Resonant Vibrations of Bluff Bodies Cause Multivortex Shedding and High Frequency Forces

J. M. Dahl,* F. S. Hover, and M. S. Triantafyllou

Massachusetts Institute of Technology, Department of Mechanical Engineering, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA

S. Dong

Purdue University, Department of Mathematics, 150 N. University Street, West Lafayette, Indiana 47907, USA

G. E. Karniadakis

Brown University, Division of Applied Mathematics, 182 George Street, Providence, Rhode Island 02912, USA

(Received 9 January 2007; published 5 October 2007)

A flexibly mounted circular cylinder in cross-flow, with natural frequencies in the inline and transverse directions having a ratio close to 2:1, exhibits drastic changes in the vortex structures in its wake, the frequency content of the fluid forces, and the orbital shape of its resulting motions. Stable multivortex patterns form in the cylinder wake, associated with large high-frequency force components.

DOI: 10.1103/PhysRevLett.99.144503 PACS numbers: 47.32.ck, 47.27.ek, 47.54.De

Introduction.—A flexibly mounted circular cylinder in cross flow serves as the canonical problem for studying the theory and properties of vortex-induced vibrations (VIV) of bluff bodies [1]. It is also the representative system for a great number of ocean structures such as marine risers, cables, umbilicals, and undersea pipelines. Recent field observations [2] have shown that despite extensive study, novel properties of VIV can be identified in the form of high-frequency force harmonics that can have a great impact on the fatigue life of ocean systems. In this Letter, we show that the cause of such forces is the appearance in the wake of multiple vortex patterns under conditions of two-degree of freedom resonance, explored through experimental and computational fluid dynamic (CFD) studies.

The wake behind a long circular cylinder in cross flow is unstable above low Reynolds numbers ($\mathrm{Re} > 50$) [3], resulting in the appearance of large-scale periodic vortex patterns, such as the Kármán street that typically forms at the characteristic Strouhal frequency [4–6]. These vortices induce unsteady forces on the cylinder, and if the structure is flexible or flexibly mounted, vibrations result [1,7,8].

The Kármán vortex street that forms behind a nonvibrating cylinder consists of two single (“2*S*”) counter-rotating vortices per cycle. When the cylinder vibrates in the transverse-only direction, this “2*S*” mode may persist or a “2*P*” pattern consisting of two pairs of counter-rotating vortices per cycle may appear, depending on the amplitude and frequency of oscillation [9]. In turn, the strength of the vortices, their number, and their proximity to the structure all directly influence the amplitude and frequency content of the fluid forces exerted on the structure [9–11].

When the frequency of oscillation is close to the Strouhal frequency, the lift (transverse) force contains a dominant fluctuating component with the vortex-shedding period, while the drag (inline) forces contain a steady drag component and a fluctuating component with half the vortex-shedding period. Vortex formation locks on to the frequency of cylinder vibration within a relatively narrow range around the Strouhal frequency (called the wake capture region). Under free vibration conditions, the transverse response is typically larger than the inline response by a factor of at least two. For these reasons, laboratory testing or numerical simulations allowing transverse-only oscillations at a single frequency are widely used to simplify the study of the problem.

Despite its widespread use, analysis of transverse-only motions can lead to different results than the actual vibrations of flexibly mounted bodies, which vibrate in both inline and transverse directions. The effect of inline motion on the magnitude of the transverse oscillation of a freely vibrating cylinder was recognized by Sarpkaya [12], while the effect of the inline motion on the wake was studied for forced cylinder motions in [13]. For very large transverse amplitudes, triplets of vortices can form in the wake of a free cylinder [14,15]; in the experiments of [14,15], the natural frequencies of the cylinder in the transverse and inline directions are equal, and the measured lift forces are dominated by the fundamental shedding frequency.

A bluff body undergoes *perfect vortex-shedding resonance* if its inline natural frequency (f_x) is twice the transverse natural frequency (f_y), and the Strouhal frequency is near or at the transverse natural frequency, resulting in fluid excited motions in both inline and transverse directions. Such a condition is virtually unavoidable for very long, stringlike structures that have a large number of natural frequencies. In the present Letter, we allow the cylinder to vibrate in both streamwise and cross-flow directions, while altering systematically the ratio f_x/f_y between a value of 1, when perfect resonance is impossible, and a value close to 2, when perfect resonance is possible. We find that under perfect resonance, a complex vortex wake results, consisting of patterns of multiple vortices, causing high-amplitude odd harmonics in lift force, which often dominate the fundamental frequency component. Until now, lift excitation forces associated with vortex shedding were believed to peak at the fundamental vibration frequency; here, we show that this model is not true.

