

Scaling and Universality in Proportional Elections

Santo Fortunato¹ and Claudio Castellano²

¹*Complex Networks Lagrange Laboratory (CNLL), ISI Foundation, Torino, Italy*

²*SMC, INFN-CNR and Dipartimento di Fisica, “Sapienza” Università di Roma, P.le A. Moro 2, 00185 Roma, Italy*

(Received 16 February 2007; revised manuscript received 14 August 2007; published 25 September 2007)

A most debated topic of the last years is whether simple statistical physics models can explain collective features of social dynamics. A necessary step in this line of endeavor is to find regularities in data referring to large-scale social phenomena, such as scaling and universality. We show that, in proportional elections, the distribution of the number of votes received by candidates is a universal scaling function, identical in different countries and years. This finding reveals the existence in the voting process of a general microscopic dynamics that does not depend on the historical, political, and/or economical context where voters operate. A simple dynamical model for the behavior of voters, similar to a branching process, reproduces the universal distribution.

DOI: [10.1103/PhysRevLett.99.138701](https://doi.org/10.1103/PhysRevLett.99.138701)

PACS numbers: 89.65.-s, 87.23.Ge, 89.75.Da

Many social nontrivial phenomena emerge spontaneously out of the mutual influence of a large number of individuals [1–4], similarly to large-scale thermodynamic behavior resulting from the interaction of a huge number of atoms or molecules. However, human interactions are neither purely mechanical nor reproducible, both typical requirements for a physical description of a process. Nevertheless the collective behavior of large groups of individuals may be independent of the details of social interactions and individual psychological attributes, and be instead the consequence of generic properties of the elementary interactions, allowing for a simple “statistical physics” modeling.

In this spirit, microscopic models have been recently proposed to account for collective social phenomena, like the formation of consensus on a specific topic [5–7], the creation of common cultural traits and their dissemination [8], the origin and evolution of language [9,10], etc. While models are studied quantitatively in great detail, the comparison with real-world social phenomena is often merely qualitative and on this account it is not possible to make a real discrimination between competing models. This in turn limits their predictive power, making it unclear if there is at all a gain in the understanding of social dynamics through statistical physics.

Elections are an ideal playground for a quantitative validation of the approach to social dynamics inspired by physics. They constitute a precise global measurement of the state of the opinions of the electorate. A large number of individuals are involved and big data sets are available for many countries, thus allowing accurate quantitative investigations.

In this Letter we present compelling evidence that elections data display properties of more traditional physical phenomena characterized by collective behavior and self-organization, i.e., scaling and universality. We show that, in proportional elections, the distribution of the number of votes received by candidates is universal; i.e., it is the same

function in different countries and years, when the number of votes is rescaled according to the strength of the party to which each candidate belongs. We claim that the universal voting behavior is due to the spreading of the word of mouth from the candidate to the voters, which we model as a sort of branching process involving the acquaintances of a candidate.

Early studies revealed that the histogram of the fraction ν of voters supporting a candidate within a constituency in Brazilian parliamentary elections is described by a $1/\nu$ law, in the central part of the range of the variable ν [11–13]. A successive analysis of Indian elections [14] found a similar yet different histogram, hinting that the distribution of the fraction of votes ν may exhibit some degree of universality. We have performed the same analysis on German, French, Italian, and Polish elections [15], finding marked differences between the various countries: the $1/\nu$ pattern is not general.

This lack of universality is a consequence of the fact that the number of votes a candidate receives is the combination of two distinct factors: how many of the total number of electors vote for the candidate’s party and the personal appeal of the candidate within the restricted pool of voters for his or her party. The first factor strongly depends on policy-related issues: typically voters know the position of all parties with respect to the political issues they deem more relevant and they select the party that best matches their personal views. The second factor is instead practically independent of political issues. Since candidates of the same party mostly share a common set of opinions on ethical, social and economical issues, the selection of a specific candidate has not to do with such issues, rather it depends on a “personal” interaction between the candidate and the voters. Typically voters know at most a few of the candidates in their party list, and in this small subset they select the one they will support. Successful candidates are those able to establish some form of direct or indirect contact with many potential voters during the electoral

campaign. This type of opinion dynamics is likely to give rise to universal phenomena. The histogram of the total fraction ν of votes may conceal the actual regularities due to the voter dynamics, as it entangles the two factors: a very popular candidate of a small party can have the same number of votes (but for completely different reasons) of a relatively unpopular candidate of a very large party.

