Reentrant Quantum Hall Effect and Anisotropic Transport in a Bilayer System at High Filling Factors

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We report on the measurements of the quantum Hall effect states in double quantum well structures at the filling factors $\nu = 4N + 1$ and $\nu = 4N + 3$, where N is the Landau index number, in the presence of the in-plane magnetic field. The quantum Hall states at these filling factors vanish and reappear several times and exhibit anisotropy. Repeated reentrance of the transport gap occurs due to the periodic vanishing of the tunneling amplitude in the presence of the in-plane field. Anisotropy demonstrates the existence of the stripes in the ground states.

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Double quantum wells or bilayer systems consist of two parallel quantum wells with high mobility electron gas separated by the tunneling barrier. The quantum tunneling induces hybridization of the subband energies and introduces subband splitting energy Δ_{SAS} with a typical value of 0.1–1 meV [1]. Such a symmetric-antisymmetric energy gap is controllable through the height and the width of the tunneling barrier. When a perpendicular magnetic field is applied, the quantum Hall states are formed, and the minima in the resistance at total Landau filling factors $\nu = 2N + 1$ and $\nu = 2N + 3$, where N is Landau level number, are ascribed to Δ_{SAS} energy gap. The quantum Hall states at $\nu = 2N + 1$ and $\nu = 2N + 3$ are strongly modified by the presence of the magnetic field component parallel to the plane of the layers. For example, it has been predicted that in high Landau levels the tunneling amplitude oscillates with the in-plane magnetic field [2]. Such nonmonotonic behavior of the tunneling gap leads to the repeated reentrance of the quantum Hall effect and allows one to probe the phase transitions in a bilayer system in the presence of the in-plane field.

The quantum Hall bilayer systems attract much attention because of the experimental and theoretical evidence for a coherent quantum Hall state at $\nu = 1$, which is driven by Coulomb interaction and occurs even for the vanishing symmetric-antisymmetric energy gap [3–5]. In principle, the interlayer coherent state could be observed in high Landau levels $N > 1$ at $\nu = 4N + 1$ and $\nu = 4N + 3$. Recently, it was recognized that not only uniform coherent states are energetically favored at small layer separations, but also anisotropic coherent states, such as stripe states, are possible in the perpendicular or tilted magnetic field [6–10]. Stripe states in a single well system in a perpendicular magnetic field have been predicted [11] and observed in transport experiments for quantum Hall systems at half-filled Landau filling factors [12]. In this phase, electrons cluster into parallel stripes of the alternative integer filling factor, and longitudinal resistance exhibits large anisotropy.

Stripe states in bilayer quantum Hall systems can be very different from stripe phases in a single layer. In a perpendicular field it has been argued that as the separation between layers is increased from zero, the bilayer quantum Hall state transforms from a uniform interlayer coherent state to the isospin coherent stripe state, and then into a modulated stripe state [7]. In the coherent stripe state the charge distribution oscillates between layers, while the total density is fixed. In-plane magnetic field induces additional stripe phases in bilayer systems at integer total filling factors $\nu = 4N + 1$ [6,8–10].

In this Letter we report on the observations of the periodic vanishing of the transport gap at $\nu = 4N + 1$ and $\nu = 4N + 3$ with tilt angle, which results in the reentrance of the quantum Hall bilayer states in high mobility GaAs double well structures. We observe strong anisotropy of the resistance induced by the in-plane magnetic field.

The samples are symmetrically doped GaAs double quantum wells separated by a $Al_xGa_{x-1}As$ tunneling barrier with different widths d_b varying from 1.4 to 5 nm. The width W of the GaAs well is 14 nm. Samples have high total density 9×10^{11} cm^{-2} (4.5×10^{11} cm^{-2} in each layer), and average mobility $\mu = 0.97 \times 10^6$ cm^2/V s. Both layers are shunted by Ohmic contacts. The density is 3–4 times larger than the density in the samples studied previously [4], and this might explain why the effect reported here has not been observed in earlier experiments. The energy separation between bonding and antibonding subbands varies from 3.2 to 0.4 meV. We measured longitudinal and Hall resistances for different tilt angles Θ at $T = 50$ mK up to 15 T using conventional ac-locking techniques with a bias current of 10 nA. Both the Hall bar and square van der Pauw geometries were studied.

