nuclear matter remains a focus of experimental and theoretical investigations because of its fundamental importance in defining the equation of state (EOS) for nuclear matter. The latter describes a number of interesting phenomena from collective excitations of nuclei to supernova explosions and radii of neutron stars [1]. The giant monopole resonance (GMR) provides a direct means to experimentally determine this quantity.

Experimental identification of the GMR requires inelastic scattering of an isoscalar particle at extremely forward angles, including 0°, where the cross section for exciting the GMR is maximal. Such measurements have improved considerably over the years, and it is now possible to obtain inelastic spectra virtually free of all instrumental background directly [2] and in coincidence with proton and neutron decay [3]. In recent work, the GMR strength distributions have been extracted in many nuclei from a multipole-decomposition analysis (MDA) of such "background-free" inelastic α-scattering spectra [2,4–9].

The excitation energy of the GMR is expressed in the scaling model [10] as

$$E_{\mathrm{GMR}} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}, \quad (1)$$

where m is the nucleon mass, $\langle r^2\rangle$ is the ground-state mean-square radius, and K_A is the incompressibility of the nucleus. In order to determine the incompressibility of infinite nuclear matter, K_∞, from the experimental GMR energies, one builds a class of energy functionals, $E(\rho)$, with different parameters that allow calculations for nuclear matter and finite nuclei in the same theoretical framework. The parameter set for a given class of energy functionals is characterized by a specific value of K_∞. The GMR strength distributions are obtained for different energy functionals in a self-consistent RPA calculation. The K_∞ associated with the interaction that best reproduces the GMR energies is, then, considered the "correct" value. This procedure, first proposed by Blaizot [11], is now accepted as the best way to extract K_∞ from the GMR data, and it has been established that both relativistic and nonrelativistic calculations are now in general agreement with $K_\infty = 240 \pm 10$ MeV [12–14].

The determination of the asymmetry term, K_τ, associated with the neutron excess ($N - Z$), remains very important because this term is crucial in obtaining the radii of neutron stars in EOS calculations [15–18]. Indeed, the radius of a neutron star whose mass is between about 1 and 1.5 solar masses ($M_\odot$) is mostly determined by the density dependence of the symmetry-energy term [19,20]. Previous attempts to extract this term from experimental GMR data have resulted in widely different values, from -320 ± 180 MeV in Ref. [21] to a range of -566 ± 1350 MeV to 139 ± 1617 MeV in Ref. [22]. Measurements of the GMR over a series of isotopes provide a way to "experimentally" determine this asymmetry term in a direct manner. The Sn isotopes ($A = 112$–124) afford such an opportunity since the asymmetry ratio, $[(N - Z)/A]$, changes by more than 80% over this mass range.

In this Letter, we report on new measurements on GMR in the even-A Sn isotopes. The GMR has been identified previously in some of the Sn isotopes as a compact peak in measurements with inelastic α scattering [7,21,23,24], and although resonance parameters for GMR in the Sn isotopes close to the values reported here have been extracted in the past using less accurate techniques [21], the potentially large systematic errors in those values necessitated the present measurements where such problems have been eliminated. We find that the GMR energies in the Sn isotopes are lower than the values predicted in recent theoretical calculations even though the interactions used in these calculations reproduce the GMR energies in the "standard" nuclei, ^{208}Pb and ^{90}Zr, very well. Also, we obtain a value $K_\tau = -550 \pm 100$ MeV from these data.

The experiment was performed at the ring cyclotron facility of the Research Center for Nuclear Physics, Osaka University, using inelastic scattering of 400-MeV α particles over the angular range 0°–8.5°. Details of the experimental technique and the data analysis procedure have been provided previously [5,6,8] and are only briefly described here. Inelastically scattered α particles were momentum analyzed with the magnetic spectrometer "Grand Raiden" [25] and detected in the focal-plane detector system composed of two multiwire drift chambers and two scintillators, providing particle identification as well as the trajectories of the scattered particles. The vertical position spectrum obtained in the double-focused mode of the spectrometer was exploited to eliminate all instrumental background [5,6,8]. The background-free "0°" inelastic spectra for the Sn isotopes are presented in Fig. 1. In all cases, the spectrum is dominated by the GMR peak near $E_x \sim 15$ MeV.