In the following we focus on the second factor, how the electors of each party select candidates in their party list. This rules out systems based on single member constituencies, where every party or coalition presents a single candidate in each electoral district, as well as proportional elections with closed lists, where voters are not allowed to express preferences among party candidates: in this case the ranking of candidates of a party is predetermined by the party.

The most suitable elections to investigate the elementary voter dynamics are proportional elections with multiple-seat constituencies and open lists. In this electoral system, the country is divided in districts, and each of them allocates a certain number of seats, Q_{\max} , typically between 10 and 30. Within each district, each party l presents a list of $Q_l \leq Q_{\max}$ candidates. Voters choose one of the parties and also express their preference among the candidates of the selected party. Each party gets n_l of the total number of seats, in proportion to the number of votes it has received in the district. The n_l most voted candidates of party l are elected. In this way, the party plays no role as to which of its candidates will be eventually elected, their success depending only on the free choice of voters.

We have considered three countries with such type of electoral system: Italy (until 1992), Poland, and Finland. We use publicly available [16] data sets for three elections in Italy (1958, 1972, 1987), one in Poland (2005), and one in Finland (2003). The total number of candidates ranges from 2029 for the Finnish elections in 2003 to 10 658 for the Polish elections in 2005.

To factor out the policy-related role of the parties, we keep track, for candidate i that receives ν_i votes, also of two other parameters: Q_i , i.e., the number of candidates of the party list l_i , where i belongs, and N_{l_i} , total number of votes collected by the Q_i candidates of list l_i .

The distribution of the number of votes collected by candidates is in general a function of the three variables $P(\nu, Q, N)$. We show instead that $P(\nu, Q, N)$ is actually a function of a single rescaled variable. We start by showing that $P(\nu, Q, N)$ does not depend on N and Q separately, but only on the ratio $\nu_0 = N/Q$, which is the average number of votes collected by a candidate in his or her list. The curves of Fig. 1(a) correspond to three different values of ν_0 . Since ν_0 is a continuous variable, fixing ν_0 actually means selecting those lists with values of ν_0 within a narrow range. For each value of ν_0 we fix a threshold for the total number N of votes and further filter the data by separating the lists with N larger or smaller than the

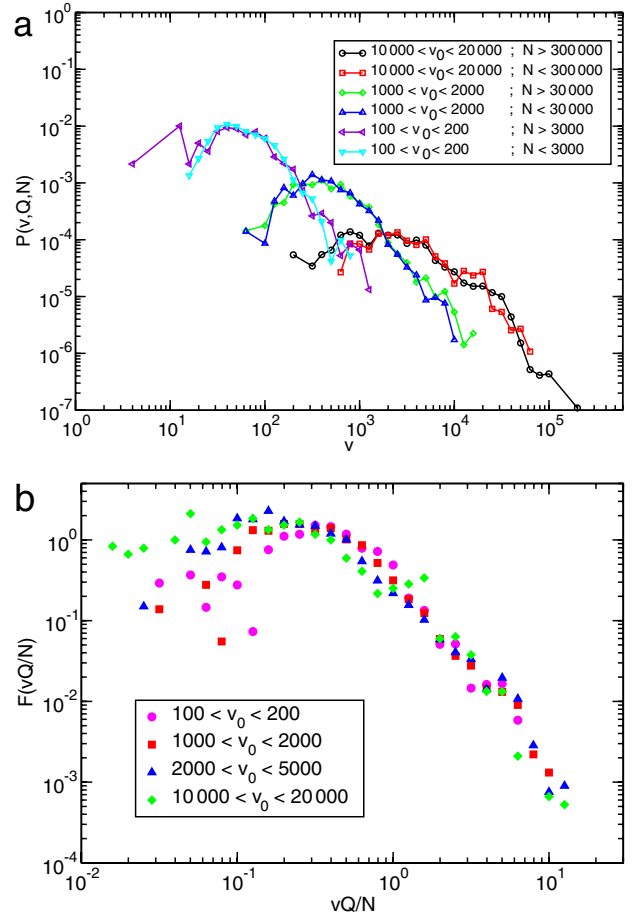


FIG. 1 (color online). (a) Scaling behavior of the distribution of votes received by candidates. Data refer to the Italian parliamentary elections in 1972, but we obtained very similar results from the analysis of each data set. The histogram $P(\nu, Q, N)$ only depends on the ratio $\nu_0 = N/Q$, so $P(\nu, Q, N) = P_0(\nu, \nu_0)$. (b) The function $P_0(\nu, \nu_0)$ shown in (a) only depends on the ratio $\nu/\nu_0 = \nu Q/N$.

threshold. For a fixed ν_0 , the resulting histograms are the same for both data samples, proving that the distribution $P(\nu, Q, N)$ is actually only a function of the arguments ν and ν_0 , $P_0(\nu, \nu_0)$.

