Using Compound Kinoform Hard-X-Ray Lenses to Exceed the Critical Angle Limit

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We have fabricated and tested a compound lens consisting of an array of four kinoform lenses for hard x-ray photons of 11.3 keV. Our data demonstrate that it is possible to exceed the critical angle limit by using multiple lenses, while retaining lens function, and this suggests a route to practical focusing optics for hard x-ray photons with nanometer scale resolution and below.

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Introduction and background.—Hard x-ray optics have applications that range from astronomical x-ray telescopes to synchrotron based microscopes. For high resolution microscopy applications, there have been a number of important developments [1–7]. There have been recommendations [8–12] and implementations [13,14] of kinoform optics for these and other hard x-ray applications. However, there appears to be some uncertainty in the literature [6,15] as to the ultimate resolution limit for hard x-ray focusing optics in the far field limit. For visible light optics, which have a refractive index $n > 1$, one expects the familiar classical result $0.61\lambda/(\mathrm{NA})$, where λ is the wavelength of radiation and NA is the numerical aperture of the optic. For large NA of order 1, the optic resolution can approach the wavelength of the illuminating radiation. For hard x rays, with the real and imaginary parts of the refractive index defined as $n \equiv 1 - \delta + i\beta$ with δ of order 10^{-6}, and large relative absorption (β/δ), it has not been clear that there are feasible and/or practical routes to high resolution optical elements with large NAs. For example, for a single reflective optic, a resolution limit due to the critical angle $\theta_c = (2\delta)^{1/2}$ has long since been known [16]. It has been pointed out [14] more recently that for a refractive or kinoform optical element, NA is limited to the critical angle, and a similar result was also obtained in the context of waveguide [15]. Based on the waveguide result, it was suggested that this was a fundamental limit, applicable to all hard x-ray optics, a serious limitation for improving the spatial resolution of all hard x-ray based methods. For astronomical applications, the critical angle limit results in telescopes with large focal lengths, and it can be impractical to build such large telescopes. A limited deflection angle also limits the performance of prisms and beam splitters and other refractive optical elements familiar from visible light optics. However, it had already been suggested [14] that compound kinoform lenses and compound refractive lenses would allow one to exceed this limit. In this Letter, we physically implement and demonstrate compound kinoform lenses that can exceed the critical angle limit, thus opening a potentially practical path for hard x-ray optics to achieve nanometer scale resolutions with potentially relatively large NAs.

One fundamental drawback of a purely refractive, hard x-ray optic in comparison with a kinoform is that the relatively high absorption in the lens imposes a trade off between demagnification and aperture, ultimately limiting the achievable resolution. For example, if one considers the familiar parabolic profile typically chosen for a purely refractive optic, $t = y^2/(2\delta F)$, the absorption as a function of the radial lens coordinate is $e^{-(4\pi\beta t/\lambda)}$. One finds that the aperture is limited and the effective lens aperture is proportional to $\sqrt{F}$. Consequently the resolution is also proportional to $\sqrt{F}$, resulting in a gain that is dependent only on the values of δ and β [9,17]. In contrast, for a lens with absorption independent of radial coordinate, the resolution is $\propto F$, and the gain $\propto 1/F$. So, by choosing a small enough F, a lens with poor but uniform transmission, one can always outperform the gain of a purely refractive lens. One obvious way to obtain an absorption independent of radial lens coordinate is to use kinoform lenses, as shown in Fig. 1(a).

Some basic features of a single kinoform lens for hard x-ray applications have been considered [13,14]. Since the optic is very sensitive to phase errors, careful attention must be paid to determining the ideal profile for the optic. For the particular case considered [14] of a source essentially infinitely far away from a lens focusing to a point, the correct phase profile was shown to be an elliptical kinoform shape, using Fermat's theorem. Other source-to-lens and source-to-object distances will require other lens profiles which can also be deduced from Fermat's theorem. The elliptical shape immediately leads one to the result that for a single lens there is a resolution limit $\sim\lambda/\theta_c$, because

