Resonant Einstein–de Haas Effect in a Rubidium Condensate

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We theoretically consider a spin polarized, optically trapped condensate of ^{87}Rb atoms in $F = 1$. We observe a transfer of atoms to other Zeeman states due to the dipolar interaction which couples the spin and the orbital degrees of freedom. Therefore the transferred atoms acquire an orbital angular momentum. This is a realization of the Einstein–de Haas effect in systems of cold gases. We find resonances which make this phenomenon observable even in very weak dipolar systems, when the Zeeman energy difference on transfer is fully converted to rotational kinetic energy.

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Magnetic effects in ultracold quantum gases have been subject to intense theoretical and experimental studies during recent years. So far most of these investigations have concentrated on short-range interactions as the dominant spin exchange process in spinor condensates of ^{23}Na [1] and ^{87}Rb [2–4]. These interactions result in rich multicomponent physics as demonstrated by the observation of phenomena like magnetic phases [1,3,4], coherent spin dynamics [3–6], domain formation [7], and a magnetically tuned spin-mixing resonance [8]. Including magnetic dipole-dipole interactions would further enhance the richness of these systems and, in particular, their anisotropic nature is expected to add completely new aspects. For relatively weak dipolar interactions phenomena like the Einstein–de Haas effect [9], spontaneous magnetization [10,11], squeezing, and entanglement [11] have been predicted. Most of these studies concentrate on the recently achieved case of a chromium Bose-Einstein condensate [12], as it is commonly believed that these effects are practically unobservable in the widely available alkali condensates due to the smallness of dipolar interactions in these systems. However, as pointed out in [11], for ^{87}Rb in the $F = 1$ state the size of the dipolar interactions as compared to the spin-mixing part of the short-range interactions reaches 10%, such that dipolar effects might be observable in this system.

In this Letter, we show that under the right conditions the dipolar interactions can even dominate the dynamics of a ^{87}Rb spinor condensate, making it a promising candidate for the observation of the Einstein–de Haas effect [13]. Our calculations demonstrate the existence of resonances that amplify the effect of dipolar interactions and can be tuned by the magnetic field or by the trap geometry. They occur when the Zeeman energy fits the rotational kinetic energy per particle. The resonances we find explore a new regime in comparison with that considered in Ref. [9] for a ^{52}Cr condensate, where the dipolar energy (not a kinetic one) is related to the Zeeman energy.

In the second quantization notation, the Hamiltonian of the system we investigate is given by

$$\begin{aligned} H = \int d^3r \Big[& \hat{\psi}_i^\dagger(\mathbf{r})\Big(-\frac{\hbar^2}{2m}\nabla^2 + U_{\rm ext}(\mathbf{r})\Big)\hat{\psi}_i(\mathbf{r}) \\ & - \gamma \hat{\psi}_i^\dagger(\mathbf{r})\mathbf{B}\mathbf{F}_{i,j}\hat{\psi}_j(\mathbf{r}) + \frac{c_0}{2}\hat{\psi}_j^\dagger(\mathbf{r})\hat{\psi}_i^\dagger(\mathbf{r})\hat{\psi}_i(\mathbf{r})\hat{\psi}_j(\mathbf{r}) \\ & + \frac{c_2}{2}\hat{\psi}_k^\dagger(\mathbf{r})\hat{\psi}_i^\dagger(\mathbf{r})\mathbf{F}_{ij}\mathbf{F}_{kl}\hat{\psi}_j(\mathbf{r})\hat{\psi}_l(\mathbf{r})\Big] + \frac{1}{2} \\ & \times \int d^3r d^3r' \hat{\psi}_k^\dagger(\mathbf{r})\hat{\psi}_i^\dagger(\mathbf{r}')V^d_{ij,kl}(\mathbf{r}-\mathbf{r}')\hat{\psi}_j(\mathbf{r}')\hat{\psi}_l(\mathbf{r}), \end{aligned} \tag{1}$$

