Optical Bistability in a Nonlinear Optical Coupler with a Negative Index Channel

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We discuss a novel kind of nonlinear coupler with one channel filled with a negative index metamaterial. The opposite directionality of the phase velocity and the energy flow in the negative index metamaterial channel facilitates an effective feedback mechanism that leads to optical bistability and gap soliton formation.

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Nonlinear optical couplers have attracted significant attention owing to their strong potential for all-optical processing applications, including switching and power-limiting devices. Transmission properties of a nonlinear coherent directional coupler were originally studied by Jensen [1], who concluded that a coupler consisting of two channels made of conventional homogeneous nonlinear materials is not bistable.

Bistability (or multistability) is a phenomenon in which the system exhibits two (or more) output intensities for the same input intensity [2,3]. Optical bistability has been predicted and experimentally realized in various settings, including a Fabry-Perot resonator filled with a nonlinear material [3], layered periodic structures [4], and nonlinear couplers with external feedback mechanisms [5–8]. In this Letter we describe a novel nonlinear optical coupler structure that utilizes a negative index metamaterial (NIM) [9,10] in one of the channels and a conventional positive index material (PIM) in another channel as shown in Fig. 1. The linear transmission properties of a similar optical structure were previously studied by Alu and Engheta [11]. We show that such a nonlinear coupler (NLC) can be bistable. Bistability occurs owing to the effective feedback mechanism enabled by a fundamental property of NIMs—opposite directionality of the wave vector and the Poynting vector. Moreover, our results suggest that the entirely uniform PIM-NIM coupler structure supports gap solitons—a feature commonly associated with periodic structures [4,12–18].

Continuous wave propagation in a nonlinear coupler can be described by the following system of equations:

$$i\sigma_1 \frac{\partial a_1}{\partial z} + \kappa_{12} a_2 \exp\{-i\delta z\} + \gamma_1 |a_1|^2 a_1 = 0,$$
$$i\sigma_2 \frac{\partial a_2}{\partial z} + \kappa_{21} a_1 \exp\{i\delta z\} + \gamma_2 |a_2|^2 a_2 = 0, \quad (1)$$

where a_1 and a_2 are the complex normalized amplitudes of the modes in the PIM and NIM channels, respectively; κ_{12} and κ_{21} are the coupling coefficients defined as in Ref. [1]; $\gamma_j \equiv \frac{\omega_0 n_{2j} \mu_j(\omega_0) P_0}{c A_{\mathrm{eff}}}$ is a normalized nonlinearity coefficient; $n_{2j} = \frac{12\pi^2 \chi_j^{(3)}}{\varepsilon_j(\omega_0) c}$, $\chi_j^{(3)}$ is the nonlinear (electric) susceptibility; ε_j and μ_j are linear frequency-dependent dielectric permittivity and magnetic permeability; ω_0 is carrier frequency; A_{eff} is the effective area; c is the speed of light in a vacuum; $\delta = \beta_1 - \beta_2$ is the mismatch between the propagation constants in the individual channels; and σ_j is the sign of the refractive index $n_j = \sqrt{\varepsilon_j(\omega_0)\mu_j(\omega_0)}$. In the case of the PIM-NIM coupler σ_1 is positive, while σ_2 is negative.

Assuming the following form for the solutions of Eqs. (1), $a_{1,2} = u_{1,2} \exp(iqz) \exp(\mp i \frac{\delta}{2} z)$, in the linear regime, we find the following relation between q and δ for the PIM-NIM coupler:

$$q^2 = \left(\frac{\delta}{2}\right)^2 - \kappa_{12}\kappa_{21}, \quad (2)$$

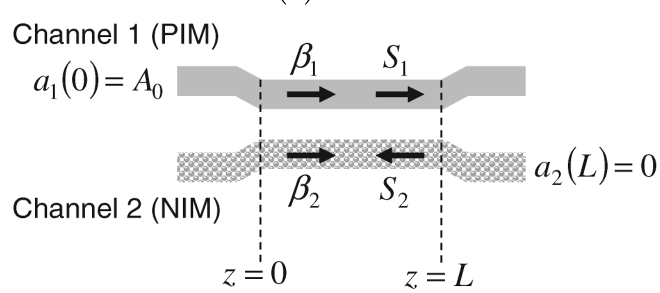

FIG. 1. A schematic of a nonlinear PIM-NIM coupler. Light is initially launched into channel 1 (PIM). A wave vector β and a Poynting vector S are parallel in the PIM channel and antiparallel in the NIM channel, enabling a new backward-coupling mechanism.