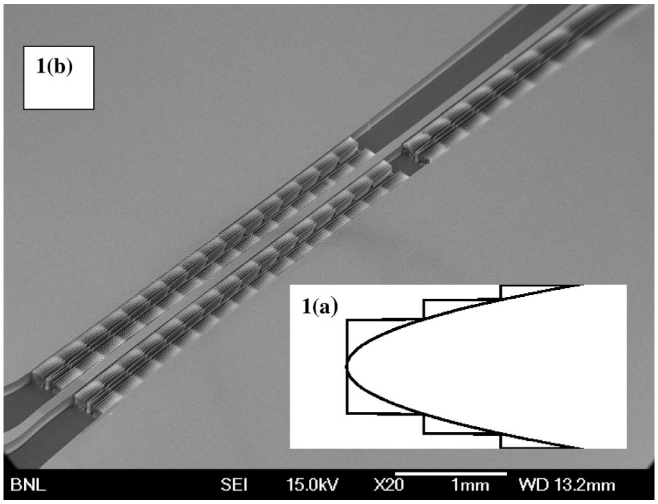

FIG. 1. (a) Inset showing the typical profile of a kinoform structure that results in reduced lens absorption. (b) An electron microscope image of the lenses, showing the single lens in one channel and the first two lenses of the four-lens array.

for a given focal length F there is a maximum lens aperture $F\theta_c$. For a single silicon lens at 11.3 keV, $0.61\lambda/\theta_c \sim$ 24 nm.

The path around the critical angle limitation is to consider compound kinoform lenses, as was suggested initially [14] and then later [6]. In fact, compound kinoform lenses for hard x rays have already been fabricated [13]; however, the phase profile was not optimized, and a simple array of parabolic kinoform lenses will not perform adequately. Compound refractive lenses with a varying focal length have also been discussed and demonstrated [18], with the goal of a more rapid reduction of focal length per unit lens, but the critical angle limit was not discussed, and ultimately solid refractive lenses will be limited by absorption.

Experiment.—Four kinoform lenses were designed in series with each other such that each lens was designed to focus the virtual source created by the prior lens in the array. The first lens was designed to approximate the synchrotron as a source infinitely far away, and had a focal length of 0.1 m. Lenses 2, 3, and 4 had focal lengths of 0.05, 0.033, and 0.025 m, respectively. The figure of each lens was deduced from Fermat's theorem and fabricated in silicon using fabrication techniques similar to those described previously [14]. A single kinoform lens was fabricated in parallel, just next to the compound lenses on a single chip. The lenses sit centered in a channel open on each end with a final etch depth of approximately 80 μm. Scanning electron micrographs of the lens array are shown in Fig. 1(b).

The focal properties of the lens were assessed at the National Synchrotron Light Source (NSLS, 2.8 GeV, 200 mA) X13B MGU beam line. The lens was mounted on a rotation/translation stage (6 degrees of freedom) and was arranged to provide focusing in the vertical direction. A motorized Huber slit was placed before the lens to define the incoming x-ray dimensions. An incident x-ray energy of 11.3 keV was selected using a double-crystal Si(111) monochromator and the lens was rotated until the best line focus was obtained visually using a charge-coupled device camera. A lithographically patterned copper target was positioned near the focal point lens, mounted on encoded translation stages with 80 nm resolution (Fig. 2, item E).

Results.—We performed two types of scans for both the single and compound kinoform lenses, outlined in Fig. 2. For the first type of scan, a slit typically with a 10 μm gap on the upstream side of the lens defines a pencil beam (Fig. 2, item B) on the lens. The pencil beam is scanned across the aperture of the lens while the microfabricated copper wire (Fig. 2, item E) is positioned at or near the lens focal plane. The copper fluorescence from the wire provides the detected signal and is recorded as a function of the distance of the pencil beam from the optical axis (Fig. 2, item C). Shown in Figs. 3(a) and 4(a) are results of such a scan. For both Figs. 3(a) and 4(a) we indicate the aperture of the lens (300 μm) and the location of the critical angle limit $F\theta_c$ as viewed from the focus. For the single lens [Fig. 3(a)], the critical angle limit is outside the aperture of the lens, but for the compound lens [Fig. 4(a)]

