Observation of Inertial Energy Cascade in Interplanetary Space Plasma

L. Sorriso-Valvo,[1] R. Marino,[2] V. Carbone,[2] A. Noullez,[3] F. Lepreti,[2] P. Veltri,[2] R. Bruno,[4] B. Bavassano,[4] and E. Pietropaolo[5]

[1]*Licryl Regional Laboratory - INFM/CNR, Ponte P. Bucci, Cubo 33C, 87036 Rende (CS), Italy*
[2]*Dipartimento di Fisica, Università della Calabria, Ponte P. Bucci Cubo 31C, 87036 Rende (CS), Italy*
[3]*Observatoire de la Côte d'Azur, Nice, France*
[4]*IFSI/CNR, Via Fosso del Cavaliere, Roma, Italy*
[5]*Dipartimento di Fisica, Università dell'Aquila, L'Aquila, Italy*
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Direct evidence for the presence of an inertial energy cascade, the most characteristic signature of hydromagnetic turbulence (MHD), is observed in the solar wind by the Ulysses spacecraft. After a brief rederivation of the equivalent of Yaglom's law for MHD turbulence, a linear relation is indeed observed for the scaling of mixed third-order structure functions involving Elsässer variables. This experimental result firmly establishes the turbulent character of low-frequency velocity and magnetic field fluctuations in the solar wind plasma.

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Space flights have shown that the interplanetary medium is permeated by a supersonic, highly turbulent plasma flowing out from the solar corona, the solar wind [1,2]. The turbulent character of the flow, at frequencies below the ion gyrofrequency $f_{ci} \simeq 1$ Hz, has been invoked since the first Mariner mission [3]. In fact, velocity and magnetic fluctuations power spectra are close to the Kolmogorov's $-5/3$ law [2,4]. However, even though fields fluctuations are usually considered within the framework of magnetohydrodynamic (MHD) turbulence [2], a firm established proof of the existence of an energy cascade, the main characteristic of turbulence, remains a conjecture so far [5]. Here we show, comparing data analysis to theoretical predictions, that solar wind fields are in a state of fully developed turbulence. We have rederived for the MHD case a proportionality relation between the mixed third-order moment of the longitudinal increments of the fields and the increment scale that is the equivalent of Kolmogorov's law [6], the only exact and nontrivial theoretical result on turbulence. Using Ulysses spacecraft measurements, we have observed the existence of such relation in solar wind, which firmly puts low-frequency solar wind fluctuations within the framework of MHD turbulence. Since solar wind is the only natural plasma accessible for *in situ* measurements, the importance of our finding stands beyond the understanding of the basic physics of solar wind turbulence, but it also has implications in a wide number of areas of more practical interest, such as plasma fusion, space weather, or solar physics.

Incompressible MHD equations are more complicated than the standard neutral fluid mechanics equations because the velocity of the charged fluid is coupled with the magnetic field generated by the motion of the fluid itself. However, written in terms of the Elsässer variables defined as $\boldsymbol{z}^{\pm} = \boldsymbol{v} \pm (4\pi\rho)^{-1/2}\boldsymbol{b}$ ($\boldsymbol{v}$ and $\boldsymbol{b}$ are the velocity and magnetic field, respectively, and ρ the mass density), they have the same structure as the Navier-Stokes equations [5]

$$\partial_t \boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp} \cdot \boldsymbol{\nabla} \boldsymbol{z}^{\pm} = -\boldsymbol{\nabla} P/\rho + \left(\frac{\nu + \kappa}{2}\right)\nabla^2 \boldsymbol{z}^{\pm} - \left(\frac{\nu - \kappa}{2}\right)\nabla^2 \boldsymbol{z}^{\mp}, \tag{1}$$