First, we focus on the observation of the oscillations of the tunneling gap with tilt angle. Figure 1 shows the phase diagram, or the plot of the longitudinal resistance R_{xx} as a function of the perpendicular component of the magnetic field B and the tilt angle Θ for a Hall bar containing double well structure with $d_b = 2.0$ nm. We may see that the minima in the resistance corresponding to the filling factors $\nu = 4N + 1$ and $\nu = 4N + 3$ vanish when the magnetic field is tilted. The vanishing of the resistance minima occurs in a small range of the tilt angle and is accompanied by vanishing Hall quantization. The value of the tilt angle, when the minima starts to disappear, is slightly different for different N. When Θ increases, the energy gap at $\nu = 4N + 1$ and $\nu = 4N + 3$ vanishes and is reestablished several times for $N = 2, 3, 4, 5 \ldots$ From this map we measure the resistance at fixed filling factor or the value of the perpendicular magnetic field as a function of the in-plane field. Figure 2 shows the dependence of R_{xx} on parallel field for the 2.0 nm barrier sample. We may see a nearly periodic oscillation of the longitudinal resistance with in-plane field with periodicity proportional to the filling factor. Similar dependencies were obtained for $d_b = 1.4$ nm and $d_b = 3.0$ nm double quantum wells. Reentrance of the quantum Hall state at the filling factors $\nu = 4N + 1$ and $\nu = 4N + 3$ originates from the oscillations of the single-particle tunneling gap Δ_{SAS}. Below we focus on the results obtained for $\nu = 4N + 1$, since the results for $\nu = 4N + 3$ are almost identical. We determine Δ_{SAS} from the thermally activated behavior of the resistance $R_{xx} \sim \exp(-\Delta_{SAS}/2kT)$ for different tilt angles. Indeed, that tunneling gap behavior is correlated with the behavior of the resistance with in-plane field (see Fig. 2): a resistance peak corresponds to the vanishing of the tunneling gap, and the resistance minimum corresponds to the maximum of the tunneling amplitude. In the tight-binding approximation it has been predicted that the tunneling amplitude is given by [2,6]

$$T_N = \Delta_{SAS} \exp\left(-\frac{Q^2 l_\perp^2}{4}\right) L_N^0\left(\frac{Q^2 l_\perp^2}{2}\right), \quad (1)$$

where L_N^0 is a generalized Laguerre's polynomial, the wave vector Q is defined as $Q = d/l_\parallel^2$, where $l_\perp = \sqrt{\hbar c/eB_\perp}$ and $l_\parallel = \sqrt{\hbar c/eB_\parallel}$ are magnetic lengths associated with the perpendicular and parallel magnetic field, respectively. It is worth noting that in realistic samples, when the finite layer width is taken into account, d should be substituted by $d_b + W$. For $N = 1, 2, 3, \ldots$ the tunneling amplitude oscillates with $B_\parallel$ and becomes negative in some range of the tilt angle (the energy per particle is simply proportional to the absolute value of the tunneling amplitude). We may speculate here that this effect was absent in previous experiments in samples with lower density [3] only because of the smaller energy gaps (cyclotron and Zeeman) at corresponding filling factors, which leads to strong Landau level mixing. Figure 2 shows a comparison between our data and Eq. (1). It can be seen, that indeed the tunneling amplitude vanishes, when the longitudinal resistance has a maximum. However, we have to note that the agreement is obtained when we substitute the wave vector Q by $Q^* = \alpha Q$, where $\alpha = 0.7$ for filling factors 5,9,13 and $\alpha = 0.6$ for $\nu = 17$. We may see that for $\alpha = 1$ the tunneling amplitude vanishes at lower in-plane field. This discrepancy might be due to the limitation of the tight-binding model. The activation energy gap decreases with increasing filling factor, and finally, the symmetric-

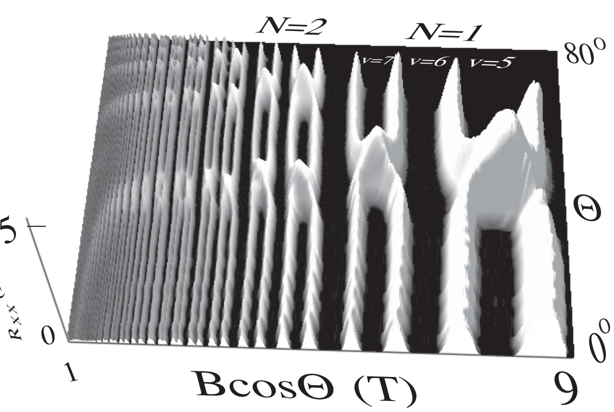

FIG. 1. Experimentally determined plot of the resistance in the magnetic field -tilt angle plane for double well structure with barrier thickness $d_b = 2$ nm. Filling factors determined from Hall resistance are labeled. Filling factors $\nu = 4N + 1$ and $\nu = 4N + 3$ correspond to the tunneling gap.

