Rotating Vortex Dipoles in Ferromagnets

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(Received 5 April 2007; revised manuscript received 26 May 2007; published 11 September 2007)

Vortex-antivortex pairs are spontaneously created in magnetic elements. In the case of opposite vortex polarities the pair has a nonzero topological charge, and it behaves as a rotating vortex dipole. We find theoretically and confirm numerically its energy as a function of angular momentum and the associated rotation frequencies. The annihilation process of the pair changes the topological charge while the energy is monotonically decreasing. The change of topological charge affects the dynamics profoundly. We finally discuss the implications of our results for Bloch point dynamics.

DOI: 10.1103/PhysRevLett.99.117202 PACS numbers: 75.10.Hk, 11.27.+d, 75.60.Ch, 75.75.+a

Magnetic vortices have a nontrivial topology which is responsible for their stability and for the fact that they are central in theoretical studies for the micromagnetic description of two-dimensional (2D) magnets and thin films [1]. Similar topological structures arise in many physical systems, as in quantum field theory [2]. However, experimental evidence for their existence and properties had been rare. The situation has dramatically changed in the last years. It was realized that a disk-shaped mesoscopic magnetic element provides an excellent geometry for a magnetic vortex configuration. In a few words, the interest in the vortex stems from the fact that this is a nontrivial magnetic state which can, nevertheless, be spontaneously created in magnetic elements [3].

This leads naturally to the question of whether there are any further nontrivial states which would play an important role in magnetic elements [4,5]. An answer comes from a somewhat unlikely direction. Recent experiments have shown a peculiar dynamical behavior of vortices and magnetic domain walls when these are probed by external fields. Vortices may switch their polarity under the influence of a very weak external field of the order of a few mT [6,7] or by passing an electrical current in a nanodisk [8]. Since the polarity of the vortex contributes to its topological structure, the switching process clearly implies a discontinuous change of the magnetic configuration. This is certainly a surprise since the external field is only very weak. The key to the phenomenon is the appearance of vortex pairs which are spontaneously created in the vicinity of existing vortices [7,9]. The creation of topological excitations under alternating external fields has been anticipated by a collective coordinate study [10].

We will study vortex-antivortex (VA) pairs and argue that these are nontrivial magnetic states which play an important role in dynamical phenomena in magnetic elements. They are localized configurations—unlike a single vortex. Specifically, we will study a VA pair where the vortex and the antivortex have opposite polarities. This behaves as a rotating vortex dipole, and its topology and its dynamics are radically different than a pair with same polarities [11]. Despite its nontrivial topology it can be destroyed by a quasicontinuous process. This opens the possibility for switching between topologically different states in ferromagnets. On account of a direct link between topology and dynamics in magnets [12,13], a change of the topological magnetization structure leads to a dramatic change of the magnetization dynamics observed in elements as the VA pair is created or annihilated [7,8].

A ferromagnet is characterized by the magnetization vector $\boldsymbol{m} = (m_x, m_y, m_z)$ which is a function of position and time $\boldsymbol{m} = \boldsymbol{m}(\boldsymbol{r}, t)$. It has a constant length which we choose unity for convenience: $\boldsymbol{m}^2 = 1$. Its dynamics is given by the Landau-Lifshitz equation (LLE)

$$\frac{\partial \boldsymbol{m}}{\partial t} = \boldsymbol{m} \times \boldsymbol{f}, \qquad \boldsymbol{f} \equiv \Delta \boldsymbol{m} - Q m_z \hat{\boldsymbol{e}}_z, \tag{1}$$

where $\hat{\boldsymbol{e}}_z$ is the unit vector in the z magnetization direction. The vector $\boldsymbol{m}$ is normalized to the saturation magnetization M_s. Distances are measured in exchange length units $\ell_{\rm ex} \equiv \sqrt{A/(2\pi M_s^2)}$, where A is the exchange constant. The unit of time is $\tau_0 \equiv 1/(4\pi\gamma M_s)$, where γ is the gyromagnetic ratio. Typical values are $\ell_{\rm ex} \sim 5$ nm and $\tau_0 \sim 10$ ps.

