Predictive Flavor Symmetries of the Neutrino Mass Matrix

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Here we propose an A_4 flavor symmetry model that implies a lower bound on the neutrinoless double beta decay rate, corresponding to an effective mass parameter $M_{ee} \gtrsim 0.03$ eV, and a direct correlation between the expected magnitude of CP violation in neutrino oscillations and the value of $\sin^2\theta_{13}$, as well as a nearly maximal CP phase δ.

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Unless flavor symmetries are assumed, particle masses and mixings are generally undetermined in gauge theories. Understanding mass and mixing constitutes one of the biggest challenges in elementary particle physics. Current observations do not determine all elements of the effective neutrino mass matrix $\mathcal{M}_\nu$ completely, and this will be a great challenge even for future experiments. Therefore theoretical ideas restricting the structure of $\mathcal{M}_\nu$ are needed in order to guide future searches. One such input studied extensively is the assumption that some entries in the neutrino mass matrix vanish [1]. While the phenomenological implications of the assumed zeros in the texture of $\mathcal{M}_\nu$ are straightforward to derive [2], it is a nontrivial task to produce a good symmetry leading to such zeros and a diagonal charged lepton mass matrix simultaneously. Although for any desired texture structure of the mass matrices such a symmetry is in principle always present, this symmetry and the associated Higgs content are sometimes discouragingly complex [3].

Here we propose a predictive flavor symmetry for leptons based on a relatively small and simple flavor group, namely A_4 or its Z_3 subgroup, and briefly analyze its phenomenological implications. We show how this provides a simple means of understanding some of the two-zero textures of $\mathcal{M}_\nu$ studied earlier [2].

The discrete group A_4 is a 12 element group consisting of even permutations among four objects. The group is small enough to lead to a simple model but large enough to give interesting predictions. The distinguishing feature of A_4 compared to other smaller discrete groups is the presence of a three-dimensional irreducible representation appropriate to describe the three generations. This has been exploited in a number of variants. Originally, the A_4 was proposed [4,5] for understanding degenerate neutrino spectrum with nearly maximal atmospheric neutrino mixing angle. More recently, predictions for the solar neutrino mixing angle have also been incorporated in so-called tri-bi-maximal [6] neutrino mixing schemes [7–12]. There also exist attempts at unified A_4 models [13]. The resulting models, however, are not always simple and usually require many Higgs fields. Here we show that a very simple model based on A_4 leads to two-zero textures for $\mathcal{M}_\nu$.

The lepton doublets L_i are assigned as the triplet representation in all the A_4 models proposed so far. Here we propose the opposite assignment indicated in Table I, where the L_i are assigned to the 1, 1′, 1″ representations. The l_{Ri} as well as the Higgs doublets responsible for lepton masses transform as A_4 triplets, while the (undisplayed) quarks and the $SU(2)$ Higgs doublet that gives their masses are all singlets under A_4. This leads to the following terms responsible for the lepton masses:

$$\begin{aligned} -\mathcal{L} &= h_1\bar{L}_1(l_R\Phi)_1 + h_2\bar{L}_2(l_R\Phi)'_1 + h_3\bar{L}_3(l_R\Phi)''_1 \\ &\quad + h_{1D}\bar{L}_1(\nu_R\Phi)_1 + h_{2D}\bar{L}_2(\nu_R\Phi)'_1 \\ &\quad + h_{3D}\bar{L}_3(\nu_R\Phi)''_1 + \frac{M}{2}\nu^T_{Ri}C\nu_{Ri} + \text{H.c.}, \end{aligned} \tag{1}$$

where the quantities in parenthesis denote products of two A_4-triplets l_R (or ν_R) and Φ forming the representations 1, 1′, 1″, respectively. Note that Eq. (1) includes the most general terms allowed by the symmetry and field content in Table I. Hence, in contrast to many other A_4 models, here one does not need to impose any additional symmetry to forbid unwanted terms.

Earlier studies on A_4 have shown that it is possible to obtain a minimum of the Higgs potential with equal vacuum expectation values (VEVs) [4]

$$\langle\Phi^0_1\rangle = \langle\Phi^0_2\rangle = \langle\Phi^0_3\rangle \equiv \frac{v}{\sqrt{3}}. \tag{2}$$

This minimum leads to charged lepton and Dirac neutrino mass matrices M_l and m_D given by, respectively

TABLE I. Lepton multiplet structure of the model.