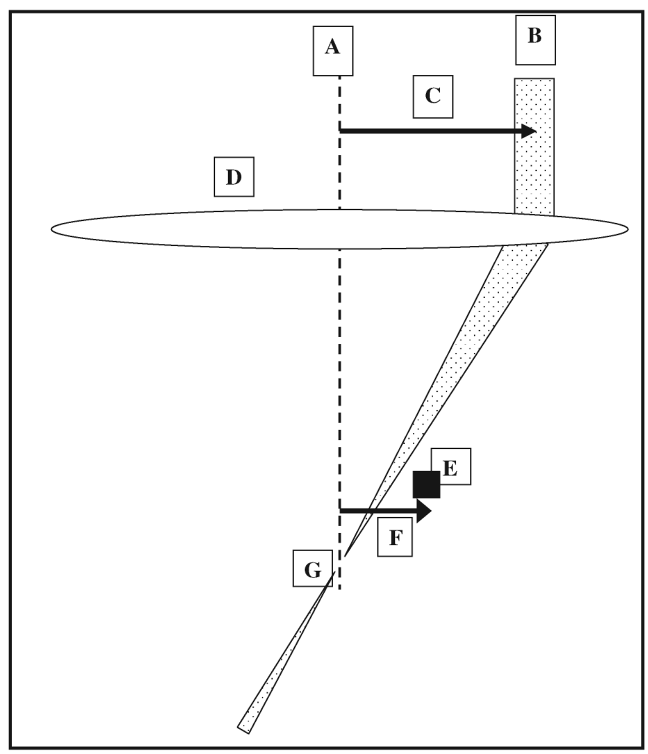

FIG. 2. Main features of the measurements discussed in this Letter. A: Lens optical axis; B: incident pencil beam; C: distance of incident slit center from optical axis which defines the position of the incident pencil beam; D: lens; E: copper fluorescence target; F: detector slit center which defines the distance of the copper target from the optic axis; G: the focal point of the lens.

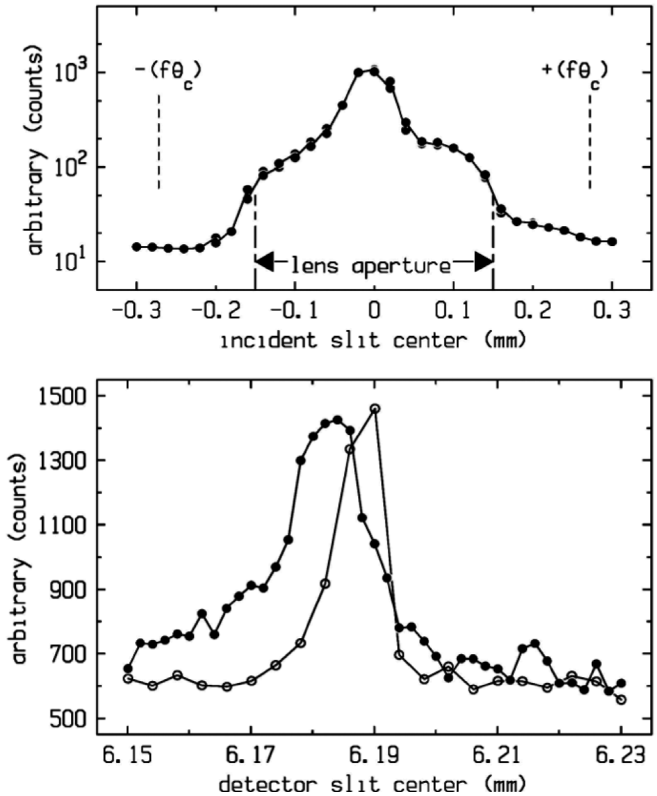

FIG. 3. Top: Scan of incident slit across the lens aperture while monitoring the fluorescence intensity from a lithographically defined target at the focus position. Bottom: Incident slit is fixed near the periphery of the lens, and the Cu target is scanned through the focus. The open circles and the solid circles are two trials with different positions of the incident slit.

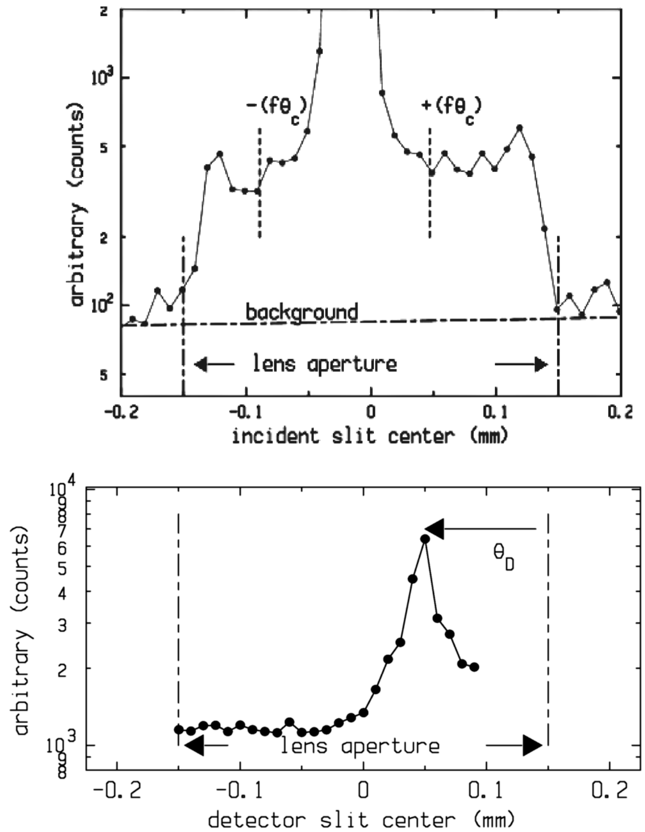

FIG. 4. Top: Scan of incident slit across lens. Note that the dashed lines at $\pm f\theta_c$ are indicated and fall within the lens aperture. Clearly the lens focuses light out to nearly the full aperture of the lens, and exceeds the critical angle. Bottom: A scan in the focal plane, with a slit placed near the periphery of the lens. The arrow labeled θ_D indicates the magnitude of the deflection of the beam with the incident slit placed at the tail of the arrow, and the deflected focused light at the head of the arrow.