where repeated indices (each of them going through the values $+1$, 0, and -1) are to be summed over. The field operator $\hat{\psi}_i(\mathbf{r})$ annihilates an atom in the hyperfine state $|F = 1, i\rangle$ at point $\mathbf{r}$. The first term in (1) is the single-particle Hamiltonian (H_0) that consists of the kinetic energy part (with m being the mass of an atom) and the trapping potential $U_{\rm ext}(\mathbf{r})$. The second term describes the interaction with the magnetic field $\mathbf{B}$ with γ being the gyromagnetic coefficient which relates the effective magnetic moment with the hyperfine angular momentum ($\boldsymbol{\mu} = \gamma\mathbf{F}$). The terms with coefficients c_0 and c_2 describe the spin-independent and spin-dependent parts of the contact interactions, respectively—c_0 and c_2 can be expressed with the help of the scattering lengths a_0 and a_2 which determine the collision of atoms in a channel of total spin 0 and 2. One has $c_0 = 4\pi\hbar^2(a_0 + 2a_2)/3m$ and $c_2 = 4\pi\hbar^2(a_2 - a_0)/3m$ [14], where $a_0 = 5.387$ nm and $a_2 = 5.313$ nm [15]. $\mathbf{F}$ are the spin-1 matrices. Finally, the last term describes the magnetic dipolar interactions. The interaction energy of two magnetic dipole moments $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ positioned at $\mathbf{r}$ and $\mathbf{r}'$ equals

$$V_d = \frac{\boldsymbol{\mu}_1\boldsymbol{\mu}_2}{|\mathbf{r}-\mathbf{r}'|^3} - 3\frac{[\boldsymbol{\mu}_1(\mathbf{r}-\mathbf{r}')][\boldsymbol{\mu}_2(\mathbf{r}-\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|^5} \tag{2}$$

and since $\boldsymbol{\mu} = \gamma \mathbf{F}$ one has $V^d_{ij,kl}(\mathbf{r} - \mathbf{r}') = \gamma^2 \mathbf{F}_{ij}\mathbf{F}_{kl}/|\mathbf{r} - \mathbf{r}'|^3 - 3\gamma^2[\mathbf{F}_{ij}(\mathbf{r} - \mathbf{r}')][\mathbf{F}_{kl}(\mathbf{r} - \mathbf{r}')]/|\mathbf{r} - \mathbf{r}'|^5$.

The equation of motion reads

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \hat{\psi}_1 \\ \hat{\psi}_0 \\ \hat{\psi}_{-1} \end{pmatrix} = (\mathcal{H}_c + \mathcal{H}_d) \begin{pmatrix} \hat{\psi}_1 \\ \hat{\psi}_0 \\ \hat{\psi}_{-1} \end{pmatrix}, \tag{3}$$

where the operator $\mathcal{H}_c$ originates from the the first four terms in the Hamiltonian Eq. (1) whereas $\mathcal{H}_d$ corresponds to the dipole-dipole interactions [the last term in Eq. (1)]. The diagonal part of $\mathcal{H}_c$ is given by $\mathcal{H}_{c11} = H_0 + (c_0 + c_2)\hat{\psi}_1^\dagger \hat{\psi}_1 + (c_0 + c_2)\hat{\psi}_0^\dagger \hat{\psi}_0 + (c_0 - c_2)\hat{\psi}_{-1}^\dagger \hat{\psi}_{-1}$, $\mathcal{H}_{c00} = H_0 + (c_0 + c_2)\hat{\psi}_1^\dagger \hat{\psi}_1 + c_0 \hat{\psi}_0^\dagger \hat{\psi}_0 + (c_0 + c_2)\hat{\psi}_{-1}^\dagger \hat{\psi}_{-1}$, $\mathcal{H}_{c-1-1} = H_0 + (c_0 - c_2)\hat{\psi}_1^\dagger \hat{\psi}_1 + (c_0 + c_2)\hat{\psi}_0^\dagger \hat{\psi}_0 + (c_0 + c_2)\hat{\psi}_{-1}^\dagger \hat{\psi}_{-1}$. The off-diagonal terms that describe the collisions not preserving the projection of spin of each atom (although the total spin projection is conserved) equal $\mathcal{H}_{c10} = c_2 \hat{\psi}_{-1}^\dagger \hat{\psi}_0$, $\mathcal{H}_{c0-1} = c_2 \hat{\psi}_0^\dagger \hat{\psi}_1$. Moreover, $\mathcal{H}_{c1-1} = 0$. On the other hand, for the $\mathcal{H}_d$ term one has $\mathcal{H}_{dij} = \int d^3 r' \psi_n^\dagger(\mathbf{r}') V^d_{ij,nk} \psi_k(\mathbf{r}')$. This term is responsible for the change of total spin projection of colliding atoms. It turns out that when two atoms interact the total spin projection (ΔM_F) can change at most by 2. In particular, the diagonal elements of $\mathcal{H}_d$ lead to the processes with $\Delta M_F = \pm 1$. In addition to such processes, the off-diagonal terms of $\mathcal{H}_d$ introduce the interaction that changes the spin projection ΔM_F by ± 2. It happens when both atoms initially in the same state go simultaneously to the nearest (in a sense of magnetic number m_F) state or in the case when atoms in different but neighboring components transfer to the states shifted in number m_F by $+1$ or -1. There is no way for the atom to be transferred directly from the $m_F = 1$ to the $m_F = -1$ state; therefore, the populating of the $m_F = -1$ component is a second order process.