In order to extract the GMR strengths, we have employed the now standard MDA procedure [26]. The cross-section data were binned into 1-MeV energy intervals between 8.5 and 31.5 MeV and the experimental 17-point angular distribution $\frac{d\sigma^{\rm exp}}{d\Omega}(\theta_{\rm cm}, E_x)$ for each excitation-energy bin was fitted by means of the least-squares method with a linear combination of calculated distributions $\frac{d\sigma_L^{\rm cal}}{d\Omega}(\theta_{\rm cm}, E_x)$, so that

$$\frac{d\sigma^{\rm exp}}{d\Omega}(\theta_{\rm cm}, E_x) = \sum_{L=0}^{7} \alpha_L(E_x)\frac{d\sigma_L^{\rm cal}}{d\Omega}(\theta_{\rm cm}, E_x), \qquad (2)$$

where $\frac{d\sigma_L^{\rm cal}}{d\Omega}(\theta_{\rm cm}, E_x)$ is the calculated distorted-wave Born approximation (DWBA) cross section corresponding to 100% energy-weighted sum-sure (EWSR) for the Lth multipole. The DWBA calculations were performed following the method of Satchler and Khoa [27] using the density-dependent single folding model, with a Gaussian α-nucleon potential for the real part, and a Woods-Saxon imaginary term. We have used the transition densities and sum rules for various multipolarities as described in Ref. [28]. The optical model parameters were obtained from analysis of elastic scattering cross sections measured in a companion experiment.

Although all strength distributions up to $L = 3$ have been reliably extracted from the multipole decomposition, only the GMR strengths, the focus of this Letter, are shown in Fig. 2. The solid lines in the figure represent Lorentzian fits to the observed strength distributions. The choice of the Lorentzian shape is arbitrary; the final results are not affected in any significant way by using, for example, a Gaussian shape instead. The finite strength at the higher

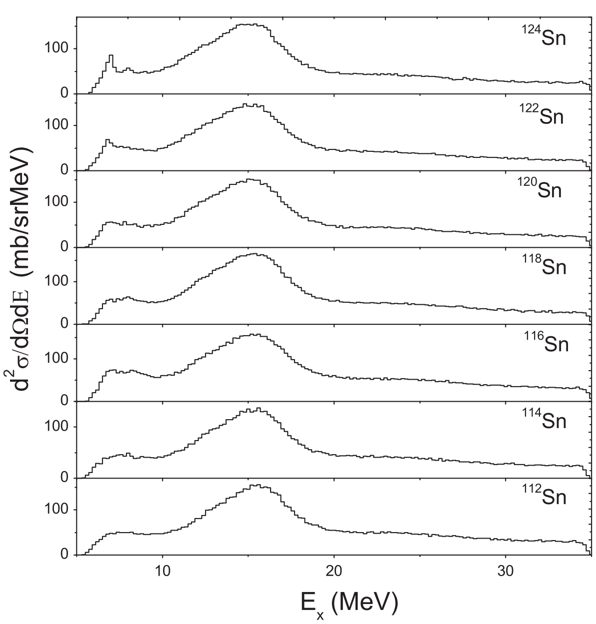

FIG. 1. Excitation-energy spectra for all even-A Sn isotopes, obtained from inelastic α scattering at $\theta_{\rm lab} = 0.69°$.

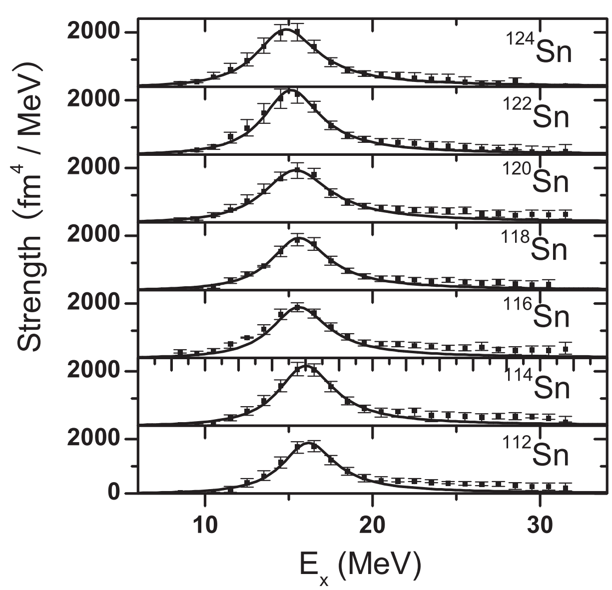

FIG. 2. GMR strength distributions obtained for the Sn isotopes in the present experiment. Error bars represent the uncertainty due to the fitting of the angular distributions in MDA. The solid lines show Lorentzian fits to the data.