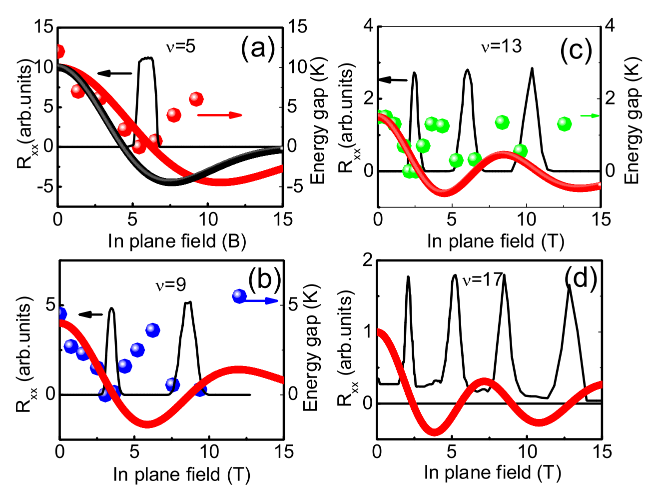

FIG. 2 (color online). Dependence of R_{xx} at $\nu = 5$, 9, 13, 17 on an in-plane magnetic field at $T = 50$ mK for 2.0 nm barrier bilayer structure. Circles show the activation energy gaps as a function of the in-plane field. The thick line (red) show the variation of the tunneling amplitude T_N calculated from the Eq. (1) with the parallel magnetic field for filling factors $\nu = 5$, 9, 13, 17, corresponding to Landau levels $N = 1$ (a), 2(b), 3(c), and 4(d) for $Q^* = \alpha Q$, where $\alpha = 0.7$ ($\nu = 5$, 9, 13) and $\alpha = 0.6$ ($\nu = 17$). The thick black line shows T_N for $\alpha = 1$.

asymmetric splitting vanishes at low field in contrast to the magnetic field independent behavior expected for the single-particle picture. Indeed, such behavior has been observed in all previous studies [3].

The states in the quantum Hall bilayers at total filling factor $\nu = 1$ are best described in the language of ferromagnetism [5,13], where the right (left) well is associated with a pseudospin up (down) state. When the electron might be in either layer, the system is described by a linear combination of pseudoispin up and down states. The resulting state is aligned along arbitrary direction in the pseudospin XY plane, and the system has the symmetry of an easy-plane ferromagnet [5]. Most previous studies of quantum Hall ferromagnetism have focused on the pseudospin and spin alignment at $\nu = 1$ [4] and fractional regime [14]. However, the situations becomes even richer, when in addition to the pseudospin, real spin and orbital quantum numbers are included [13].

Tuning of the tunneling gap by application of a parallel magnetic field might be used for testing of the quantum Hall states in a bilayer system. Spontaneous interlayer coherence occurs at a small $d/l_\perp$ ratio, which is given by $d/l_\perp = 0.739d(100\ \text{Å})\sqrt{n_s(10^{11}\ \text{cm}^{-2})}/\sqrt{\nu}$. In our structure with barrier thickness 2 nm we have $d/l_\perp = 1.7$ for $\nu = 5$. The uniform interlayer coherent state at $\nu = 5$ is predicted for $d/l_\perp < 0.8$ in Ref. [7] in the limit $\Delta_{\text{SAS}} \rightarrow 0$. The wider barrier sample with $d_b = 5$ nm exhibits no quantum Hall minima at $\nu = 4N + 1$. The state at $\nu = 5$ in this sample is not a coherent state, since the layer separation $d/l_\perp = 2.0$ is too large. The ratio $d/l_\perp$ decreases with decreasing magnetic field. We attribute the absence of the coherent state in the $d_b = 5$ nm sample at high filling factors to the mixing between Landau levels.