the critical angle limit falls well inside the aperture of the lens. For the single lens in Fig. 3(a) it is clear that there is intensity at the focal spot even as the incident slit is scanned across the entire lens, with some extra intensity when the slit is near the optical axis, because the average lens thickness decreases near the optical axis. For the ideal kinoform one expects enhanced intensity near the optic axis and a uniform response out to the edge of the optic aperture. For the compound lens results in Fig. 4(a) it is clear that there is focused light out past the critical angle, as expected. The enhanced intensity on axis is more pronounced due to the transmission through four lenses instead of one lens.

One alternative interpretation of the data from the first type of scan is that the intensity in the focal spot might just be scattered light, and the lens may not in fact be focusing. In order to rule out this alternate interpretation of the data, we also carried out a second type of scan. For this type of scan we position the incident slit near the lens periphery and vary the distance of the copper target from the optical axis (Fig. 2, item F). In Fig. 3(b) we show focusing behavior for the single lens, and in Fig. 4(b) we show the focusing for the compound lens. For the single lens we show two scans corresponding to different locations of the incident slit on the lens. The fact that the two scans do not overlap exactly implies that the knife edge was not at the optimal focal distance. Subsequent measurements on the single lens have verified submicron resolution performance, but these measurements will be discussed at a later date. In Fig. 4(b), the incident slit is held at constant position near the periphery of the compound lens (i.e., $\theta > \theta_c$) and the copper wire is scanned through the focal spot. Figure 4(b) shows a deflection of $91 \pm 1\ \mu$m at a measured distance of 30.5 ± 0.5 mm, which corresponds to a measured deflection of $\sim(1.08 \pm 0.03)\theta_c$, clearly greater than the critical angle limit for a single lens.

Discussion.—The compound lens fabricated did not perform exactly as designed; the expected deflection angle at the measured point was $1.8\theta_c$ and we obtained $(1.08 \pm 0.03)\theta_c$. We attribute this primarily to roughness of the etched walls, and we are studying ways to improve the smoothness and metrology of the etched structures. However, the imperfection of the lens does not detract from the central observation that we have obtained lens function for deflection angles greater than the critical angle. While the deflection angle appears to be only marginally above the critical angle, in fact it far exceeds the fabrication tolerance. The smallest feature that was reliably fabricated on this wafer was of order 1 μm wide and 90 μm deep, and for a single lens would have limited the maximum deflection to $0.3\theta_c$, far smaller than the $1.08\theta_c$ we obtained. Presumably similar methods can be applied to prisms, and refractive beam splitters.

For a NA of $1.08\theta_c$ one could have expected a spot size of 30 nm. However, vibrations at the beam line completely dominate the observed spot size, making it very difficult to make this more direct measurement. We have since verified that these limitations are due to the beam line by comparing results for similar single element lenses at X13B at NSLS and beam line 8-ID at the Advanced Photon Source, and these measurements will be discussed at a later date. For this reason, we chose the experimental approach here of demonstrating basic lens function and measuring the deflection angle.

In principle then, by stacking an array of kinoform lenses one can create numerical apertures as large as desired, with the penalty of insertion loss for each lens. Unfortunately, the kinoform structures we can fabricate are cylindrical lenses that focus in one dimension, to a line focus. We have generated a spot by using a crossed pair arrangement of lenses, similar to the Kirkpatrick-Baez arrangement for reflective optics [19]. This approximation will be valid for small numerical apertures but will break down for larger NAs. Our simulations suggest that this approximation is valid up to NAs of order 0.1. To progress to larger NAs than this we suggest the fabrication of an array of radially symmetric compound kinoform lenses based on the approach outlined here. The fabrication of such radially symmetric lenses remains a challenging problem.

Summary.—It had been suggested theoretically that refractive hard x-ray lenses are limited to a minimum spot size of 10 nm due to the critical angle of the lens material. Here, we have shown experimentally that a compound kinoform lens can focus light with angles that exceed the critical angle limit, thus removing a potential barrier for high resolution refractive and kinoform hard x-ray optics.

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