TABLE I. Lorentzian-fit parameters and various moment ratios for the GMR strength distributions in the Sn isotopes, as extracted from MDA in the present work. m_k is the kth moment of the strength distribution: $m_k = \int E_x^k S(E_x) dE_x$. All moment ratios have been calculated over $E_x = 10.5$–20.5 MeV. The errors quoted for EWSR are statistical only.

Target	E_{GMR} (MeV)	Γ (MeV)	EWSR	m_1/m_0 (MeV)	$\sqrt{m_3/m_1}$ (MeV)	$\sqrt{m_1/m_{-1}}$ (MeV)
^{112}Sn	16.1 ± 0.1	4.0 ± 0.4	0.92 ± 0.04	16.2 ± 0.1	16.7 ± 0.2	16.1 ± 0.1
^{114}Sn	15.9 ± 0.1	4.1 ± 0.4	1.04 ± 0.06	16.1 ± 0.1	16.5 ± 0.2	15.9 ± 0.1
^{116}Sn	15.8 ± 0.1	4.1 ± 0.3	0.99 ± 0.05	15.8 ± 0.1	16.3 ± 0.2	15.7 ± 0.1
^{118}Sn	15.6 ± 0.1	4.3 ± 0.4	0.95 ± 0.05	15.8 ± 0.1	16.3 ± 0.1	15.6 ± 0.1
^{120}Sn	15.4 ± 0.2	4.9 ± 0.5	1.08 ± 0.07	15.7 ± 0.1	16.2 ± 0.2	15.5 ± 0.1
^{122}Sn	15.0 ± 0.2	4.4 ± 0.4	1.06 ± 0.05	15.4 ± 0.1	15.9 ± 0.2	15.2 ± 0.1
^{124}Sn	14.8 ± 0.2	4.5 ± 0.5	1.03 ± 0.06	15.3 ± 0.1	15.8 ± 0.1	15.1 ± 0.1

excitation energies is attributable to the mimicking of $L = 0$ angular distribution by components of the continuum [4,8]. The extracted GMR-peak parameters and the various moment ratios typically used in theoretical calculations are presented in Table I.

The moment ratios, m_1/m_0, for the GMR strengths in the Sn isotopes are shown in Fig. 3 and are compared with recent theoretical results from Colò (nonrelativistic) [12,29] and Piekarewicz (relativistic) [13,30]. As can be seen, the calculations overestimate the experimental GMR energies significantly (by almost 1 MeV in case of the higher-A isotopes). This is very surprising since the interactions used in these calculations are those that very closely reproduce the GMR centroid energies in ^{208}Pb and ^{90}Zr. Admittedly, there are uncertainties associated with the assumptions inherent in the calculations regarding widths; however, the calculations reported here are identical in all respects to those performed for ^{208}Pb and ^{90}Zr, and the experimental and theoretical centroids reported here have been calculated over exactly the same excitation-energy range. This disagreement remains a challenge for the theory: Why are the tin isotopes so "soft"? Are there any nuclear structure effects that need to be taken into account to describe the GMR energies in the Sn isotopes?

The incompressibility of a nucleus, K_A, may be expressed as

$$K_A \sim K_{vol}(1 + cA^{-1/3}) + K_\tau[(N - Z)/A]^2 + K_{Coul}Z^2A^{-4/3}. \quad (3)$$

Here, $c \approx -1$ [31], and K_{Coul} is essentially model independent (in the sense that the deviations from one theoretical model to another are quite small), so that the associated term can be calculated for a given isotope. Thus, for a series of isotopes, the difference $K_A - K_{Coul}Z^2A^{-4/3}$ may be approximated to have a quadratic relationship with the asymmetry parameter, of the type $y = A + Bx^2$, with K_τ being the coefficient, B, of the quadratic term. It should be noted that it has been established previously [22,32] that fits to the above equation do not provide good constraints on the value of K_∞. However, this expression is being used here not to obtain a value for K_∞, but, rather, only to demonstrate the approximately quadratic relationship between K_A and the asymmetry parameter.