At a larger $d/l_\perp$ ratio, the uniform coherent phase is unstable to the formation of the anisotropic state [7–9]. Several distinct stripe states have been predicted in quantum Hall bilayers. For example, Fig. 2 of Ref. [8] demonstrates 5 quantum phases classified by the behaviors of the expectation values of the their isospin components (note, that Cote *et al.* [6] use a different terminology for anisotropic and uniform state denomination). However, it is worth noting that, as was indicated by the authors [8], the energies of the stripe states are very close, so when the finite layer width is taken into account, some of these states may be favored energetically. We should emphasize here that the in-plane field plays two important roles. First, it destroys the single-particle tunneling gap and allows formation of states with interlayer coherence. Second, the in-plane field induces the transport anisotropy. For example, it stabilizes the preferred direction or nematic phase.

In order to probe the anisotropic states associated with interlayer coherence, we perform the tilted field measurements in samples with square van der Paw geometry. The resistance anisotropy is strongly enhanced in this geometry, since an anisotropy in the resistivities causes an inhomogeneous current density distribution in the sample [15].

In Fig. 3(a) we present measurements of the longitudinal resistance for the current flowing in the perpendicular directions and for different tilt angles. No significant anisotropy is observed in the perpendicular magnetic field. However, in the tilted field the resistance shows pronounced anisotropy at $\nu = 5$ when the tunneling gap vanishes. Figure 3(b) shows the detailed temperature dependence of the longitudinal resistance for two direction of the current. Following the literature, we use the expressions "hard-axis" and " easy-axis" to specify the directions of the current flow along the axis with high and low resistances, respectively. We may see that the hard-axes resistance R_{yy} increases with decreasing temperature, whereas R_{xx} remains constant. Figure 3(c) shows low field R_{xx} and R_{yy} traces for different tilt angles. We may see that the resistance anisotropies at large filling factors occurs at sufficiently high tilt angles. Figure 4 shows the summary of the tilted field measurements for two samples with 1.4 and 2.0 nm barrier widths. We measure resistance anisotropy at half integer filling factors when the longitudinal resistance exhibits peaks, and at $\nu = 4N + 1$, when the tunneling gap vanishes in the presence of the in-plane field. We may see that the anisotropy exhibits nonmonotonic behavior: it increases periodically following the vanishing of the tunneling gap. Note that anisotropy occurs not only at filling factors $\nu = 2N + 1$, but at half filling factors as well. Since the alternative explanation of the anisotropic trans-

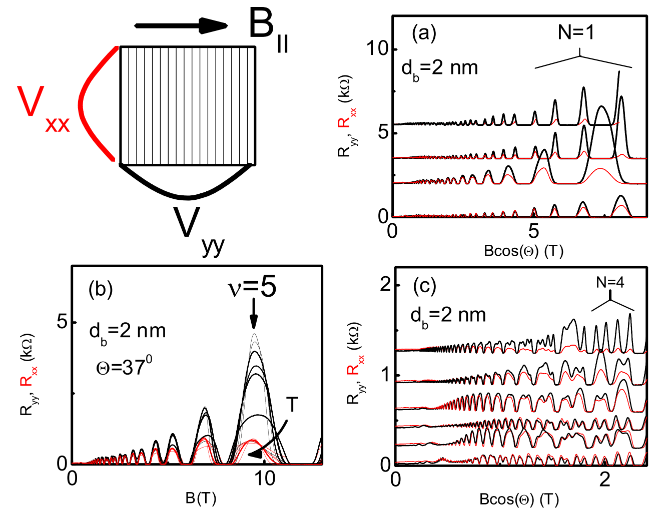

FIG. 3 (color online). (a) Longitudinal resistance measurements in the van der Paw geometry for sample with barrier thickness $d_b = 2.0$ nm with current directed along hard axes (thick black lines) and easy axis (red lines) for different tilt angles Θ (from bottom to top): 0°, 38.7°, 50.6°, 57.6°. (b) R_{yy} (thick black lines) and R_{xx} (thin red lines) at $\Theta = 37°$ as a function of the magnetic field for different temperatures T (K): 1.3, 0.7, 0.57, 0.45, 0.25, 0.1. (c) Low field part of the resistance for different Θ (from bottom to top): 0°, 33.9°, 48.5°, 60.3°, 72.2°, 80.3°.

