Exact Solution for the Generalized Bohm Criterion in a Two-Ion-Species Plasma

Dongsoo Lee,[1] Lutfi Oksuz,[1,2] and Noah Hershkowitz[1,*]
[1]*Department of Engineering Physics, University of Wisconsin at Madison, Madison, Wisconsin 53706, USA*
[2]*Department of Physics, Suleyman Demirel Universitesi, Isparta 32260, Turkey*
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For a weakly collisional two-ion species plasma, it is shown that the minimum phase velocity of ion acoustic waves (IAWs) at the sheath-presheath boundary is equal to twice the phase velocity in the bulk plasma. This condition provides a theoretical basis for the experimental results that each ion species leaves the plasma with a drift velocity equal to the IAW phase velocity in the bulk plasma [D. Lee *et al.*, Appl. Phys. Lett. **91**, 041505 (2007)]. It is shown that this result is a consequence of the generalized Bohm criterion and fluid expressions for the IAW phase velocities.

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Plasma-wall interactions are important to the characteristics of all bounded plasmas. In many situations, low temperature plasmas with an electron temperature of a few eV are present. Low temperature, weakly collisional plasmas are essential to the fabrication of many semiconductor devices [1–3]. They also can be used to produce novel surface coatings [4,5]. The scrape-off layers of magnetically confined fusion plasmas often consist of low temperature plasmas [6,7], as do space plasmas [8,9] and many other plasmas. All of these involve multiple species of ions. All require an understanding of the ion flux to the plasma boundary in multiple-ion species plasmas. This understanding currently does not exist.

In weakly collisional plasmas, the ion collisional mean free paths are significantly larger than the Debye length. There are three regions of interest near plasma boundaries: bulk plasma, presheath, and sheath [10,11]. Ions gain most of their energy from acceleration over the sheath. An important question is how fast ions are leaving the plasma at the sheath-presheath boundary. For single-ion species plasmas, the answer is well known. The ion drift velocity v_d at the sheath-presheath boundary is given by the Bohm criterion with equality,

$$v_d \geq C_s = \sqrt{T_e/m_i}, \tag{1}$$

where C_s is the Bohm velocity, T_e is the electron temperature, and m_i is the ion mass. Here, T_e is assumed to be much greater than the ion temperature T_i. This result was first proven theoretically almost 60 years ago [12], and demonstrated experimentally [13,14].

For multiple-ion species plasmas, a consensus agreement has not yet emerged. Riemann [15] derived a generalized Bohm criterion,

$$\sum_j \frac{q_j^2 n_j}{m_j v_j^2 - \gamma_j T_j} \leq \frac{e^2 n_e}{T_e}, \tag{2}$$

where j numbers the ion species, q is the ion charge, v is the ion drift velocity, n is the ion density at the sheath-presheath boundary, γ is the specific heat ratio, and n_e denotes the electron density. However, the generalized Bohm criterion does not specify each ion's drift velocity at the boundary. Among the infinite solutions, two simple solutions are easily considered. One is that all ions reach the sheath edge with the same velocity called the "system sound velocity," and the other is that each ion species has its own C_s. This problem has been investigated both theoretically [16,17] and experimentally [14,18–21]. Franklin showed that in the presence of uniform ionization all ions left with their own C_s [17]. A recent experiment in a weakly collisional two-ion species plasma where the ionization could be ignored demonstrated that each ion species did, in fact, leave with the system sound velocity [21]. However, the question still remains as to why. In this Letter, we show that this result is a consequence of the generalized Bohm criterion and the ion acoustic wave (IAW) dispersion relation.

Laser-induced fluorescence (LIF) provides a way to determine ion velocity distribution functions (IVDFs) if a suitable laser is available [22–25]. Measurements near a sheath-presheath boundary were made with a diode laser in a single-ion species argon plasma [13,14,20]. It was shown that the Bohm criterion is well satisfied at the sheath-presheath boundary and that the equality applies, i.e., $v_d \approx C_s$. In a two-ion species Ar-He plasma, IAW data [19] combined with argon LIF data [14] showed that the argon ion drift velocity at the sheath-presheath boundary was faster than the argon Bohm velocity, and the helium ion drift velocity was slower than the helium Bohm velocity. In fact, the data suggested that at the sheath-presheath boundary, both ion species approached the same velocity, the IAW phase velocity in the bulk plasma (system sound velocity).