Results.—We present results from laboratory experiments at the MIT Towing Tank [16,17] and from three-dimensional Direct Numerical Simulation (DNS) using the spectral element method [18]. In the experiments, a rigid circular cylinder was mounted on a two dimensional guide-rail system and was connected to mechanical springs, allowing motion in the inline and cross-flow directions and allowing tuning of two directional natural frequencies. The natural frequency and damping were determined by the cylinder's perturbed response in still water. A complete description of the experimental apparatus, methods, and an error analysis is given in [17]. Forces were measured at the ends of the cylinder; Particle Image Velocimetry (PIV) visualizations were performed separately [19]. All experiments were performed in the Reynolds number range of 11 000 to 60 000, whereas the Reynolds number for the simulations was set at 10 000. The data are parameterized with the ratio of the measured natural frequencies in water in the streamwise (f_x) and cross-flow (f_y) directions, f_x/f_y. This ratio was varied systematically over the range from one to two.

Figure 1 (in color) shows the cylinder motion orbits (transverse versus inline motion) for each test run, as a function of the reduced velocity, V_{rn} [20] (horizontal axis), and the frequency ratio f_x/f_y (vertical axis). The effect of varying f_x/f_y on the lift force frequency content is striking, as shown through the background contours [21]. The lift forces contain increasingly larger third harmonic components as the frequency ratio increases, to the point that these higher harmonics dominate the total lift force.

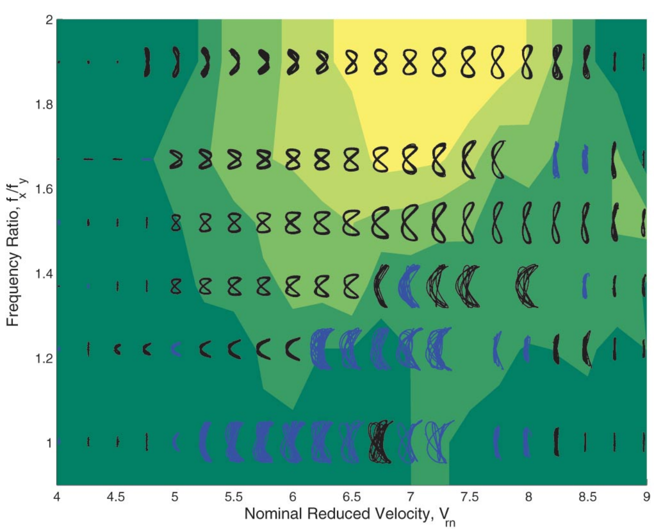
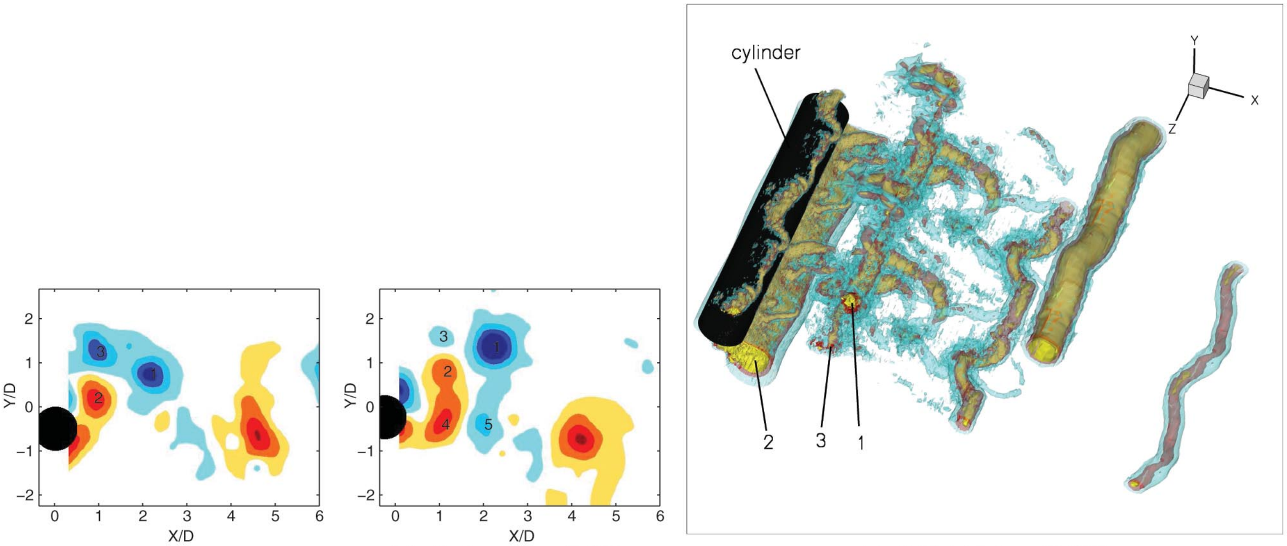