	L_1	L_2	L_3	l_{Ri}	ν_{Ri}	Φ_i	Δ
$SU(2)$	2	2	2	1	1	2	3
$U(1)$	−1	−1	−1	−2	0	1	2
A_4	1	1′	1″	3	3	3	1′ or 1″

$$\begin{aligned} M_l &= v\,\mathrm{diag}(h_1, h_2, h_3)U \\ m_D &= v\,\mathrm{diag}(h_{1D}, h_{2D}, h_{3D})U, \end{aligned}$$

with

$$U = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \qquad \omega \equiv e^{2\pi i/3}. \tag{3}$$

The above M_l and m_D imply that the symmetry basis L_i also corresponds to the mass basis and only the right-handed fields need to be redefined. As a result, the neutrino mass matrix following from Eqs. (1) and (2), after the seesaw diagonalization [14], is already in the flavor basis and is given by

$$\mathcal{M}^{\,I}_{\nu f} = m_D M_R^{-1} m_D^T = \frac{v^2}{M}\begin{pmatrix} h_{1D}^2 & 0 & 0 \\ 0 & 0 & h_{2D}h_{3D} \\ 0 & h_{2D}h_{3D} & 0 \end{pmatrix}. \tag{4}$$

This has the same zero textures as obtained in [5] except that only two (instead of three) neutrinos are degenerate. As noted in [5], this texture by itself is not complete and one needs to modify it. For example, one can supersymmetrize the above scenario and use radiative corrections to split the degeneracy and obtain predictions for the mixing angles and masses as in [5].

Here we choose a different approach, introducing a triplet field Δ [15] transforming either as a $1''$ or as a $1'$ under A_4, as in Table I. In the first case a small induced VEV $\langle \Delta^0 \rangle \equiv u$ for its neutral component leads to a type-II neutrino mass matrix contribution given as

$$\mathcal{M}^{\,\mathrm{II}}_{\nu} = \begin{pmatrix} 0 & \lambda u & 0 \\ \lambda u & 0 & 0 \\ 0 & 0 & \lambda' u \end{pmatrix}, \tag{5}$$

where λ, λ' are two Yukawa couplings (another hybrid model based on A_4 and using both type-I and type-II contributions to neutrino masses has been considered in [16]). The total neutrino mass matrix is given by the sum of Eq. (4) and (5) and has the form

$$\mathcal{M}_{\nu} = \begin{pmatrix} a & x & 0 \\ x & 0 & b \\ 0 & b & y \end{pmatrix}, \tag{6}$$

where a, b and x, y refer to the type-I and type-II contributions, respectively. The above arguments provide a simple derivation of the two-zero texture classified as B_1 in Ref. [1]).

Alternatively, had the triplet been assigned to the $1'$ representation of A_4 then we would have obtained

$$\mathcal{M}_{\nu} = \begin{pmatrix} a & 0 & x \\ 0 & y & b \\ x & b & 0 \end{pmatrix}, \tag{7}$$

a texture classified as B_2 in [1]. One can modify the assignment of various L_i fields among different singlet representations of A_4. This results either in one of the two above textures or in a texture that is not viable phenomenologically. Thus, the realization of the A_4 flavor symmetry proposed here leads to just two viable two-zero textures, which are quite predictive as we will show.

While the full A_4 symmetry is used in Eq. (1), the resulting two-zero textures follow essentially from a Z_3 subgroup of A_4 that remains unbroken by the vacuum structure in Eq. (2) [9]. This Z_3 is generated by $(1, z, z^2)$, $z^3 = 1$ with the leptons transforming as

$$L_i \to Z^L_{ij} L_j, \qquad (l_{Ri}, \nu_{Ri}) \to Z^R_{ij}(l_{Rj}, \nu_{Rj}), \tag{8}$$

where $Z^L = \mathrm{diag}(1, \omega, \omega^2)$ and

$$Z^R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{9}$$

Note that the fields which earlier transformed as triplets under A_4 are now put into a reducible representation of the Z_3 group. Let us now demand that M_l, m_D, and M_R are invariant under the above defined Z_3. This implies

$$Z^{L\dagger} M_l Z^R = M_l; \qquad Z^{L\dagger} m_D Z^R = m_D; \qquad Z_R^T M_R Z^R = M_R.$$

It is straightforward to show that the above invariance implies that both M_l and m_D must have the form

$$\begin{pmatrix} X & X & X \\ A & \omega A & \omega^2 A \\ B & \omega^2 B & \omega B \end{pmatrix}. \tag{10}$$

The above form coincides with that obtained in Eq. (3) with proper identification of parameters. The right-handed neutrino mass matrix now has the following general form [17]

$$\begin{pmatrix} M_1 & M_2 & M_2 \\ M_2 & M_1 & M_2 \\ M_2 & M_2 & M_1 \end{pmatrix}. \tag{11}$$