which indicates the presence of a bandgap for $|\delta| < 2\sqrt{\kappa_{12}\kappa_{21}}$. The photonic bandgap is a feature typical for periodic or distributed feedback (DFB) structures such as fiber Bragg gratings or thin film stacks [12]. Formation of the bandgap in a uniform structure considered here is one of the unique properties of the PIM-NIM coupler arising from introduction of the NIM into one channel of the coupler. Introducing two parameters $a^2 = u_1^2 + u_2^2$ and $f = u_2/u_1$, the nonlinear counterpart of the relation (2) can be written in the form

$$\delta = -\frac{\kappa_{21} + f^2\kappa_{12}}{f} - \frac{a^2}{1+f^2}(\gamma_1 + \gamma_2 f^2)$$
$$q = -\frac{\kappa_{21} - f^2\kappa_{12}}{2f} - \frac{a^2}{2}\frac{1}{1+f^2}(\gamma_2 f^2 - \gamma_1). \quad (3)$$

Figure 2 shows linear (a) and nonlinear δ-q curves (b)–(d) for the case of $\gamma_1, \gamma_2 > 0$. In Fig. 2(b) both PIM and NIM channels are nonlinear, with the same nonlinear coefficients. Beyond a critical power level, the lower branch of the δ-q curve forms a loop. Both linear and nonlinear δ-q curves in Figs. 2(a) and 2(b) resemble dispersion relations found in the case of linear and nonlinear fiber Bragg gratings [12,19], with the difference that in the nonlinear case the effect of cross-phase modulation has been neglected and forward and backward waves are spatially separated. However, importantly, in the case of PIM-NIM NLC both channels are made of homogeneous material with no periodicity or external feedback mechanism. The

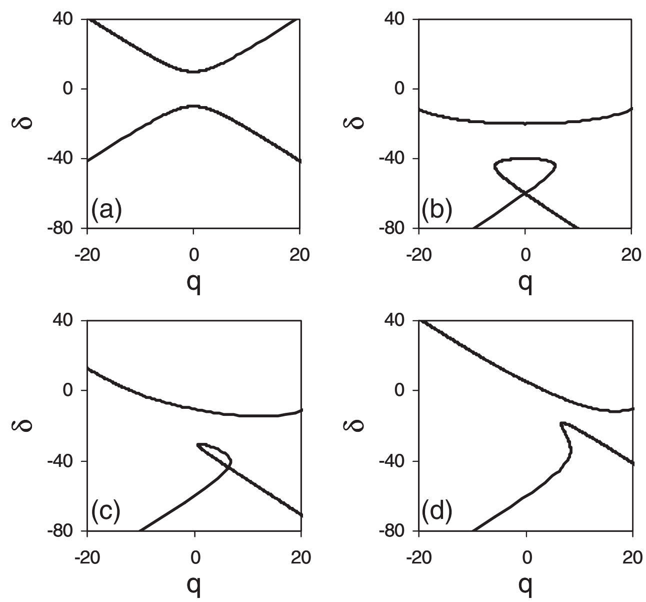

FIG. 2. PIM-NIM coupler δ-q relations: (a) linear ($\kappa_{12} = \kappa_{21} = 5$), (b) two channels with the same nonlinear susceptibilities $\gamma_1 = \gamma_2 = \gamma$ ($\kappa_{12} = \kappa_{21} = 5$, $a^2\gamma/\kappa = 6$), (c) two channels with different nonlinear susceptibilities ($\kappa_{12} = \kappa_{21} = 5$, $a^2\gamma_1/\kappa = 6$, $a^2\gamma_2/\kappa = 3$), (d) the NIM channel is linear, while the PIM channel is nonlinear ($\kappa_{12} = \kappa_{21} = 5$, $a^2\gamma_1/\kappa = 6$, $a^2\gamma_2/\kappa = 0$).

effective feedback mechanism is provided by the inherent property of the NIMs—opposite directionality of the phase velocity and the Poynting vector. As shown in Fig. 1, while the propagation vectors of the waves propagating in both NLC channels point in the same direction, assuring the necessary phase-matching condition, the Poynting vectors corresponding to the energy flow direction point in opposite directions. As light propagates in the PIM channel in the forward direction, it continuously couples to the NIM channel, where it flows in the backward direction. Therefore, the PIM-NIM coupler acts as an effective DFB structure. Figure 2(c) corresponds to the case of different nonlinear coefficients in the two channels, while Fig. 2(d) corresponds to the case of the nonlinear PIM and linear NIM channel. Figures 2(c) and 2(d) suggest that there are more degrees of design freedom in the NLC case in comparison with conventional DFB structures since in the general case the nonlinear coefficients γ_1 and γ_2 as well as the coupling coefficients κ_{12} and κ_{21} may be not identical and can be tailored.