FIG. 1 (color). Cylinder trajectories overlaid on the third harmonic magnitude of the lift force. Blue—trajectory moves downstream at the top of the figure-eight (C: Clockwise); black—trajectory moves upstream at the top of figure-eight (CC: Counterclockwise); flow is left to right. Contours indicate the third harmonic lift coefficient magnitude over the total lift coefficient magnitude. Darkest green denotes a third harmonic magnitude less than 25% of the total lift; lightest yellow denotes a third harmonic magnitude greater than 75% of the total lift.

The appearance of higher harmonics in the lift force measurements is related to multivortex patterns forming in the wake of the cylinder. Images of the wake obtained through PIV in laboratory experiments are given in Figs. 2(a) and 2(b), where clearly defined vortex triplets and quintuplets are evident [22]. When the inline motions of the cylinder are significant and properly phased with the transverse motions, shed vortices come in close proximity to the cylinder, leading to the higher harmonics observed in the lift force. The spatial persistence of vortex triplets is demonstrated in Fig. 2(c), obtained through DNS. Here, the three-dimensional vortex tubes forming the triplet are well defined, stable structures.

When we study in detail the orbital motion of the cylinder, we find that the path direction and the orbit shape are important indicators of the appearance of higher harmonic forcing. When both inline and transverse free oscillations occur, a figure-eight pattern is typical, but a crescentlike shape may also appear, as seen in Fig. 1. In the case of a figure-eight pattern, we define the path trajectory to be either a counterclockwise (CC) or a clockwise (C) trajectory, by noting the direction of cylinder motion at the top of the figure-eight path, with the cross flow coming from left to right. Black orbits in Fig. 1 exhibit CC paths, while blue orbits exhibit (C) paths. A figure-eight pattern with a CC path direction (black) is highly repeatable and stable, while it is accompanied by a strong third harmonic forcing. For example, as shown in Fig. 3(a) for $f_x/f_y = 1.9$, the third harmonic appears first at $V_{rn} \approx 5.2$, increases in magnitude, becoming the dominant lift component at a value of $V_{rn} = 7.0$, then decreases until the first harmonic prevails again at $V_{rn} > 8.5$. In contrast, clockwise paths are typically erratic and possess a negligible third harmonic lift component. The motion of the cylinder contains insignificant third harmonic components, since there is no structural natural frequency near these high frequencies.

DNS studies show that for prescribed motions with clockwise direction, the third harmonic of the lift force is more pronounced at low Reynolds number (e.g. 1000), and that its intensity decreases as the Reynolds number increases to 10 000. On the other hand, for CC trajectories, the intensity of the third harmonic component increases with increasing Reynolds number. Simulations with free cylinder vibrations at $f_x/f_y = 2$ confirm that the motion is in the CC direction, in agreement with the experimental results which are obtained at higher Reynolds number. Figures 3(b) and 3(c) show the motion and force time