In spite of this more complicated form, it is easy to see that the type-I contribution has exactly the same zero texture as in Eq. (4), which is therefore more general than its derivation through the seesaw mechanism used here. It simply follows from the Z_3 invariance of the effective neutrino mass matrix:

$$Z^{LT} \mathcal{M}_\nu Z^L = \mathcal{M}_\nu \tag{12}$$

irrespective of the underlying dynamics. For example, the same form would arise in a model without the right-handed neutrinos but containing a Z_3-singlet Higgs triplet with a nonzero VEV. As in the A_4 case, one can introduce a triplet Δ transforming as ω^2 under z, and whose VEV will now break Z_3 to give the required two-zero texture as in Eq. (5).

We now turn to the phenomenological implications. The main feature of two-zero texture models, such as the ones

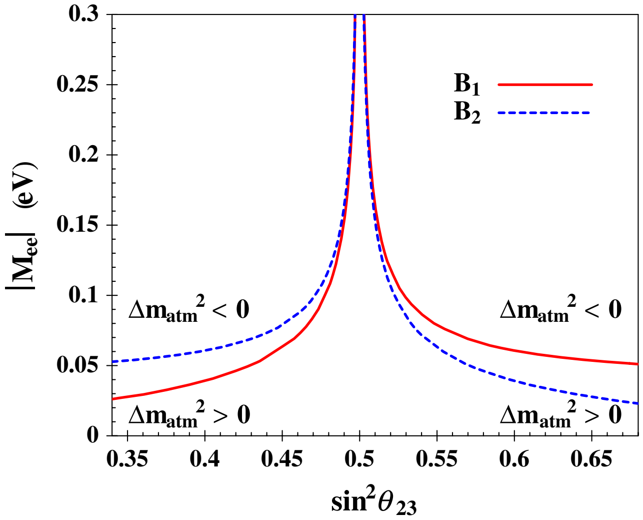

FIG. 1 (color online). Lower bound on neutrinoless double beta decay.

derived here, is their power in predicting the as yet undetermined neutrino parameters. Current neutrino oscillation experiments determine two mass splittings $\Delta m^2_{\mathrm{atm}}$ and $\Delta m^2_{\mathrm{sol}}$ and the corresponding mixing angles θ_{12} and θ_{23}, with some sensitivity on θ_{13}, which is bounded [18]. The Dirac *CP* phase will be probed in future oscillation experiments. Similarly, the absolute neutrino mass scale will be probed by future cosmological observations [19], tritium beta decays [20], and neutrinoless double beta decay experiments [21] with improved sensitivity. The latter will also shed light on the two Majorana *CP* phases that are hard to test otherwise, as they do not affect lepton number conserving processes. The general 3×3 light neutrino mass matrix $\mathcal{M}_\nu$ in the flavor basis contains *a priori* nine independent real parameters, once the three unphysical phases associated with the charged lepton fields are removed. In contrast, in the proposed model all the above nine parameters are given in terms of only five unknowns. Hence the number of physical parameters characterizing the charged current weak interaction is reduced with respect to what is expected in the general case [15].

We now illustrate these predictions. We first consider the mass parameter characterizing neutrinoless double beta decay $|M_{ee}|$, which depends mainly on θ_{23}, as illustrated in Fig. 1. A remarkable feature of our A_4 flavor symmetry model is that it implies the lower bound $|M_{ee}| \gtrsim 0.03$ eV, as seen in Fig. 1. This prediction correlates with the maximality of the atmospheric mixing angle and lies within the range of planned experiments. The bound hardly depends on other parameters. For example, in contrast to Ref. [10], it shows no strong dependence with the value of the relevant Majorana phase. This follows from the more stringent lower bound on the lightest neutrino mass obtained in the present model. We note that $|M_{ee}|$ has, however, some dependence on the value of $\Delta m^2_{\mathrm{atm}}$ and the bound corresponds to $\Delta m^2_{\mathrm{atm}} = 2 \times 10^{-3}$ eV^2.

We now turn to the predictions for *CP* violation and the parameter δ.

As seen in Fig. 2, for both the B_1 (left panel) and B_2 cases (right panel), our model predicts the near maximality of the *CP* violation in neutrino oscillations. The predicted *CP* violating parameter δ depends mainly on θ_{13}, which is currently bounded only by oscillation data [18].