where P is the total hydromagnetic pressure, ν is the viscosity, and κ the magnetic diffusivity. In particular, the nonlinear term appears as $\boldsymbol{z}^{\mp} \cdot \boldsymbol{\nabla} \boldsymbol{z}^{\pm}$, suggesting the form of a transport process, in which Alfvénic MHD fluctuations $\boldsymbol{z}^{\pm}$ propagating along the background magnetic field are transported by fluctuations $\boldsymbol{z}^{\mp}$ propagating in the opposite direction. This transport is *active* as $\boldsymbol{z}^{\pm}$ and $\boldsymbol{z}^{\mp}$ are clearly not independent. Still, following the same procedure as in [7,8], and assuming local (small scale) homogeneity, a relation for mixed third-order structure function, similar to the Yaglom equation for the transport of a *passive* quantity [9], can be obtained in the stationary state

$$\begin{aligned}\partial_{\parallel} Y^{\pm}(r) = &-\tfrac{4}{3}\epsilon^{\pm} + 2\nu\nabla^2\langle|\Delta \boldsymbol{z}^{\pm}|^2\rangle \\ &- 2\langle\Delta \boldsymbol{z}^{\pm} \cdot (\boldsymbol{\nabla} + \boldsymbol{\nabla}')\Delta P/\rho\rangle \\ &+ \langle \boldsymbol{z}^{\mp} \cdot (\boldsymbol{\nabla} + \boldsymbol{\nabla}')|\Delta \boldsymbol{z}^{\pm}|^2\rangle.\end{aligned} \tag{2}$$

Here, $\Delta \boldsymbol{z}^{\pm} \equiv \boldsymbol{z}^{\pm}(\boldsymbol{x}') - \boldsymbol{z}^{\pm}(\boldsymbol{x})$ are the (vector) increments of the fluctuations between two points $\boldsymbol{x}$ and $\boldsymbol{x}' \equiv \boldsymbol{x} + \boldsymbol{r}$, $\boldsymbol{\nabla}$ and $\boldsymbol{\nabla}'$ are the gradients at the corresponding two points, $\partial_{\parallel}$ is the longitudinal derivative along the separation $\boldsymbol{r}$, while $Y^{\pm}(r)$ are the mixed third-order structure function $\langle|\Delta \boldsymbol{z}^{\pm}|^2 \Delta z_{\parallel}^{\mp}\rangle$ and $\epsilon^{\pm} \equiv \nu\langle|\boldsymbol{\nabla} \boldsymbol{z}^{\pm}|^2\rangle \overset{\text{hom}}{=} 3\nu\langle|\partial_{\parallel} \boldsymbol{z}^{\pm}|^2\rangle$ are the pseudoenergy average dissipation rates, namely, the dissipation rates of both $\langle|\boldsymbol{z}^{\pm}|^2\rangle/2$, respectively. Finally, ΔP represent the increment of the total pressure fluctuations and the kinematic viscosity ν is here assumed to be equal to the magnetic diffusivity κ.

The last term on the right-hand side of Eq. (2) is related to large-scale inhomogeneities and disappears if the flow is globally homogeneous. Also, assuming local isotropy, the term containing pressure correlation vanishes, so that after longitudinal integration of (2) and in the inertial range of MHD turbulence (i.e., when $\nu \rightarrow 0$), a linear scaling law

$$Y^{\pm}(r) = -\tfrac{4}{3}\epsilon^{\pm} r \qquad (3)$$

is obtained, characterizing a turbulent cascade with a well-defined finite energy flux $\epsilon^{\pm}$. An alternative derivation of this result using correlators instead of structure functions had been first obtained in [10,11] and checked in numerical simulations [12], while a similar relation involving the vorticity and velocity in helical flows was derived in [13,14]. When neutral fluid turbulence is considered, Eq. (3) becomes [7] $\langle|\Delta \boldsymbol{v}|^2 \Delta v_{\parallel}\rangle = -4/3\epsilon r$ (ϵ being the average kinetic energy dissipation rate), from which Kolmogorov's $-4/5$ law for the longitudinal third-order structure function can be recovered if there is full isotropy as $\langle(\Delta v_{\parallel})^3\rangle = -4/5\epsilon r$.