Subsequent argon LIF data for Ar-Xe plasma found the Ar^+ drift velocity was slower than its own Bohm velocity at the sheath-presheath boundary [20]. Unlike the previous experiment with argon-helium, the argon ions were the lighter species. Again, these results suggested that the measured argon ion speed approached the ion sound speed of the system. Recently, a suitable transition was found to study xenon ions with a diode laser in an argon-xenon plasma [26]. LIF measurements were made for both argon and xenon IVDFs near a negatively biased plate. These measurements directly demonstrated for the first time that each ion species reached the system sound velocity at the sheath-presheath boundary [21], the result that the previous measurements had inferred.

IAWs are electrostatic waves with electron-ion oscillations and electric field fluctuation in the direction of the wave propagation. The IAW phase velocity v_{ph} for weakly collisional, nondrifting single-ion species plasmas derived from the fluid equations is

$$v_{\mathrm{ph}} = \omega/k = C_s, \tag{3}$$

where ω is the wave frequency and k is the wave number. The ion acoustic wave dispersion relation can be derived from the fluid equations for multiple-ion species plasmas, and can be shown to be [27]

$$1 = \sum_j \frac{\omega_{\mathrm{pj}}^2}{(\omega - k v_j)^2 - k^2 v_{\mathrm{thj}}^2} \tag{4}$$

The sum is over all charged particles (e = electron, 1, 2 = ions) and ω_{pj}, v_j, and v_{thj} are the plasma frequency, drift velocity, and thermal velocity of jth charged species, respectively. The LIF experimental results, carried out in low temperature weakly collisional plasmas, showed that the ion temperatures were approximately room temperature (i.e., much less than the electron temperature) [13,20,24].

For cold, nondrifting bulk plasma with Maxwellian electrons and two-ion species, Eq. (4) becomes

$$1 \approx \frac{\omega_{\mathrm{pe}}^2}{\omega^2 - k_b^2 v_{\mathrm{the}}^2} + \frac{\omega_{p1}^2 + \omega_{p2}^2}{\omega^2}, \tag{5}$$

where k_b is the wave number in the bulk plasma. Because the phase velocity in the bulk plasma (ω/k_b) is much slower than the electron thermal velocity, Eq. (5) can be arranged as

$$1 + \frac{\omega_{\mathrm{pe}}^2}{k_b^2 v_{\mathrm{the}}^2} = 1 + \frac{1}{k_b^2 \lambda_D^2} = \frac{\omega_{p1}^2 + \omega_{p2}^2}{\omega^2}, \tag{6}$$

where λ_D is the Debye length. In the limit of small Debye length ($\lambda_D \ll 1/k_b$), Eq. (6) can be approximated by

$$\frac{\omega_{\mathrm{pe}}^2}{k_b^2 v_{\mathrm{the}}^2} \approx \frac{\omega_{p1}^2 + \omega_{p2}^2}{\omega^2} \Rightarrow \frac{1}{v_{\mathrm{the}}^2} = \frac{\omega_{p1}^2/\omega_{\mathrm{pe}}^2 + \omega_{p2}^2/\omega_{\mathrm{pe}}^2}{v_{\mathrm{ph},b}^2}, \tag{7}$$

where $v_{\mathrm{ph},b}(= \omega/k_b)$ is the IAW phase velocity in the bulk plasma (system sound velocity). Equation (7) describes the IAW dispersion relation in the bulk plasma. The phase velocity $v_{\mathrm{ph},b}$ can be measured by launching an IAW and from Eq. (7) is given by

$$v_{\mathrm{ph},b}^2 = \frac{\omega^2}{k_b^2} = \frac{v_{\mathrm{the}}^2(\omega_{p1}^2 + \omega_{p2}^2)}{\omega_{\mathrm{pe}}^2} = \frac{n_1}{n_e}\frac{T_e}{m_1} + \frac{n_2}{n_e}\frac{T_e}{m_2}. \tag{8}$$

Given charge neutrality ($n_e = n_1 + n_2$) in the case of singly ionized gases, the ratios of each ion density to the electron density can be calculated with the electron temperature (T_e) measured by the Langmuir probe.