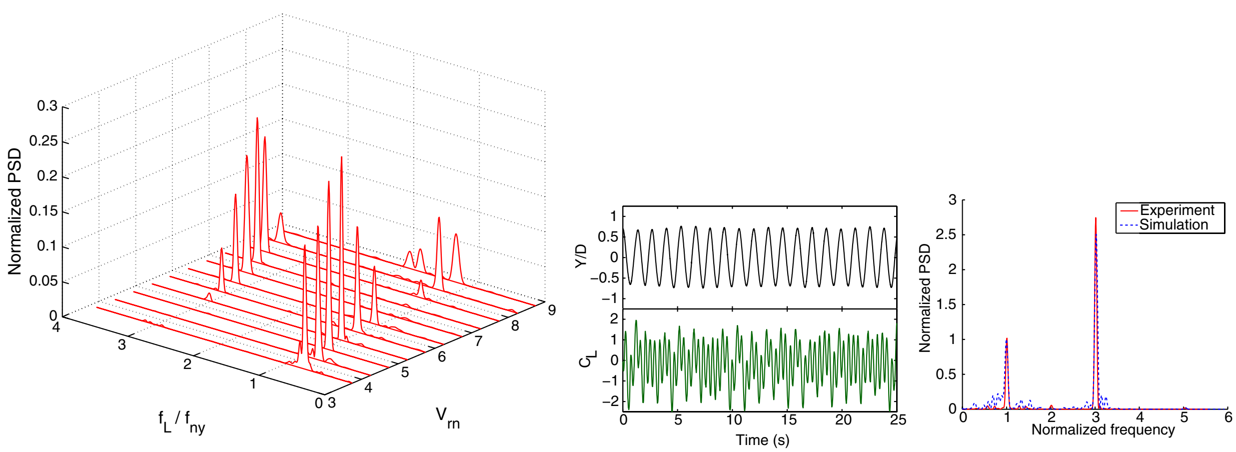


(a) Triplet (PIV) (b) Quintuplet (PIV) (c) 3-D Triplet (DNS)

FIG. 2 (color). Visualization of the modes of vortex shedding in the wake of a vibrating circular cylinder at Re = 10 000. PIV planar images show (a) vortex triplets, and (b) quintuplets—red denote counterclockwise and blue clockwise vorticity. Labeled vortices represent structures that remained coherent in the time-resolved wake near the cylinder over one half cycle. Figure (c) depicts three-dimensional DNS results in the form of instantaneous pressure isocontours, demonstrating the spatial persistence of vortex triplets; $f_x/f_y = 2$, $V_{\mathrm{rn}} = 5.45$. Numbers 1,2, and 3 indicate the vortices in one vortex triplet.

traces as well as the lift force spectrum for an example case, where the third harmonic component is twice as large as the fundamental component. Hence, for Reynolds numbers typically above 5000, we find that CC motion is closely associated with the appearance of large high harmonics of the lift force.

Discussion.—High-frequency stress harmonics have been observed in recent field experiments [2]. Fatigue life of structures is affected significantly by additional high-frequency harmonics because fatigue curves exhibit a high-power functional dependence on the stress level. In steel structures, for example, it is typical to model the fatigue process as $N \times S^b = A$, where N is the number of cycles to fatigue, S is the stress amplitude, and A and b are constants, with $b = 4$ [24]. Hence, when a high-frequency stress is added to the basic harmonic, of roughly



(a) Experimental Lift Coefficient Spectra $f_x/f_y = 1.9$ (b) Time Trace (c)Lift Coefficient Spectrum Comparison

FIG. 3 (color online). Lift force characteristics when vortex triplets prevail. Figure (a) shows a composite of normalized lift coefficient spectra as functions of reduced velocity, demonstrating the dominance of the third harmonic in a region around $V_{\mathrm{rn}} \approx 7$. Figure (b) shows the time trace of the cylinder response (Y/D) and the corresponding lift coefficient (C_L) for $f_x/f_y = 1.9$, $V_{\mathrm{rn}} = 6.5$. Figure (c) shows the comparison between experiment and 3-D DNS of the normalized lift coefficient spectrum.

the same magnitude as the first harmonic, fatigue life is decreased by a factor of 2^4. The importance of the present Letter is to show the root cause for this phenomenon.