In the form (1) the LLE has only one free parameter which is the quality factor $Q \equiv K/(2\pi M_s^2)$, where K is the anisotropy constant. We typically use $Q = 1$ in the following, which corresponds to easy-plane anisotropy, unless stated otherwise. Equation (1) is associated with the energy functional $E = E_e + E_a$, where

$$E_e = \frac{1}{2}\int (\boldsymbol{\nabla} \boldsymbol{m})^2 dxdy, \qquad E_a = \frac{Q}{2}\int (m_z)^2 dxdy, \tag{2}$$

where we consider, for simplicity, a 2D infinite system. Energy is measured in units of $4\pi M_s^2 \ell_{\rm ex}^2$. The magnetocrystalline anisotropy E_a models an intrinsic interaction in materials, but it can also serve as a simplified model for the magnetostatic term in thin films. We defer discussion of a magnetostatic term until we derive our main results.

A magnetic vortex is an axially symmetric solution of Eq. (1) of the form

$$m_z = \lambda \cos\Theta(\rho), \qquad m_x + i m_y = \sin\Theta(\rho) e^{iS(\phi - \phi_0)}, \tag{3}$$

where (ρ, ϕ) are polar coordinates, S is an integer called the *winding number*, $\lambda = \pm 1$ is the vortex *polarity*, and ϕ_0 is a constant. The magnetization angle $\Theta(\rho = 0) = 0$ at the vortex center, while $\Theta(\rho \to \infty) = \pi/2$ away from it. The winding number S is a topological invariant. The usual case for a vortex observed in experiments is $S = 1$, while in the case $S = -1$ it is termed an *antivortex*.

A further topological invariant is defined by [2]

$$\mathcal{N} \equiv \frac{1}{4\pi} \int n dx dy, \qquad n \equiv \frac{1}{2} \epsilon_{\mu\nu} (\partial_\nu \boldsymbol{m} \times \partial_\mu \boldsymbol{m}) \cdot \boldsymbol{m}, \quad (4)$$

where $\epsilon_{\mu\nu}$ is the totally antisymmetric tensor (μ, $\nu = 1$, 2), and n is the topological density. For a vortex of the form (3) we have $\mathcal{N} = -1/2 S\lambda$.

A convenient formulation for the description of vortices but also multivortex states is obtained through the stereographic variable

$$\Omega \equiv \frac{m_x + i m_y}{1 + m_z}. \quad (5)$$

A simple configuration for a two-vortex solution is written as

$$\Omega = i \frac{\bar{\zeta} + a}{\bar{\zeta} - a}, \qquad \zeta \equiv x + iy, \qquad \bar{\zeta} \equiv x - iy, \quad (6)$$

where a is a constant which will be considered real for simplicity. It gives the distance between vortices as well as the size of each vortex core. At $\zeta = a$ we have a vortex of the form (3) with $S = 1$ and $\lambda = -1$, while at $\zeta = -a$ we have an antivortex ($S = -1$) with opposite polarity $\lambda = 1$. At large distances $|\zeta| \to \infty$ we have $\Omega \to i$. In other words, we have chosen the boundary condition

$$\boldsymbol{m} = (0, 1, 0) \quad \text{as} \quad |\zeta| \to \infty. \quad (7)$$

Figure 1 shows examples of VA pairs.

The form (6) will be used as an ansatz for a VA pair with opposite polarities. It belongs to a family of exact static solutions of the LLE for an isotropic 2D ferromagnet ($Q = 0$) [2,14]. Its energy is $E_e = 4\pi$ for every a (due to the scale invariance of the exchange interaction), and its topological charge is $\mathcal{N} = 1$. We have $n(x, y) = n(-x, y)$, and thus precisely half the topological charge comes from the one half plane ($x > 0$) and the second half comes from the second half plane ($x < 0$). The vortex and the antivortex are not overlapping irrespectively of the distance between them.

In the presence of easy-plane anisotropy ($Q > 0$) the vortex profile is modified. Its size is set to $R_c \sim 1/\sqrt{Q}$, or $R_c \sim \sqrt{A/K}$ in the usual units. Otherwise, the comments on the topological density and charge of the VA pair given in the previous paragraph remain valid.