Hence, the dipolar interaction does not conserve the projection of total spin of two interacting atoms, nor is the projection of total orbital angular momentum preserved [see (4)]. However, the dipolar interaction couples the spin and the orbital motion of atoms as revealed by the last relation in (4)

$$\begin{gathered} [V_d, F_{1z} + F_{2z}] \neq 0, \qquad [V_d, L_{1z} + L_{2z}] \neq 0, \\ [V_d, L_{1z} + L_{2z} + F_{1z} + F_{2z}] = 0. \end{gathered} \tag{4}$$

Therefore, going to $m_F = 0, -1$ states atoms acquire the orbital angular momentum and start to circulate around the center of the trap. This is the realization of the famous Einstein–de Haas effect in cold gases.

To solve Eq. (3) we neglect the quantum fluctuations and replace the field operator $\hat{\psi}_i(\mathbf{r})$ by an order parameter $\psi_i(\mathbf{r})$ for each component and apply the split-operator method. All integrals appearing in $\mathcal{H}_{dij}$ are the convolutions and we use the Fourier transform technique to calculate them. To find analytical formulas for the Fourier transforms of the components of the convolutions that do not change during the evolution we apply the regularization procedure described in Ref. [16].

The gyromagnetic coefficient for ^{87}Rb atoms in an $F = 1$ hyperfine state is positive and equals $\gamma = \frac{1}{2}\mu_B/\hbar$. We prepare an initial state of the condensate as the one with all magnetic moments aligned along the magnetic field; i.e., all atoms are in $m_F = 1$ component. To this end, we run the mean-field version of Eq. (3) in imaginary time while the magnetic field is turned on (and equal to $B = 0.73$ mG for a spherically symmetric trap with the frequency $\omega = 2\pi \times 100$ Hz). Then we reverse the direction of the magnetic field and look for the transfer of atoms to other Zeeman states.

Our starting condition (all atoms in $m_F = 1$ state) suppresses the short-range spin dynamics and initially the $m_F = 1$ state is depleted only due to the dipolar interaction. Usually we observe a small number of atoms going from the Zeeman state $m_F = 1$ to the $m_F = 0, -1$ states. However, on resonance (see Fig. 1) the transfer to the other states can be of the order of the initial population of $m_F = 1$ component. This transfer is as large as in the case of chromium condensate [9] despite the fact that the dipolar energy ($\mu^2 n$, where n is the atomic density) is approximately 100 times smaller. The only difference is that the time scale corresponding to the maximal transfer is about 100 ms, i.e., 100 times longer than for chromium. This can be understood as follows. For ^{87}Rb the dipolar energy is the