But a close inspection of the function $P_0(\nu, \nu_0)$ reveals that the dependence on two variables is actually only apparent: the distribution of the rescaled variable $\nu/\nu_0 = \nu Q/N$ turns out to be independent of ν_0 . Again, we filter the data by putting together candidates belonging to lists such that the ratio $\nu_0 = N/Q$ falls in one of four narrow windows. For each set of candidates we derive the histogram of the rescaled variable ν/ν_0 : the four curves have a remarkable overlap, so there is no dependence on ν_0 but only on ν/ν_0 [Fig. 1(b)]. We conclude that $P(\nu, Q, N)$ is actually a function of the single variable $\nu Q/N$; i.e.,

$$P(\nu, Q, N) = F(\nu Q/N). \quad (1)$$

Since $v_0 = N/Q$ is the average number of votes collected by a candidate in his or her list, the ratio $vQ/N = v/v_0$ is an index of the performance of a candidate against his or her competitors in the same list. If $v/v_0 < 1$, the candidate has received less votes than average; if $v/v_0 \gg 1$, he/she performed much better than average.

Equation (1) indicates that each election can be characterized by a single function $F(vQ/N)$. A comparison between the scaling functions F for all five data sets is presented in Fig. 2 and gives an even more striking result: the scaling function $F(vQ/N)$ is the same for different countries and years. The universal curve is very well reproduced by a *log-normal* function; i.e.,

$$F(vQ/N) = \frac{N}{\sqrt{2\pi}\sigma vQ} e^{-(\log(vQ/N) - \mu)^2 / 2\sigma^2}, \quad (2)$$

with $\mu = -0.54$, $\sigma^2 = -2\mu = 1.08$. The relation $\sigma^2 = -2\mu$ is due to the fact that the expected value of the variable $vQ/N = v/v_0 = 1$ and that the expected value of a lognormal distributed variable is $\exp(\mu + \sigma^2/2)$.

The universality of the distribution $F(vQ/N)$ is truly remarkable. The elections considered span a period of 30 years, in which deep cultural, economic, and social transformations have occurred: there is no hint of that in the data pattern. Likewise, differences between countries as diverse as Italy, Poland, and Finland do not play any role. This calls for a modelization in terms of simple mechanisms of interaction between voters (and candidates), regardless of the details of the social, cultural, and economic environment.

The spreading of word of mouth is known to be a very effective vehicle of diffusion of new products among potential buyers [17]. We interpret the electoral results using a simple opinion dynamics model based on word of mouth:

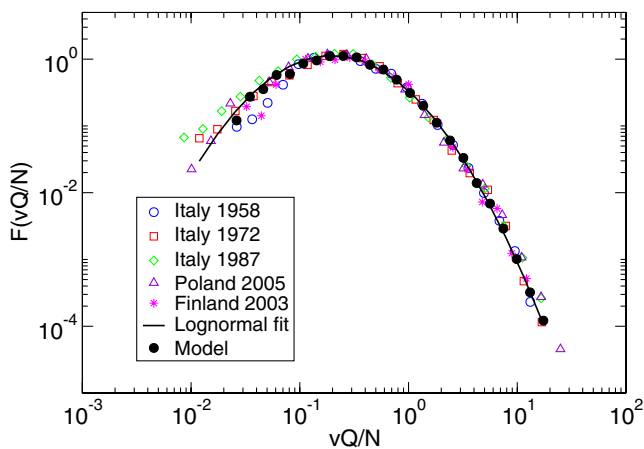


FIG. 2 (color online). Universality of the scaling function $F(vQ/N)$ across different countries and years. The lognormal fit, performed on the Polish curve, describes very well the data. The universal curve is well reproduced by our model, where the dynamics of the voters' opinions reflects the spreading of the word of mouth in the party's electorate.

electors that have already chosen a candidate try to convince their peers to vote for the same candidate (Fig. 3). At the beginning, only candidates have an opinion (they vote for themselves). The dynamics starts with the candidates trying to convince their acquaintances. The people convinced by each candidate become activists and in turn try to convince their contacts to vote for their candidate, and so on. Only undecided voters can be convinced. Not all interactions result in an undecided voter being convinced: persuasion occurs only with probability r . Models of opinion spreading with similar features have been introduced recently [18,19].