A more detailed study of the VA pair requires information on its dynamics. For this purpose we write the conserved linear momentum associated with the LLE (1) [12] $P_x = -\int y n dx dy$, $P_y = \int x n dx dy$, which gives a measure of the position of the VA pair. Since the vortex and the antivortex have a mirror image topological density distribution we find $(P_x, P_y) = (0, 0)$. The important result is that the mean position is conserved in time and thus the VA pair is spontaneously pinned at the position where it is created.

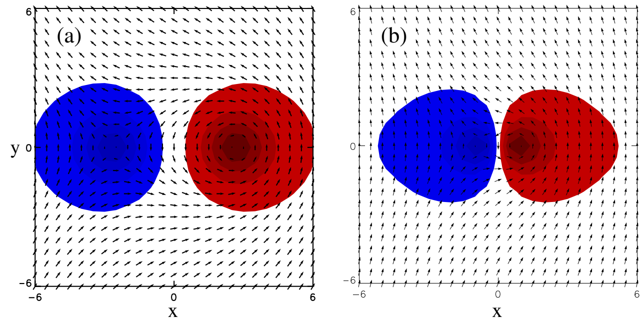

FIG. 1 (color online). VA pairs with opposite polarities. The vectors show (m_x, m_y); m_z is color-coded. (a) Relatively large distance between vortices ($d = 2.7$). The pair rotates with $\omega = 0.06$ ($\ell = 68$). (b) Small distance between vortices ($d = 1.85$), corresponding to $\omega = 0.17$ ($\ell = 15$).

Further information is obtained by considering the angular momentum of the system [12]

$$\ell = \tfrac{1}{2} \int \rho^2 n dx dy, \quad (8)$$

which is also written with the aid of the topological density n, and it is conserved within model (1). For a VA pair ℓ is clearly nonzero (the integrand is positive definite), and it gives a measure of its size. The distance d between the vortex and the antivortex can be defined from

$$\left(\frac{d}{2}\right)^2 \equiv \frac{\int \rho^2 n dx dy}{\int n dx dy} = \frac{\ell}{2\pi\mathcal{N}} = \frac{\ell}{2\pi} \Rightarrow d^2 = \frac{2}{\pi}\ell, \quad (9)$$

where we assume that the VA is centered at the origin. A nonzero angular momentum ℓ indicates that the VA should be a rotating object.

The rotational dynamics is fully confirmed by numerical simulations of the LLE (1). However, the rotating pair is apparently unstable and short lived due to radiation of energy in analogy to a rotating electric dipole. The conservative system will be studied further here. The results will then be used to understand the full dynamics of a more realistic system including mechanisms for energy dissipation.

For a simpler numerical investigation we assume a steady state of a rotating vortex pair. This is a stationary point of the extended energy functional $F = E - \omega\ell$, where ω is the angular frequency of rotation measured in units of $4\pi\gamma M_s$. A standard scaling argument [15] readily gives, for a stationary state, the virial relation

$$E_a = \omega\ell. \quad (10)$$

A useful result is now obtained if we assume well separated vortex and antivortex where each of them behaves as an isolated one. The anisotropy energy for an isolated static vortex is $\pi/2$ [16]. Substituting this in

Eq. (10) we obtain

$$\omega = \frac{\pi}{\ell} = \frac{2}{d^2}, \tag{11}$$

where Eq. (9) was used in the second equality. The limit of large vortex separation was studied in Ref. [10] and the angular frequency (11) was obtained. The rotation frequency goes to zero in the limit of large vortex separation. As a concrete example, suppose that $d = 10\ell_{\rm ex}$ apart and $Q = 1$, to obtain $\omega = 0.01$, or $\omega \sim 10^9$ Hz.