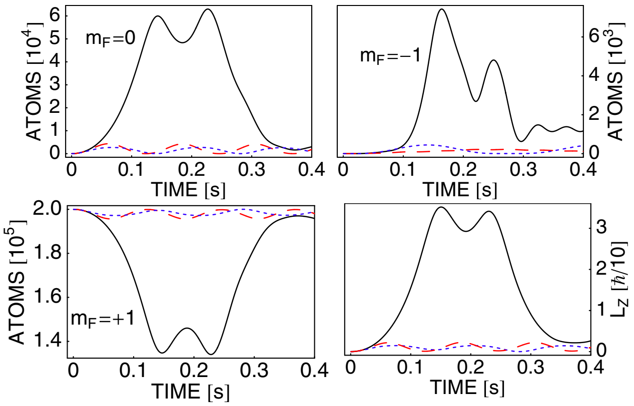

FIG. 1 (color online). Transfer of atoms to $m_F = 0, -1$ Zeeman states as a function of time. Initially, $N = 2 \times 10^5$ atoms were prepared in the $m_F = 1$ component in a spherically symmetric trap with the frequency $\omega = 2\pi \times 100$ Hz. The residual magnetic field equals -0.029 mG (solid black lines, on-resonance case) and $B = -0.015$ mG, -0.036 mG (blue dotted lines and red dashed lines, respectively, off-resonance case). The lower panel shows the number of atoms in $m_F = 1$ state and the time dependence of the orbital angular momentum per atom. The maximum of the latter ($\approx 0.35\hbar$) is consistent with the transfer of 30% of atoms to $m_F = 0$ state (where a singly quantized vortex is formed) and 3% of atoms to $m_F = -1$ state with the doubly quantized vortex.

smallest energy of the problem and is only a perturbation as compared to the kinetic rotational energy and the Zeeman energy. An efficient population transfer from $m_F = 1$ to $m_F = 0$ due to the dipolar interactions is only possible when the total energies (the sum of mean-field, trap, kinetic, and Zeeman energies) in these states are approximately equal. Figure 2(a) clearly shows that the total energy tends to equalize on resonance, which is not true in the off-resonant case. Numerics also shows [Fig. 2(b)] that the above condition can be fulfilled only when the Zeeman energy fits the rotational kinetic energy: $\mu B_{\mathrm{res}} = \epsilon_{\mathrm{rot}}$. It means that the resonant magnetic field is inversely proportional to the magnetic moment of an atom,

$$B_{\mathrm{res}} \propto 1/\mu. \tag{5}$$

Surprisingly, the smaller the atomic magnetic moment the larger the value of the resonant magnetic field. For chromium condensate, however, the dipolar energy is larger than the kinetic energy. Therefore, in this case the resonance condition should be derived by relating the Zeeman and the dipolar (not the kinetic one) energies, i.e., $\mu B_{\mathrm{res}} = \mu^2 n$ [9]. This results in a condition $B_{\mathrm{res}} \propto \mu$ which differs qualitatively from (5).

The maximal transfer is reached at a time which is of the order of the characteristic time scale determined by the dipolar interactions ($\hbar/\mu^2 n$). Since the magnetic moment of the ^{87}Rb atom is 12 times smaller than that of ^{52}Cr we have to wait hundreds of milliseconds (not a fraction of a millisecond as in Ref. [9]) to see the action of resonance.

Figure 2 illustrates the ideas just discussed. In the left frame the total energies of the $m_F = 1, 0$ components are plotted as a function of time both in on- and off-resonance cases showing that the resonances we find are dynamical phenomena. The transfer gets maximal when the energies approach each other (perhaps crossing both curves would require the dynamical tuning of the resonance by changing the magnetic field). On the contrary, almost no transfer of atoms occurs when the energy curves keep away. Simultaneously, the right frame proves that on resonance the Zeeman energy (in fact, together with the kinetic energy) is transferred to the rotational kinetic energy of atoms in the $m_F = 0$ component.