Although in the general case $\kappa_{12} \neq \kappa_{21}$ and $\gamma_1 \neq \gamma_2$, in order to highlight the most important new physical effects associated with the PIM-NIM NLC, in the following discussion we assume identical linear coupling coefficients $\kappa_{12} = \kappa_{21} \equiv \kappa$ and nonlinear coefficients $\gamma_1 = \gamma_2 \equiv \gamma$. Then, making the following substitution, $a_1 = A_1 \exp\{i\phi_1\}$ and $a_2 = A_2 \exp\{i\phi_2\}$, where A_1, A_2, ϕ_1, and ϕ_2 are real functions of z, the equations for A_1, A_2, and $\psi = \phi_1 - \phi_2 + \delta z$ can be written in the form

$$\frac{\partial A_1}{\partial z} = \kappa A_2 \sin\psi, \qquad \frac{\partial A_2}{\partial z} = \kappa A_1 \sin\psi,$$
$$\frac{\partial \psi}{\partial z} = \kappa \cos\psi\left(\frac{A_2}{A_1} + \frac{A_1}{A_2}\right) + \gamma(A_1^2 + A_2^2) + \delta. \quad (4)$$

From Eqs. (4) the constants of the motion are given by

$$C = P_1 - P_2 \qquad \Gamma = 4A_1A_2\cos\psi + 2\kappa^{-1}(\delta + \gamma A_2^2)A_1^2, \quad (5)$$

where $P_1 = A_1^2$ and $P_2 = A_2^2$. C is defined by the boundary conditions at $z = 0$ and $z = L$, where L is the length of the coupler. The expression for the first constant of motion C should be compared to that of conventional PIM-PIM NLC in which case $C = P_1 + P_2$.

Power evolution in channel 1 is described by the following equation:

$$\left(\frac{\partial P_1}{\partial z}\right)^2 = \kappa\{4\kappa + \Gamma\gamma\}P_1P_2 + \Gamma\kappa\delta P_1 - \frac{1}{4}\Gamma^2\kappa^2 - (\delta + \gamma P_2)^2 P_1^2. \quad (6)$$

If light initially is launched into channel 1, i.e., $A_1(0) = A_0$, $A_2(L) = 0$, then $C = A_1^2(L)$. In the case of $\delta = 0$, the solutions for P_1 and P_2 are found in the form

$$P_1(z) = C\frac{dn[2\kappa(z-L)/m, m] + 1}{2dn[2\kappa(z-L)/m, m]}$$
$$P_2(z) = C\frac{1 - dn[2\kappa(z-L)/m, m]}{2dn[2\kappa(z-L)/m, m]}, \quad (7)$$

where $m = \frac{k}{\sqrt{1+k^2}} = \frac{1}{\sqrt{1+(\gamma C/4\kappa)^2}}$, $dn(z', k')$ is the Jacobi elliptic function [20]. The parameter C can be found using the transcendental equation

$$A_0^2 = C\frac{dn[2\kappa L/m, m] + 1}{2dn[2\kappa L/m, m]}. \quad (8)$$

Finally, one can define transmission and "reflection" coefficients for the nonlinear coupler as

$$\Im = \frac{P_1(L)}{A_0^2} = \frac{C}{A_0^2} = \frac{2dn[2\kappa L/m, m]}{1 + dn[2\kappa L/m, m]}$$
$$\Re = 1 - \frac{P_1(L)}{A_0^2} = 1 - \frac{C}{A_0^2} = \frac{1 - dn[2\kappa L/m, m]}{1 + dn[2\kappa L/m, m]}. \quad (9)$$

Figure 3(a) shows output power $P_1(L)$ as a function of input power $P_1(0)$ for three values of $\kappa L = 2$ (solid line), $\kappa L = 4$ (dashed line), and $\kappa L = 6$ (dot-dashed line), assuming κ is varying, the coupler length $L = 1$ (in the units of length) is fixed and $\gamma L = 6$. As the coupling κ between the channels increases and the effective feedback mechanism is established, PIM-NIM NLC becomes bistable or more generally multistable as illustrated in the case of $\kappa L = 6$. Its transmission characteristics are very similar to those of DFB structures [12–15]. Figure 3(b) shows the transmission coefficient $P_1(L)/P_1(0)$ as a function of input power $P_1(0)$ for two values of $\gamma L = 1$ (solid line) and $\gamma L = 6$ (dashed line), assuming γ is varying, the coupler length $L = 1$ is fixed and $\kappa L = 6$. As the nonlinearity coefficient decreases, the threshold of bistability shifts to higher values as expected.