Figure 4 shows the difference $K_A - K_{Coul}Z^2A^{-4/3}$ for the Sn isotopes investigated in this work vs the asymmetry parameter, $[(N - Z)/A]$. The values of K_A have been derived using the customary moment ratio $\sqrt{m_1/m_{-1}}$ for energy of the GMR in Eq. (1). A quadratic fit to the data is also shown. The fit gives $K_\tau = -550 \pm 40$ MeV, with the uncertainty attributed only to the fitting procedure. Including the uncertainties in K_A in the fit adds another ~25 MeV to this "error" (to ±67 MeV) and the uncertainty in the value of K_{Coul} (±0.7 MeV; see Ref. [33]) would contribute ~15 MeV. Considering, further, the approximation made in arriving at the quadratic expression, the actual total uncertainty would be somewhat larger still;

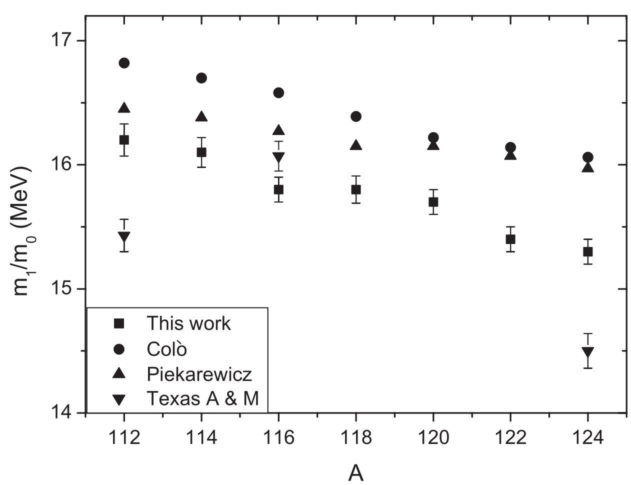

FIG. 3. Systematics of the moment ratios m_1/m_0 for the GMR strength distributions in the Sn isotopes. The experimental results (filled squares) are compared with results of nonrelativistic RPA calculations by Colò [29] (filled circles) and relativistic calculations of Piekarewicz [30] (triangles). Results for ^{112}Sn, ^{116}Sn, and ^{124}Sn reported by the Texas A&M group [23,24] are also shown (inverse triangles). The differences between the present results and the Texas A&M results for ^{112,124}Sn might be attributable to the background subtraction required in their analysis.

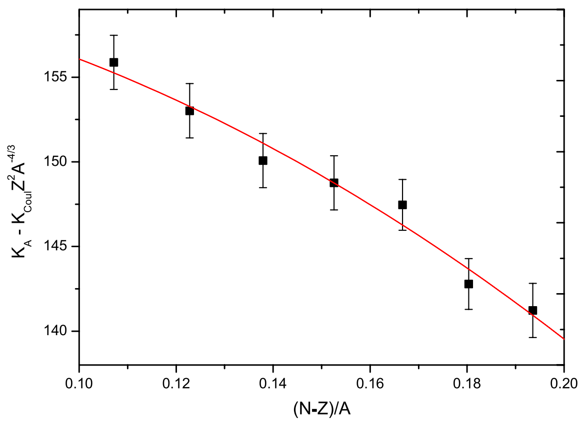

FIG. 4 (color online). Systematics of the difference $K_A - K_{\text{Coul}}Z^2A^{-4/3}$ in the Sn isotopes as a function of the "asymmetry parameter" $[(N-Z)/A)]$; $K_{\text{Coul}} = -5.2$ MeV [33]. The solid line represents a least-squares quadratic fit to the data.

hence the rounded value $K_\tau = -550 \pm 100$ MeV quoted earlier in the text. This result is consistent with the value $K_\tau = -500 \pm 50$ MeV obtained recently from an analysis of the isotopic transport ratios in medium-energy heavy-ion reactions [34]. As shown in Ref. [18], this value provides constraints on the radius of a $1.4M_\odot$ neutron star that are in rather good agreement with recent observational data. Thus, from the data on the compressional-mode giant resonances, we now have "experimental" values of both K_∞ and K_τ which, together, can provide a means of selecting the most appropriate of the interactions used in EOS calculations. For example, this combination of values for K_∞ and K_τ essentially rules out a vast majority of the Skyrme-type interactions currently in use in nuclear structure calculations [33]. A similar conclusion was reached for EOS equations in Ref. [35]. Furthermore, a more precise determination of K_τ provides additional motivation for measurement of isoscalar monopole strength in unstable nuclei, a focus of investigations at RIKEN and GANIL, for example [36,37].