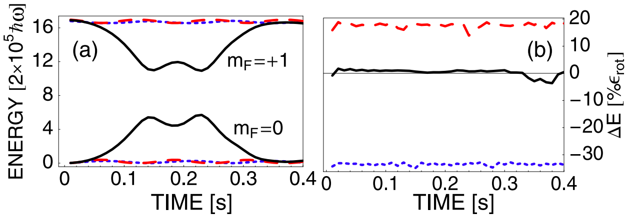

FIG. 2 (color online). Total energy for $m_F = 1, 0$ components as a function of time (left frame). All parameters are the same as in Fig. 1. The resonance happens for $B = -0.029$ mG (solid black lines) whereas the off-resonance cases are represented by $B = -0.015$ mG (blue dotted lines) and $B = -0.036$ mG (red dashed lines). The right frame shows $\Delta E = (E_{\mathrm{kin}}^{+1}/N_{+1} + \mu B - \epsilon_{\mathrm{rot}})/\epsilon_{\mathrm{rot}}$, where $\epsilon_{\mathrm{rot}} = E_{\mathrm{rot}}/N_0$.

Huge transfer of atoms to $m_F = 0, -1$ states is the realization of the Einstein–de Haas effect in cold gases. Numerical analysis of phases of spinor components shows that the vortices are generated in $m_F = 0, -1$ spin states and atoms in $m_F = 0, -1$ rotate around the quantization axis. In $m_F = 0$ and $m_F = -1$ components singly and doubly quantized vortices are formed, respectively, as a result of total angular momentum conservation. The density in these states is fragmented and the number of rings results from the symmetry of dipolar interaction, Fig. 3. Similar fragmentation was already predicted in the case of ^{52}Cr condensate in Ref. [9].

Figure 4 (upper frame) shows the position and the width of the resonance displayed in Fig. 1. Similar behavior is observed when the value of the reversed magnetic field is kept constant and the trap geometry is changed (lower frame). Here, the maximal transfer of atoms is obtained in a cigar trap with the aspect ratio $\omega_\rho/\omega_z = 4$ with almost 50% efficiency at $B = -0.073$ mG. The inset shows the resonance at an experimentally easier to control value of magnetic field $B = 0.3$ mG but still detectable number of atoms in $m_F = 0$ state. Note that in all these cases the maximal atomic density is of the order of 10^{14} cm^{-3} making the three-body losses low enough and hence allowing the observation of population transfer on a time scale of the order of hundreds of milliseconds. To understand quantitatively the resonance we start from the condition discussed earlier: $\mu B_{\mathrm{res}} = E_{\mathrm{rot}}/N_0$, where E_{rot} is the rotational energy of the $m_F = 0$ component which is assumed to be a singly quantized vortex, given within the Thomas-Fermi approximation by $\psi_0(\rho, \phi, z) = \{[\lambda - m\omega^2(\rho^2 + z^2)/2 - \hbar^2/(2m\rho^2)]/c_0\}^{1/2} e^{i\phi}$. Here, the chemical potential λ is obtained by the requirement that the number of atoms in the ψ_0 state equals N_0. One can tune to the resonance in two ways: (i) by adjusting the magnetic field B, and (ii) by changing the trap geometry that influences the rotational energy entering the resonance condition and keeping the magnetic field constant. The curve resulting from the condition $\mu B_{\mathrm{res}} = E_{\mathrm{rot}}/N_0$ in the case of the spherically symmetric trap is plotted in Fig. 5. The resonant

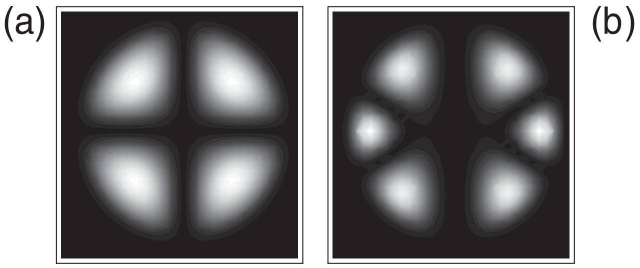

FIG. 3. Density in the xz plane (the z axis goes vertically) of $m_F = 0$ (a) and $m_F = -1$ (b) spin components in on-resonance case at 140 ms.