We implement the process by representing the electorate of a party as a set of treelike communities of voters, with candidates as roots, as shown schematically in Fig. 3. We have as many independent trees as candidates, and each candidate acts on the nodes of its own tree, representing the voters within its sphere of influence, and not on the others. The distribution $p(k)$ of the number k of contacts of a voter has to be broad, as there are very active people that try to convince as many voters as possible, as well as less active ones, that do not feel particularly involved or motivated. We assume therefore that $p(k)$ is described by a power law, i.e., that the probability $p(k)$ that a voter has k acquaintances is $p(k) \sim k^{-\alpha}$, with $\alpha > 1$. To completely fix the distribution $p(k)$, we fix the lower bound of k , that we indicate with k_{\min} .

Every iteration of the process consists in the persuaded voters trying to convince their undecided contacts, each with probability r . One keeps track of the running number of convinced voters, which increases with time. The process stops when this number equals N , where N is the size

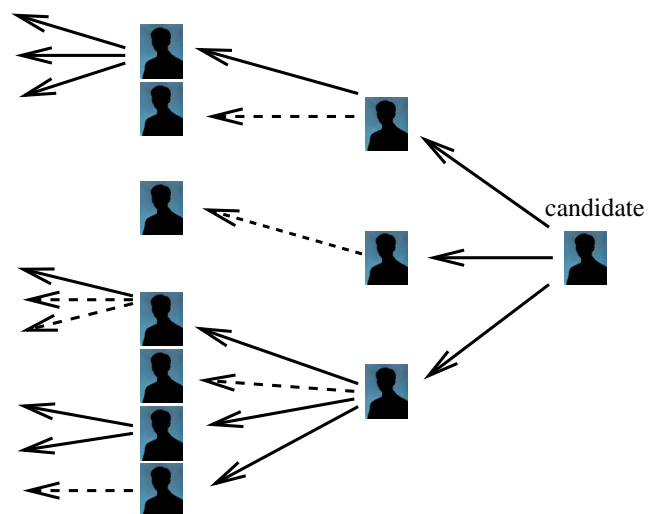


FIG. 3 (color online). Spreading of the word of mouth among voters. The candidate (right) convinces some of his or her contacts to vote for him/her. The convinced voters become "activists" and try to convince some of their acquaintances, and so on. Successful interactions are indicated by solid lines, unsuccessful interactions are displayed as dashed lines.

of the electorate of that party in the constituency. Our model is similar to a collection of branching processes evolving in parallel, coupled via the condition that they stop when the total number of convinced voters reaches N . Branching processes have a large number of applications in the physics literature, from modeling of forest fires [20], to percolation [21], to self-organized criticality [22]. It is important to stress, however, that our model is not a usual branching process, but a full-fledged new process, with different and nontrivial properties. The key point is that, while in branching processes once a node has decided how to branch it remains frozen, here convinced voters keep trying to persuade their contacts: branching events can occur at any point in the trees.

We study the dynamics of our model by means of computer simulations. For each choice of the parameters α , k_{\min} and r , that we consider, we repeat the process several times. Each time we store the number of votes received by every candidate. When enough scores are collected, the histogram $P(v, Q, N)$ is determined.

The distribution $P(v, Q, N)$, obtained via numerical simulations, exhibits the scaling properties of the empirical distribution; i.e., it obeys Eq. (1). In Fig. 2 we fit the model distribution to the empirical curve. To account for finite size effects, we ran the simulations on the same set of values for Q and N that occur in the empirical data sets, and convoluted the resulting curves. The model curve of Fig. 2 is the convolution of the distributions obtained from each pair of Q and N , for $\alpha = 2.45$, $k_{\min} = 10$ and $r = 0.25$: the agreement is remarkable.

The histogram $F(vQ/N)$ depends rather slowly on the three model parameters α , k_{\min} , and r ; besides, the decreasing part of the curve is very robust [15].