The phenomenon of bistability is closely related to the notion of gap solitons found in different settings, including DFBs and asymmetric dual-core fibers [12–19,21,22]. As shown in Fig. 3(b), the transmission coefficient approaches $\Im = 1$ at the points 1, 2, 3, and 4, suggesting the existence of transmission resonances. At the resonance corresponding to point 1 in Fig. 3(b), spatial power distributions $P_1(z)$ (solid line) and $P_2(z)$ (dashed line) peak in the middle of the structure as shown in Fig. 4. The dot-dashed line in Fig. 4 shows the constant of the motion $C = P_1 - P_2$. At this transmission resonance incident light is coupled to a solitonlike static entity that is known as a gap soliton [14,15]. Nonstationary gap solitons in PIM-NIM NLC analogous to those found by Aceves and Wabnitz [17] in the context of periodic media will be discussed elsewhere. It is notable that a gap soliton, usually existing in periodic structures, forms in a uniform structure in the case of the PIM-NIM coupler, owing to the new backward-coupling mechanism in the PIM-NIM coupler.

In summary, we found that backward coupling between the modes propagating in the PIM and NIM channels enabled by the basic property of NIMs, oppositely directed phase velocity, and the Poynting vector, results in optical bistability in PIM-NIM NLC and gap soliton formation. These effects have no analogies in conventional PIM-PIM couplers composed of uniform (homogeneous) waveguides with no feedback mechanism. In this study we have only considered time-independent properties of an ideal lossless coupler in order to highlight the essence of new phenomena in the PIM-NIM coupler—bistability and gap solitons. In realistic, currently available NIMs both dielectric permittivity and magnetic permeability typically have non-negligible imaginary parts, implying that these mate-

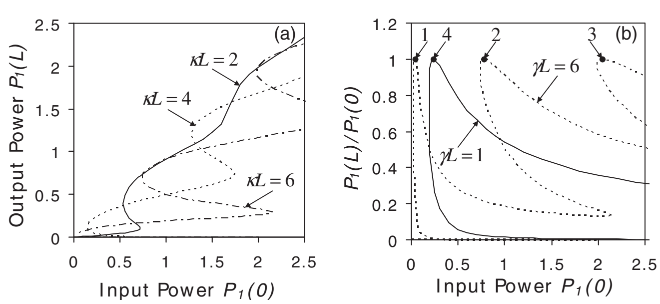

FIG. 3. (a) Output power $P_1(L)$ as a function of input power $P_1(0)$ for three values of $\kappa L = 2$ (solid line), $\kappa L = 4$ (dashed line) and $\kappa L = 6$ (dot-dashed line) when $\gamma L = 6$. (b) Transmission coefficient defined as $P_1(L)/P_1(0)$ as a function of $P_1(0)$ for $\gamma L = 1$ (solid line) and $\gamma L = 6$ (dashed line) when $\kappa L = 6$. Transmission resonances are indicated by the numbers 1, 2, 3, and 4.

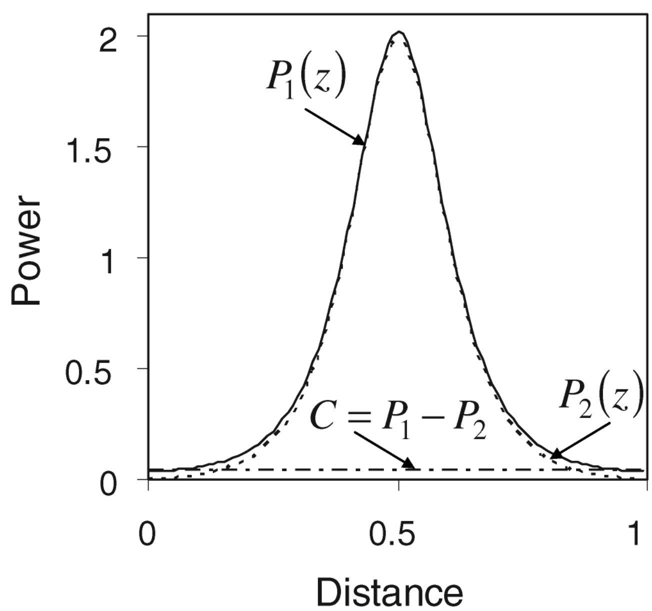

FIG. 4. Spatial distributions of P_1 (solid line), of P_2 (dashed line), and the constant of motion $C = P_1 - P_2$ (dot-dashed line) versus z at transmission resonance indicated by the number "1" in Fig. 3(b).

rials are lossy. It is expected that moderate losses would result in the increase in the input intensity required for the transition from the low-transmission state to the high-transmission state. A detailed numerical investigation of effect of losses as well as the time-dependent properties of the PIM-NIM coupler will be addressed in our future publications.

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