The IAW dispersion relation at the sheath-presheath boundary can be derived in a similar manner. Assuming cold ions and Maxwellian electrons again, the dispersion relation Eq. (4) becomes

$$1 \approx \frac{\omega_{\mathrm{pe}}^2}{\omega^2 - k_s^2 v_{\mathrm{the}}^2} + \frac{\omega_{p1}^2}{(\omega - k_s v_1)^2} + \frac{\omega_{p2}^2}{(\omega - k_s v_2)^2}, \tag{9}$$

where k_s is the wave number at the sheath-presheath boundary. Making the same assumptions used above, Eq. (9) yields

$$\frac{1}{v_{\mathrm{the}}^2} = \frac{k_s^2 \omega_{p1}^2/\omega_{\mathrm{pe}}^2}{(\omega - k_s v_1)^2} + \frac{k_s^2 \omega_{p2}^2/\omega_{\mathrm{pe}}^2}{(\omega - k_s v_2)^2}. \tag{10}$$

If it is further assumed that the ion density ratio and T_e do not change over the presheath, Eqs. (7) and (10) can be combined into

$$\frac{\omega_{p1}^2/\omega_{\mathrm{pe}}^2 + \omega_{p2}^2/\omega_{\mathrm{pe}}^2}{v_{\mathrm{ph},b}^2} = \frac{k_s^2 \omega_{p1}^2/\omega_{\mathrm{pe}}^2}{(\omega - k_s v_1)^2} + \frac{k_s^2 \omega_{p2}^2/\omega_{\mathrm{pe}}^2}{(\omega - k_s v_2)^2} = \frac{\omega_{p1}^2/\omega_{\mathrm{pe}}^2}{(v_{\mathrm{ph},s} - v_1)^2} + \frac{\omega_{p2}^2/\omega_{\mathrm{pe}}^2}{(v_{\mathrm{ph},s} - v_2)^2}, \tag{11}$$

where $v_{\mathrm{ph},s}(= \omega/k_s)$ is the IAW phase velocity at the sheath-presheath boundary. This assumption was verified for the data presented in Ref. [21]. Expressing Eq. (11) with the each ion's mass and density ratio in the case of singly ionized gases,

$$\frac{n_1/m_1}{(v_{\mathrm{ph},s} - v_1)^2} + \frac{n_2/m_2}{(v_{\mathrm{ph},s} - v_2)^2} = \frac{n_1/m_1 + n_2/m_2}{v_{\mathrm{ph},b}^2}. \tag{12}$$

The generalized Bohm criterion Eq. (2) can be simplified in the singly ionized two-ion species plasma,

$$\frac{n_1}{n_e}\frac{T_e}{m_1}\frac{1}{v_1^2} + \frac{n_2}{n_e}\frac{T_e}{m_2}\frac{1}{v_2^2} = 1, \tag{13}$$

where the equality is applied. Equation (13) has only two unknowns, v_1 and v_2 at the sheath-presheath boundary. Therefore, Eqs. (12) and (13) can be solved for v_1 and v_2 depending on the phase velocity $v_{\mathrm{ph},s}$ in Eq. (12).

In order to investigate the solutions (v_1, v_2) of the equations, we chose the parameters from Ref. [21] as representative. The neutral pressures of argon (ion species 1) and xenon (ion species 2) were 0.5 and 0.2 m Torr, respectively. The electron temperature was 0.69 eV, which gives each ion's Bohm velocity as $C_{\mathrm{Ar}} = 1290$ m/s and $C_{\mathrm{Xe}} = 710$ m/s, respectively. The plasma