We have shown that election data reveal impressive regularities when the role of policy-related issues is factored out so that the voter dynamics only relies on the contact of the candidates with the voters. This pattern of behavior is the same in different countries and times and hence is affected neither by individual features of the voters nor by the environment where the voters live. We conclude that the underlying voting dynamics is elementary and can be described by simple statistical physics models. A branchinglike process representing the propagation of word of mouth reproduces the universal distribution of votes for candidates. We expect this universality to hold for other countries where the electoral system is (or will be in the future) proportional with open lists.

As a potential application of our results, since the relative performance of a candidate in a list has the same distribution everywhere, the index vQ/N is an objective estimate of the popularity of a candidate, independently of the constituency and the year of the election; this gives parties an unbiased quantitative basis to decide internal rankings and hierarchies.

Word of mouth spreading is a crucial ingredient to explain other instances of collective social dynamics, such as the spreading of news and fads in a population and the diffusion of new products among potential consumers. From the analysis of these processes other signatures of universality may emerge. This research direction may strengthen the confidence on the applicability of statistical physics to explain large-scale social dynamics.

We thank C. Brooks, A. Flammini, M. Marsili, D. Stauffer, and A. Vespignani for useful discussions.

-
- [1] Ph. Ball, *Complexus* **1**, 190 (2003).
 - [2] Z. Nédá, E. Ravasz, Y. Brechet, T. Vicsek, and A.-L. Barabási, *Nature* (London) **403**, 849 (2000).
 - [3] D. Helbing, I. Farkas, and T. Vicsek, *Nature* (London) **407**, 487 (2000).
 - [4] D. Helbing and M. Treiber, *Science* **282**, 2001 (1998).
 - [5] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch, *Adv. Complex Syst.* **3**, 87 (2000).
 - [6] K. Sznajd-Weron and J. Sznajd, *Int. J. Mod. Phys. C* **11**, 1157 (2000).
 - [7] P. L. Krapivsky and S. Redner, *Phys. Rev. Lett.* **90**, 238701 (2003).
 - [8] R. Axelrod, *J. Conflict Resolut.* **41**, 203 (1997).
 - [9] M. A. Nowak, N. L. Komarova, and P. Niyogi, *Nature* (London) **417**, 611 (2002).
 - [10] C. Schulze and D. Stauffer, in *Econophysics and Sociophysics: Trends and Perspectives*, edited by B. K. Chakrabarti, A. Chakraborti, and A. Chatterjee (Wiley-VCH, Weinheim, 2006).
 - [11] R. N. Costa Filho, M. P. Almeida, J. S. Andrade, Jr., and J. E. Moreira, *Phys. Rev. E* **60**, 1067 (1999).
 - [12] R. N. Costa Filho, M. P. Almeida, J. E. Moreira, and J. S. Andrade, Jr., *Physica* (Amsterdam) **322A**, 698 (2003).
 - [13] A. T. Bernardes, D. Stauffer, and J. Kertész, *Eur. Phys. J. B* **25**, 123 (2002).
 - [14] M. C. Gonzalez, A. O. Sousa, and H. J. Herrmann, *Int. J. Mod. Phys. C* **15**, 45 (2004).
 - [15] S. Fortunato and C. Castellano (to be published).
 - [16] http://www.stat.fi/tk/he/vaalit/vaalit2003/vaalit2003_vaalitilastot_en.html, <http://elezionistorico.interno.it/>; http://www.wybory2005.pkw.gov.pl/index_EN.html.
 - [17] F. M. Bass, *Manage. Sci.* **15**, 215 (1969).
 - [18] A. A. Moreira, D. R. Paula, R. N. Costa Filho, and J. S. Andrade, Jr., *Phys. Rev. E* **73**, 065101 (2006).
 - [19] G. Travieso and L. da Fontoura Costa, *Phys. Rev. E* **74**, 036112 (2006).
 - [20] K. Christensen, H. Flyvbjerg, and Z. Olami, *Phys. Rev. Lett.* **71**, 2737 (1993).
 - [21] A. Mezhlumian and S. A. Molchanov, *J. Stat. Phys.* **71**, 799 (1993).
 - [22] S. Zapperi, K. Bækgaard Lauritsen, and H. E. Stanley, *Phys. Rev. Lett.* **75**, 4071 (1995).