We now turn to the case of a vortex and antivortex forming a small pair. When the size of the VA pair is smaller than the length scale $1/\sqrt{Q}$ introduced by the anisotropy energy, the latter is negligible. Then (6) is exact in the area of the vortex cores. Away from the vortex cores, we will adopt the approximation that the anisotropy will prevail and it will enforce the vacuum (7). In the limit $a \ll 1$ we have $E = E_e = 4\pi$ [17] and $\mathcal{N} = 1$ due to the contribution of the origin which becomes a singular point. The magnetic configuration becomes then fully aligned to the y axis, except for the origin, as $a \to 0$. Then, one can eliminate the singular point and assume a fully aligned state. In doing so, one eliminates the topological complexity of the system and changes the topological charge $\mathcal{N}$ by unity. Needless to say, such singular points may not exist in magnetic materials because the lattice spacing in the solid provides a natural length cutoff. The finite lattice spacing is apparently responsible for the annihilation of vanishingly small VA pairs and the subsequent surprising change of the topological charge of magnetic configurations in experiments [6–8].

One usually expects that a topologically nontrivial state ($\mathcal{N} \neq 0$) cannot be deformed continuously to the ground state of the system which has topological charge zero. In addition, the two states are separated by an infinitely high energy barrier. However, in the case of the VA pair studied here, although the former argument is correct, no energy barrier is encountered in the process. This is an unusual property but it is completely explained by the scale invariance of the exchange energy in two dimensions [2].

We go on to find numerically VA pairs in steady rotation. We use stretched coordinates and simulate the infinite system. We go to a rotating reference frame, which amounts to substituting $\boldsymbol{f} \to \boldsymbol{f} - \omega(\delta\ell/\delta\boldsymbol{m})$ in Eq. (1). We use a relaxation algorithm [i.e., add dissipation to the LLE and effectively use Eq. (14)]. We feed the algorithm with the ansatz (6) as an initial condition, and it relaxes to a roughly steady state. Our algorithm cannot give exact solutions of the equation; however, we are able to calculate good approximations for the energy and the angular momentum of the rotating state for various values of the rotation frequency ω. Figure 1 shows rotating VA-pair solutions. Figure 2 shows the results for the energy as a function of the angular momentum for $Q = 1$. We have checked that the virial relation (10) is approximately satisfied for our numerical solutions.

We can find the rotation frequency of a vanishingly small VA pair by substituting (6) in (2) and (8) to obtain

$$E_a = \frac{Q}{2}\ell = Q\int_S \frac{a^2\rho^2}{(\rho^2 + a^2)^2}(2\pi\rho d\rho), \tag{12}$$

where S denotes a circular domain which includes the vortex cores. Substituting (12) in the virial relation (10) we find $\omega = Q/2$. A more rigorous calculation would require an asymptotic analysis of the LLE. On the side of small ℓ the results in Fig. 2 are consistent with the theoretical value $\omega(\ell \to 0) = 1/2$ (at $Q = 1$).

The frequency satisfies $\omega = dE/d\ell$ and it is thus given by the slope of the curve. For large ℓ we can substitute Eq. (11) for $\omega(\ell)$ and obtain

$$E = \pi \ln(\ell/\ell_0) + 4\pi, \tag{13}$$

where the constant ℓ_0 cannot be fixed by the present calculation. The log term in the energy can be obtained by considering the exchange energy interaction between the vortices.

We now turn to discuss a more realistic system where dissipation is present. The LLE including Gilbert damping reads

$$\frac{\partial \boldsymbol{m}}{\partial t} = \boldsymbol{m} \times \boldsymbol{f} - \alpha \boldsymbol{m} \times (\boldsymbol{m} \times \boldsymbol{f}), \tag{14}$$

where α is the dissipation constant. We typically use in simulations enough dissipation which dominates the radiation effect of the rotating dipole. An initial state (6) is iterated in time using, typically, $\alpha \sim 0.1$ and $a \sim 5$. In the initial phase of the simulation the vortex and the antivortex quickly adjust to the core size R_c.