In summary, we have measured the energies of the isoscalar giant monopole resonance (GMR) in the even-A $^{112-124}$Sn isotopes via inelastic scattering of 400-MeV α particles at extremely forward angles. The GMR energies are significantly lower than those predicted for these isotopes by recent calculations. Further, the asymmetry term, K_τ, in the expression for the nuclear incompressibility has been determined to be -550 ± 100 MeV.

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[1] N. K. Glendenning, Phys. Rev. C **37**, 2733 (1988).
[2] U. Garg, Nucl. Phys. **A731**, 3 (2004).
[3] M. Hunyadi *et al.*, Phys. Rev. C **75**, 014606 (2007).
[4] M. Itoh *et al.*, Phys. Lett. B **549**, 58 (2002).
[5] M. Uchida *et al.*, Phys. Lett. B **557**, 12 (2003).
[6] M. Itoh *et al.*, Phys. Rev. C **68**, 064602 (2003).
[7] M. Uchida *et al.*, Phys. Rev. C **69**, 051301 (2004).
[8] B. K. Nayak *et al.*, Phys. Lett. B **637**, 43 (2006).
[9] U. Garg *et al.*, Nucl. Phys. **A788**, 36c (2007).
[10] S. Stringari, Phys. Lett. **108B**, 232 (1982).
[11] J. Blaizot *et al.*, Nucl. Phys. **A591**, 435 (1995).
[12] G. Colò *et al.*, Phys. Rev. C **70**, 024307 (2004).
[13] B. G. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett. **95**, 122501 (2005).
[14] B. K. Agrawal, S. Shlomo, and V. Kim Au, Phys. Rev. C **68**, 031304 (2003).
[15] J. M. Lattimer and M. Prakash, Phys. Rep. **333**, 121 (2000).
[16] J. M. Lattimer and M. Prakash, Science **304**, 536 (2004).
[17] A. W. Steiner *et al.*, Phys. Rep. **411**, 325 (2005).
[18] B.-A. Li and A. W. Steiner, Phys. Lett. B **642**, 436 (2006).
[19] J. M. Lattimer and M. Prakash, Astrophys. J. **550**, 426 (2001).
[20] C. J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. **86**, 5647 (2001).
[21] M. M. Sharma *et al.*, Phys. Rev. C **38**, 2562 (1988).
[22] S. Shlomo and D. H. Youngblood, Phys. Rev. C **47**, 529 (1993).
[23] D. H. Youngblood *et al.*, Phys. Rev. C **69**, 034315 (2004).
[24] Y.-W. Lui *et al.*, Phys. Rev. C **70**, 014307 (2004).
[25] M. Fujiwara *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **422**, 484 (1999).
[26] B. Bonin *et al.*, Nucl. Phys. **A430**, 349 (1984).
[27] G. R. Satchler and D. T. Khoa, Phys. Rev. C **55**, 285 (1997).
[28] M. N. Harakeh and A. van der Woude, *Giant Resonances: Fundamental High-Frequency Modes of Nuclear Excitation* (Oxford University Press, New York, 2001).
[29] G. Colò (private communication).
[30] J. Piekarewicz, Phys. Rev. C **76**, 031301 (2007).
[31] S. K. Patra *et al.*, Phys. Rev. C **65**, 044304 (2002).
[32] J. M. Pearson, Phys. Lett. B **271**, 12 (1991).
[33] H. Sagawa *et al.*, Phys. Rev. C **76**, 034327 (2007).
[34] B.-A. Li and L.-W. Chen, Phys. Rev. C **72**, 064611 (2005).
[35] Lie-Wen Chen, Che Ming Ko, and Bao-An Li, Phys. Rev. C **72**, 064309 (2005).
[36] H. Baba *et al.*, Nucl. Phys. **A788**, 188c (2007).
[37] C. Monrozeau *et al.*, Nucl. Phys. **A788**, 182c (2007).