In this work, we show that relation (3) is indeed satisfied in some periods within solar wind turbulence. In order to avoid variations of the solar activity and ecliptic disturbances (like slow wind sources, coronal mass ejections, ecliptic current sheet, and so on), we use high speed polar wind data measured by the Ulysses spacecraft [15,16]. In particular, we analyze here the first seven months of 1996, when the heliocentric distance slowly increased from 3 AU to 4 AU, while the heliolatitude decreased from about 55° to 30°. The fields components are given in the *RTN* reference frame, where *R* (radial) indicates the Sun-spacecraft direction, centered on the spacecraft and pointing out of the Sun, *N* (normal) lies in the plane containing the radial direction and the Sun's rotation axis, while *T* completes the right-handed reference frame. Note that, since the wind speed in the spacecraft frame is much larger than the typical velocity fluctuations, and it is nearly aligned with the *R* radial direction, time fluctuations are in fact spatial fluctuations with time and space scales (τ and r respectively) related through the Taylor hypothesis, so that $r = -\langle v_R\rangle\tau$ (note the *reversed* sign). From the 8 minutes averaged time series $z^{\pm}(t)$, we compute the time increments $\Delta z^{\pm}(\tau; t) = z^{\pm}(t+\tau) - z^{\pm}(t)$ and obtain the mixed third-order structure function $Y^{\pm}(-\langle v_R\rangle_t \tau) = \langle|\Delta z^{\pm}(\tau; t)|^2 \Delta z_R^{\mp}(\tau; t)\rangle_t$ using moving averages $\langle\bullet\rangle_t$ on the time t over periods spanning around 10 days, during which the fields can be considered stationary. Also, even if velocity and magnetic field are not exactly isotropic (the *R* direction has weaker fluctuations at large scales), their fluctuations have roughly the same amplitude at small and intermediate scales, as checked from their spectrum.

A linear scaling $Y^{\pm}(\tau) = 4/3\epsilon^{\pm}\langle v_R\rangle_t \tau$ is indeed observed in a significant fraction of the periods we examined, with an inertial range spanning as much as two decades, indicating for the first time the existence of a well-defined inertial energy cascade range in plasma turbulence. In fact, solar wind inertial ranges can even be larger than the ones reported for laboratory fluid flows [8], showing the robustness of this result. Figure 1 (two upper panels) shows some example of scaling and the extension of the inertial range for both $Y^{\pm}(\tau)$. The linear scaling law generally extends from a few minutes to 1 day or more and is present in about 20 periods of a few days in the 7 months considered. This probably reflects different regimes of driving of the turbulence by the Sun itself, and it is certainly an indication of the nonstationarity of the energy injection processes. Several other periods are found in which the scaling range is considerably reduced. In particular, the sign of $Y^{\pm}(\tau)$ is observed to be either positive or negative. Since pseudoenergies dissipation rates are positively defined, a positive sign for $Y^{\pm}(\tau)$ [negative for $Y^{\pm}(r)$] indicates a (standard) forward cascade with pseudoenergies flowing towards the small scales to be dissipated. On the contrary, a negative $Y^{\pm}(\tau)$ is the signature of an *inverse* cascade where the energy flux is being transferred on average toward larger scales. Figure 2 shows the location of the most evident scaling intervals, together with the values of the flux rate $\epsilon^{\pm}$ estimated through a fit of the scaling law (3), typically of the order of a few hundreds in $\mathrm{J\,kg^{-1}\,s^{-1}}$. For comparison, values found for ordinary turbulent fluids are 1–50 $\mathrm{J\,kg^{-1}\,s^{-1}}$ [17]. It is worth noting that, in a large fraction of cases, both $Y^{\pm}(\tau)$ switch from positive to negative linear scaling (or vice versa) within the same time period when going from small to large scales (see the two bottom panels of Fig. 1). The occurrence of both kind of cascades within the same flow is not so uncommon within hydrodynamic turbulence. This phenomenon has been attributed to some large-scale instability, as observed, for example, in geophysical flows or when the flow is affected by a strong rotation [18]. In the case of solar wind plasma, a possible explanation for the inverse cascade could be the enhanced intensity of the background magnetic field. This would make the turbulence mainly bidimensional allowing for an inverse cascade as observed in numerical simulations [19]. It should also be noticed that in most of the cases the time scale at which the cascade reverses its sign is of the order of 1 day. This scale roughly indicates where the small scale Alfvénic correlations between velocity and magnetic field are lost [20,21]. This could mean that the nature of the fluctuations changes across the break. However, these particular aspects still deserve to be adequately considered within the solar wind context.