The dynamical behavior of a VA pair can be summarized as follows. A VA pair which will be initially created will shrink, due to dissipation, as it rotates. Its energy will follow approximately the curve of Fig. 2 as its size and its angular momentum decrease. At vanishing size a sin-

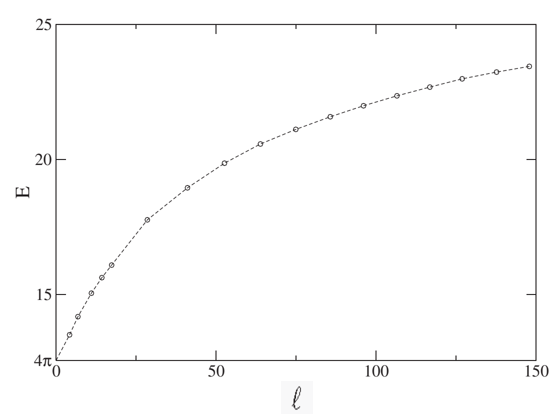

FIG. 2. The energy E of a rotating VA pair as a function of its angular momentum ℓ (for $Q = 1$). The circles indicate numerical results (the dashed line is a guide to the eye). They can be fitted by $E = \pi \ln(\ell/4.5) + 4\pi$ for large ℓ, while $E = 0.5\ell + 4\pi$ for $\ell \to 0$. The slope of the curve gives the rotation frequency.

gular point, as the one discussed earlier, would be created. This would practically disappear due to the discreteness of the solid.

The addition of the magnetostatic field in Eq. (14) is necessary in order to make the model relevant for experiments in magnets. The qualitative picture for the dynamics of a VA pair will not be affected since our analysis does not depend on the exact form of the field $\boldsymbol{f}$ in Eq. (1). The magnetostatic field will, however, modify the profile of the pair. Unlike exchange and anisotropy, the magnetostatic energy is sensitive to rotations of the spin vector. It has been pointed out in Ref. [10] that this symmetry breaking is responsible for the resonant excitation of vortex pairs by alternating external fields.

For an ultrathin film, we suppose no variation of $\boldsymbol{m}$ across the film thickness, and we assume that the magnetostatic interaction would behave as a single-ion easy-plane anisotropy (as for the ground state in thin films [18]). This amounts to the substitution $Q \rightarrow Q + 1$ in Eq. (1). The above analysis is then valid for an effective anisotropy strength $Q + 1$. In particular, a change in the topological charge occurs through a 2D singular point (no Bloch point need be formed). In the special case of a significant easy-axis magnetocrystalline anisotropy $Q = -1$ we effectively recover the isotropic model. Consequently, a VA pair which is somehow created in the magnet would be static. More realistically (experimentally) we expect that it would be slowly moving.

Numerical simulations [9,19] show that, in a film of finite thickness, a singular point is created first at one of the surfaces of the film. The VA pair vanishes by formation and annihilation of the singular point at successive film levels, until the vacuum state is reached throughout the film. A curve similar to that in Fig. 2 gives now the *linear* energy density across the film thickness. The energy released when the VA pair is annihilated is $4\pi t$ (t is the film thickness) and it is emitted through spin waves [9,17].

At the stage when the singular point has been formed and annihilated, say, near the top film surface, while a VA pair still exists in the lower film levels, we have a Bloch point (BP) in the film [20]. This lies at the interface between the levels where the VA has been annihilated and the levels where this has a finite size, at the singular point. This is a somewhat simplified realization of the BP studied in Ref. [21]. Its energy is approximately 4π times the length of the vortex or the antivortex line. More important is the unusual fact that during creation and annihilation of the BP one does not have to overcome any energy barrier (unlike the case of Ref. [22]), essentially for the same reasons explained above in connection to the singular point.

Our discussion clearly suggests a connection between VA-pair dynamics and the dynamics of a BP. In particular, a rotating motion of the magnetization appears inherent in a BP configuration. This effect is not present in the dissipative dynamics of a BP discussed in the literature [23].

The dynamics observed in both experiments in Refs. [7,8] includes three vortices. Our analysis applies to the part of the process when the vortex and antivortex with opposite polarities are relatively close together. The presence of a third vortex does not modify our main arguments. The full three-vortex system has a nonvanishing topological charge and thus they form a rotating object which is spontaneously pinned in the magnet. A study of this configuration could be done following the methods of this Letter.

I am grateful to N. R. Cooper and N. Papanicolaou for discussions and for important comments on the manuscript. I also gratefully acknowledge discussions with H. Stoll, H. Fähnle, and O. Tchernyshyov.

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