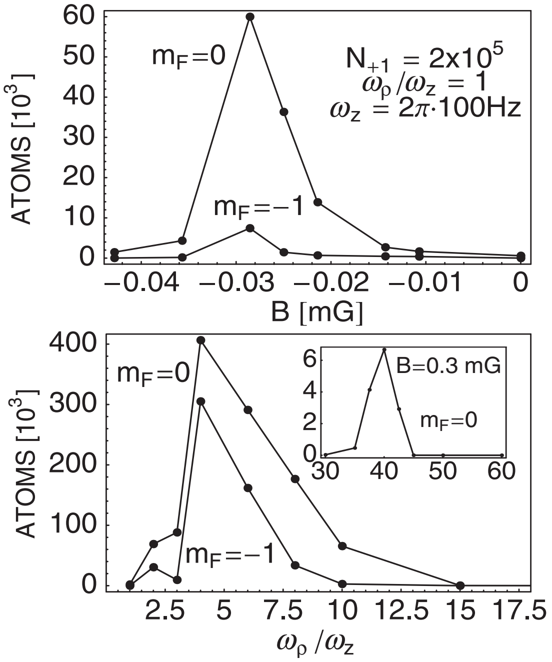

FIG. 4. Maximal transfer of atoms to $m_F = 0, -1$ states as a function of the residual magnetic field (upper frame) and the trap geometry (lower frame). For lower frame $\omega_z = 2\pi \times 100$ Hz, $B = -0.073$ mG, and $N_{+1} = 8 \times 10^5$. Inset shows the resonance at $B = -0.3$ mG for $\omega_z = 2\pi \times 20$ Hz and $N_{+1} = 10^5$.

magnetic field is uniquely related to the number of atoms in the $m_F = 0$ component. However, this number depends on the initial number of atoms in the $m_F = 1$ state and can be found only numerically. To verify the resonance condition we compare the numerical results (marked by solid circles and with additional information regarding the initial number of atoms in the $m_F = 1$ component) with the Thomas-Fermi results. The agreement is good, for example, when initially one has $N_1 = 2 \times 10^5$ atoms in the $m_F = 1$ component (in this case the maximal transfer of atoms to the $m_F = 0$ state equals 6×10^4). The solid line in Fig. 5 gives for the value of $N_0 = 6 \times 10^4$ the value of the resonant magnetic field ≈ 0.03 mG which is very close to the numerical value (see upper frame in Fig. 4). For other systems, e.g., ^{52}Cr, the condition $\mu B_{\mathrm{res}} = E_{\mathrm{rot}}/N_{m_F}$ suggests that the value of the resonant magnetic field is even $\approx$10 times smaller since $\mu_{\mathrm{Cr}}/\mu_{\mathrm{Rb}} = 12$ and E_{rot}/N_{-2} for chromium looks similar to E_{rot}/N_0 in the rubidium case.

In conclusion, we have shown the existence of dipolar resonances in rubidium spinor condensates. The resonances occur when the Zeeman energy of atoms in the $m_F = 1$ component, while transferring to the $m_F = 0$ state, is fully converted to the rotational kinetic energy. This is so far an unexplored regime. Symmetries of the dipolar interaction force the atoms in $m_F = 0, -1$ states to circulate around the quantization axis and form singly and doubly quantized vortices, respectively. Therefore, dipolar resonances is a route to the observation of the Einstein–de Haas effect (as well as other phenomena related to the dipolar interaction) in weak dipolar systems.

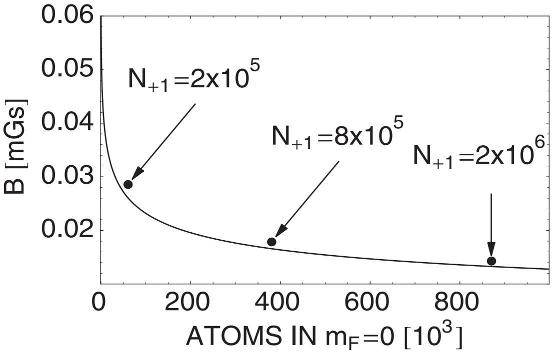

FIG. 5. Comparison between numerics (solid circles) and the Thomas-Fermi approximation. Solid line indicates the value of the magnetic field at resonance (for spherically symmetric trap) as a function of number of atoms in the $m_F = 0$ state.

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