At this point, the question is why the scaling is not observed all of the time within the solar wind. As already stated, Eq. (2) reduces to the linear law (3) only when local homogeneity, incompressiblity, and isotropy conditions are satisfied. If this is not the case, then the full relation (2) should be used and is much more complicated to check

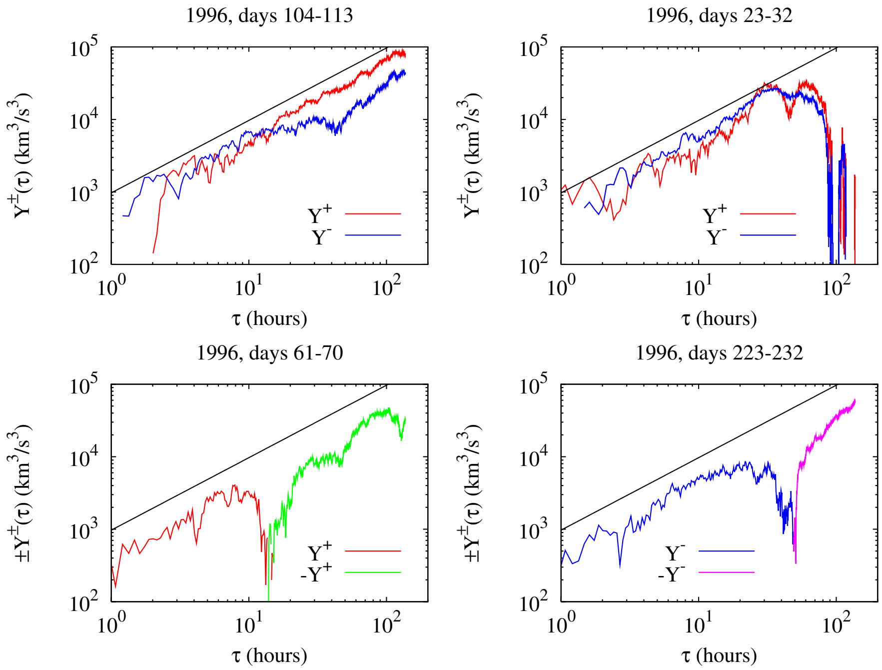

FIG. 1 (color online). The scaling behavior of $Y^{\pm}(\tau)$ as a function of the time scale τ for four different periods we examined. Different colors of the curves refer to positive and negative values of the mixed structure functions $Y^{\pm}(\tau)$ and thus of $\epsilon^{\pm}$. The full black lines correspond to linear scaling laws to guide the eye.

experimentally, as it is not even isotropic. In general, solar wind inhomogeneities play a major role at large scales so that local homogeneity is generally fulfilled within the range of interest. Regarding incompressibility, it has been shown that compressive phenomena mainly affect shocked regions and dynamical interaction regions like stream-stream interface [1,2]. However, the time interval we analyze, because of Ulysses high latitude location, is not affected by these compressive phenomena [22]. On the other hand, it has also been shown [2] that magnetic field compressibility increases mainly at very small scales within the fast wind regime. It follows that the incompressibility assumption can be considered valid to a large extent for the analyzed interval and at intermediate scales. The large-scale anisotropy, mainly due to the average magnetic field, is only partially lost at smaller scales, and a residual anisotropy is always present [23,24], generally breaking the local isotropy assumption. Thus, while inhomogeneity, compressibility, and anisotropy could all be responsible for the loss of linear scaling, anisotropy proba-