Our principal finding is that when the inline and transverse natural frequencies of a freely vibrating cylinder are close to the 2:1 ratio and under resonance conditions, a fundamental change in both the orbital motion of the structure and the frequency content of the fluid forces occurs. These changes are caused by a combination of the following phenomena: (1) the wake consists of stable, repeatable multivortex patterns near the cylinder, instead of the "2*S*" or "2*P*" modes expected for transverse-only motions, and (2) the body moves in a stable figure-eight pattern which is characterized by a counterclockwise (CC) direction of motion (as defined in Fig. 1).

The importance of the CC direction is that the cylinder moves upstream just before the vortices are shed and then moves downstream while crossing through center, coming in close contact with the recently shed vortices. The cylinder is subject to large suction forces from these multiple vortices, resulting in high-frequency lift forces. Other paths taken by the cylinder under different conditions place the vortices further away, hence weakening the ability of the wake to create such forces. These phenomena also explain the large impact of relatively small inline motions on the amplitude of the transverse oscillations.

This study shows that lift forces associated with vortex-induced vibrations are not always dominated by a fundamental frequency component near the Strouhal frequency, but can have significant higher frequency components. These large harmonic components of lift can result in higher stresses on vibrating structures and lead to reduced fatigue life.

*jdahl@mit.edu

[1] P. W. Bearman, Annu. Rev. Fluid Mech. **16**, 195 (1984).
[2] J. Vandiver, S. Swithenbank, V. Jaiswal, and V. Jhingran, *Offshore Mech. Arctic Engng. Conf.* (ASME, Hamburg, Germany, 2006).
[3] Reynolds number is defined as $\mathrm{Re} = UD/\nu$, where U is velocity, D is cylinder diameter, and ν is viscosity.
[4] G. Triantafyllou, M. Triantafyllou, and C. Chryssostomidis, J. Fluid Mech. **170**, 461 (1986).
[5] P. Huerre and P. Monkewitz, Annu. Rev. Fluid Mech. **22**, 473 (1990).
[6] Strouhal number is defined as $\mathrm{St} = fD/U$ where f is the vortex-shedding frequency, also called the Strouhal frequency.
[7] C. Williamson and R. Govardhan, Annu. Rev. Fluid Mech. **36**, 413 (2004).
[8] T. Sarpkaya, J. Fluids Struct. **19**, 389 (2004).
[9] C. Williamson and A. Roshko, J. Fluids Struct. **2**, 355 (1988).
[10] A. Ongoren and D. Rockwell, J. Fluid Mech. **191**, 197 (1988).
[11] J. Carberry, J. Sheridan, and D. Rockwell, J. Fluid Mech. **538**, 31 (2005).
[12] T. Sarpkaya, J. Offshore Mech. Arct. Eng. **117**, 232 (1995).
[13] D. Jeon and M. Gharib, J. Fluids Struct. **15**, 533 (2001).
[14] C. Williamson and N. Jauvtis, Eur. J. Mech. B, Fluids **23**, 107 (2004).
[15] N. Jauvtis and C. Williamson, J. Fluid Mech. **509**, 23 (2004).
[16] J. Dahl, F. Hover, and M. Triantafyllou, *8th Intern. Conf. on Flow Induced Vibr.*, edited by E. de Langre and F. Axisa (Ecole Polytechnique, Paris, France, 2004).
[17] J. Dahl, F. Hover, and M. Triantafyllou, J. Fluids Struct. **22**, 807 (2006).
[18] G. Karniadakis and S. Sherwin, *Spectral/hp Element Methods for CFD* (Oxford Univ. Press, New York, 1990), 2nd ed..
[19] See EPAPS Document No. E-PRLTAO-99-053738 for supplementary information for flow visualization animations. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[20] The reduced velocity is defined as $V_{rn} = U/(f_y D)$, where f_y is transverse natural frequency in still water.
[21] Lift coefficient is defined as $C_L = L/(0.5\rho U^2 DS)$ where L is lift force, ρ is fluid density, and S is cylinder span.
[22] Based on the analysis of [23], considering particle density and interrogation window size, the relative velocity error is estimated to be 6% of the actual velocity. All labeled vortices are greater than 96 pixels in diameter, for which attenuation due to spatial filtering is less than 15%.
[23] C. Willert and M. Gharib, Exp. Fluids **10**, 181 (1991).
[24] *Design of Risers for Floating Production Systems (FSPs) and Tension-Leg Platforms (TLPs) Recommended Practice*, American Petroleum Institute, 1998