The rephasing invariant magnitude $|J|$ of *CP* violation in neutrino oscillations is defined as

$$J = \mathrm{Im}[K_{11}K_{22}K_{12}^{*}K_{21}^{*}] = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^{2}\sin\delta, \qquad (13)$$

where K_{ij} are the elements of the leptonic mixing matrix. As seen in Fig. 3, which holds for both $B1$ and $B2$ models, one finds that $|J|$ is directly correlated with the value of $\sin^2\theta_{13}$, to be probed in the next generation of high sensitivity neutrino oscillation experiments such as Double

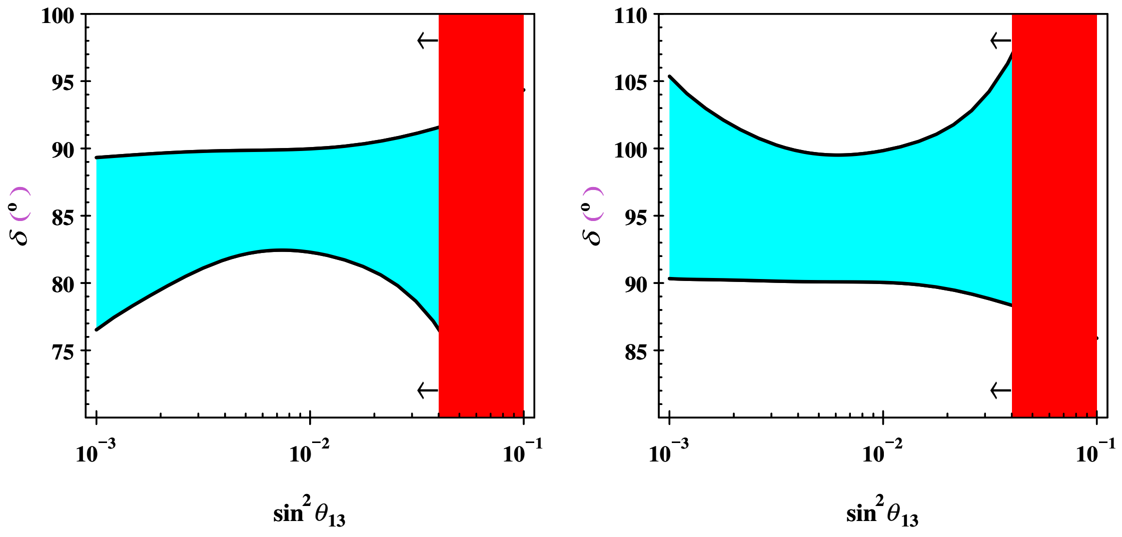

FIG. 2 (color online). Near-maximal *CP* violation in neutrino oscillations.

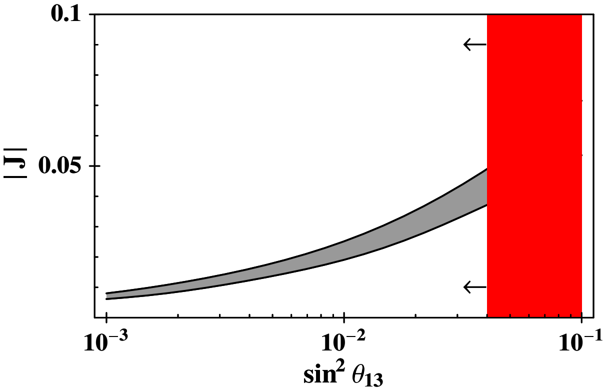

FIG. 3 (color online). *CP* violation in neutrino oscillations versus $\sin^2\theta_{13}$.

Chooz. The width of the band reflects the current uncertainties in the neutrino oscillation parameters [18].

The above predictions based on the tree level texture of $\mathcal{M}_\nu$ are expected to change very little as a result of radiative corrections. The Z_3 symmetry giving tree level texture in Eq. (4) protects it to all orders. Since Z_3 gets broken by the triplet VEV, finite radiative corrections involving triplet Higgs fields modify tree level zeros. However, these are suppressed at least by the square of the Yukawa couplings times the loop factor and hence are small.

In summary, here we have proposed an A_4 flavor symmetry for leptons that leads to a near-maximal *CP* phase δ and correlates the invariant measure of *CP* violation in neutrino oscillations with the magnitude of $\sin^2\theta_{13}$ to be probed in future neutrino oscillation experiments. Moreover, it implies a lower bound $|M_{ee}| \gtrsim 0.03$ eV for the mass parameter characterizing neutrinoless double beta decay, also accessible to planned experiments. All these features already emerge from an effective Z_3 invariance of the larger A_4 symmetry. However, the structure of M_R is different in the A_4 model and the effective Z_3 model. Hence, for example, some phenomenological details related to leptogenesis could be different. These issues will be taken up elsewhere.

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