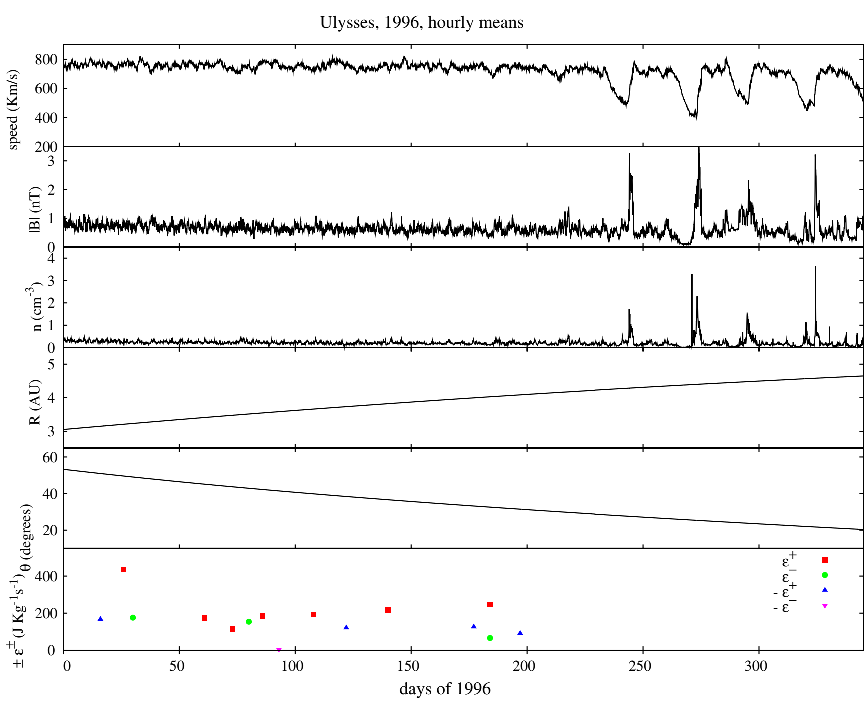

FIG. 2 (color online). Hourly averaged quantities are represented as a function of the flight time of Ulysses. The top panels represent, respectively, the solar wind speed, the magnitude of the magnetic field, the particle density, the distance from the Sun and the heliolatitude angle. In the bottom panel are plotted the values of $\epsilon^{\pm}$, calculated through a fit using the relation (3) during the periods where a clear linear scaling exists.

bly is the main candidate within high latitude regions of the solar wind. It is important to note that the presence of a Yaglom-like law that involves the third-order mixed moment is more general than the phenomenology usually involving the second order moment. Indeed, while the Yaglom MHD relation (2) involves only differences along the parallel direction that are in fact the only quantities accessible from single satellite measurements, phenomenological arguments involve the full spatial dependence of vector fields that cannot be directly measured yet. This means that our result is compatible with both Kolmogorov and Iroshnikov-Kraichnan type cascade [2,5], and it does not help in discriminating between these phenomenologies [25,26].

In conclusion, we observed, for the first time in the solar wind, the only natural plasma directly accessible, evidence of Yaglom MHD scaling law indicating the existence of a local energy cascade in hydromagnetic turbulence. The scaling holds in a number of relatively long periods of about 10 days and also provides the first estimation of the pseudoenergy dissipation rate. Although our data might not fully satisfy requirements of homogeneity, incompressibility and isotropy everywhere, the observed linear scaling extends on a wide range of scales and appears very robust. The unexpected existence of the scaling law in anisotropic, weakly compressible, and inhomogeneous turbulence still needs to be fully understood. Our result establishes a firm point within solar wind phenomenology, and, more generally, provides a better knowledge of plasma turbulence, carrying along a wide range of practical implications on both laboratory fusion plasmas and space physics.

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