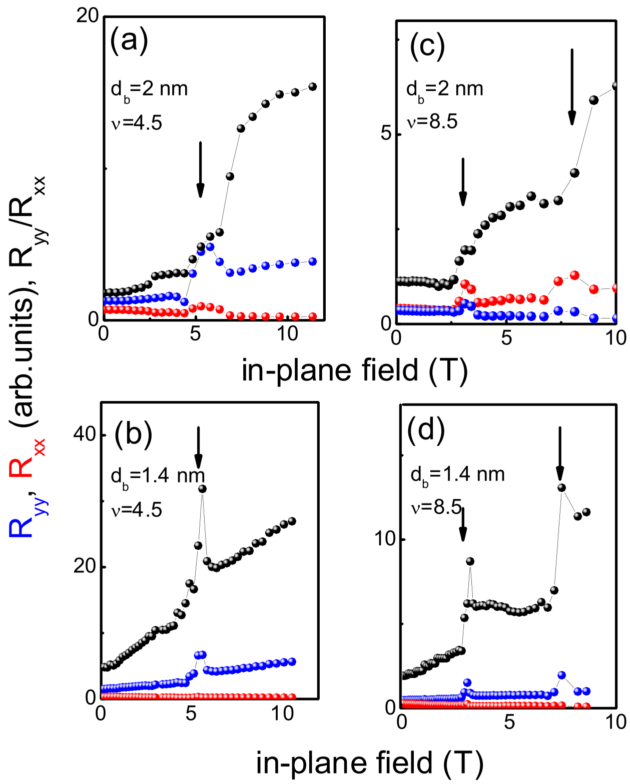

FIG. 4 (color online). Longitudinal resistance measurements in the van der Paw geometry for sample with barrier thickness $d_b = 1.4$ and 2.0 nm with current directed along hard axes R_{yy} (blue circles) and easy-axis R_{xx} (red circles) and the ratio R_{yy}/R_{xx} (black circles) as a function of the in-plane field for different filling factors. Arrows indicate the region, when the tunneling gap vanishes. In this region the resistances R_{xx} and R_{yy} are measured at integer filling factors $\nu = 4N + 1$.

port in a bilayer system might be the stripe phase in a single isolated layer, let us indicate the difference between our data and previous observations [12,16]. First, there is the clear correlation between oscillation of the tunneling gap and anisotropy coefficient, as shown in Fig. 4. For isolated layers we should observe a monotonic increase of the anisotropy with in-plane field [16]. Second, the anisotropy dramatically decreases with the increase of the barrier thickness: for the double well with $d_b = 3$ nm, only a small anisotropy is observed, while for the sample with $d_b = 5$ nm, which can be considered as two isolated wells, no clear anisotropy is found. This observation is crucial, since it supports the signature of the interlayer coherent anisotropic states in the sample with $d_b = 1.4$ nm and 2 nm barriers. Observation of the anisotropic states in a single isolated well requires extremely high mobility [16], such as $\mu \approx 1 \times 10^7$ cm^2/V s, which is 10 times larger than in our samples.

We are not able to distinguish between the 5 quantum anisotropic phases, predicted in Refs. [6,8]. However, note that the most favorable configuration is that of the stripes aligned with the in-plane field. We find the stripes aligned perpendicular to the parallel magnetic field. It is consistent with an "isospin Skyrmion stripe phase" (Fig. 1c in Ref. [8]).

Anisotropy of states at $\nu = 9/2 \ldots 13/2 \ldots 17/2 \ldots$ requires further theoretical study. The bilayer system with total half-odd integer filling factor can be regarded as two separate quantum Hall states with filling factors $\nu = 1/4$ in the absence of the tunneling. However, we may speculate, that interlayer interaction can lead to modulated stripe state or anisotropic Wigner crystal at these filling factors.

In conclusion, we observe repeated reentrance of the quantum Hall state in bilayer systems at filling factors $\nu = 4N + 1$ and $\nu = 4N + 3$ in the presence of the in-plane magnetic field due to the oscillations of the symmetric-asymmetric gap. When the gap vanishes, the transport becomes anisotropic. The anisotropy persist at half-odd filling factors, where bilayer quantum Hall states are recovered with an increase of the tilt angle.

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