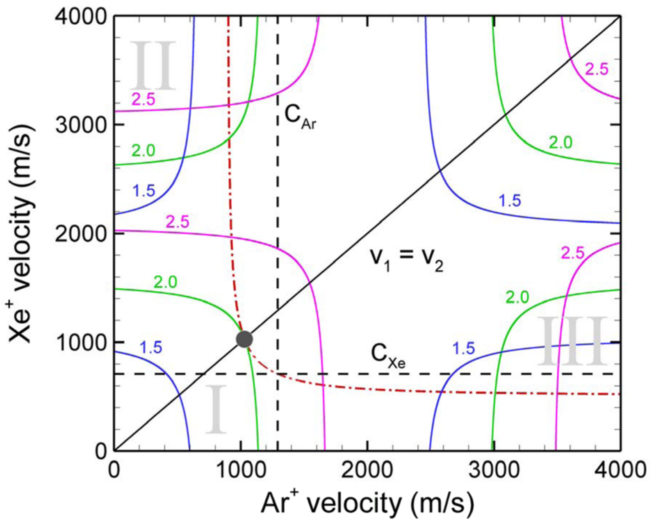

FIG. 1 (color online). Plots of the generalized Bohm criterion (dash-dotted, red) and the dispersion relation at different $v_{\mathrm{ph},s}$ values: $v_{\mathrm{ph},s} = 1.5v_{\mathrm{ph},b}$ [blue (or dark gray)], $2v_{\mathrm{ph},b}$ [green (or light gray)], and $2.5v_{\mathrm{ph},b}$ [purple (or gray)]. The numbers on the curves label each value. Both the curves meet at only one point (dot) in the region I when $v_{\mathrm{ph},s} = 2v_{\mathrm{ph},b}$. Each ion's Bohm velocity is shown for comparison.

potential changes in the presheath were less than $2T_e$. The measured $v_{\mathrm{ph},b}$ was 1030 ± 50 m/s, so the relative ion ratios from Eq. (8) were $n_{\mathrm{Ar}}/n_e = 0.48$ and $n_{\mathrm{Xe}}/n_e = 0.52$ in the bulk. A stainless steel plate, 15 cm in diameter, was positioned in the middle of the chamber and biased at -30 V to establish an ion sheath.

The ion velocities at the sheath boundary are given by the intersection of the two equations shown in Fig. 1. The generalized Bohm criterion, Eq. (13), is given by a dash-dotted curve (red in the electronic version). Three sets of curves for the IAW dispersion relation, Eq. (12), corresponding to $v_{\mathrm{ph},s} = 1.5v_{\mathrm{ph},b}$, $2v_{\mathrm{ph},b}$, and $2.5v_{\mathrm{ph},b}$ are also graphed. Figure 1 shows that the two equations have solutions with two, three, or four real roots. There are three regions where the solutions exist. First consider regions II and III. Solutions in these regions are not physically acceptable because in each of these regions, one ion species requires a velocity much too large to be achieved by acceleration in the presheath. For example, the presheath potential drop should be $\geq 9T_e$, which is much larger than the measured presheath potential drop.

In region I, it is important to note that two roots are apparent when $v_{\mathrm{ph},s} \geq 2v_{\mathrm{ph},b}$. This condition sets the minimum value of $v_{\mathrm{ph},s}$ for the generalized Bohm criterion to have physically meaningful solutions. When $v_{\mathrm{ph},s} = 2v_{\mathrm{ph},b}$, a double root exists and this corresponds to each ion species having a drift velocity equal to $v_{\mathrm{ph},b}$. This result was determined for almost equal ion density ratios of the two species. In fact, it is evident from Eqs. (12) and (13) that the condition $v_{\mathrm{ph},s} = 2v_{\mathrm{ph},b}$ does not depend on the ion ratios when $v_1 = v_2 = v_{\mathrm{ph},b}$. In that case, both the equations have the same form. This provides an understanding of the experimental result found in weakly collisional plasmas where ionization was not significant [21].

From the IAW dispersion relation and the generalized Bohm criterion in the two-ion species plasma, we have shown that meaningful solutions only exist when the IAW phase velocity at the sheath-presheath boundary is greater than or equal to twice the phase velocity in the bulk plasma. This equality condition corresponds to each ion species having the bulk ion sound velocity at the boundary. This provides the theoretical basis that has been absent in our previous experimental results. It is now apparent that the results for two-ion species plasmas are the same as for single-ion species plasmas. In both situations, the ion drift velocity at the sheath-presheath boundary is equal to the bulk ion sound velocity.

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*Author to whom all correspondence should be addressed. hershkowitz@engr.wisc.edu

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