# Nonlocal Cosmology

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We explore nonlocally modified models of gravity, inspired by quantum loop corrections, as a mechanism for explaining current cosmic acceleration. These theories enjoy two major advantages: they allow a delayed response to cosmic events, here the transition from radiation to matter dominance, and they avoid the usual level of fine-tuning; instead, emulating Dirac's dictum, the required large numbers come from the large time scales involved. Their solar system effects are safely negligible, and they may even prove useful to the black hole information problem.

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*Introduction.*—A variety of complementary data sets [1] have led to general agreement that the Universe is accelerating as if it had critical density, comprised of about 30% matter and 70% cosmological constant [2]. There is, however, no current compelling explanation for either the smallness of $\Lambda$ or for its recent dominance in cosmological history [3]. Two existing classes of models, scalars [4] and "$f(R)$" modifications of gravity [5], can be arranged to reproduce the observed (or any other) expansion history [5–8]. However, neither has an underlying rationale nor do they avoid fine-tuning [9]. Quantum scalar effects, depending on a very small mass, have also been proposed [10].

In this Letter, we account for the current phase of acceleration through nonlocal additions to general relativity. Such corrections arise naturally as quantum loop effects and have of course been studied, though in other contexts [6,11,12]. As we will see, even the simple models we explore here can both generate large numbers without major fine-tuning and deliver a delayed response to cosmic transitions, in particular, to that from radiation to matter dominance at $z \sim 2300$. We will neither attempt to derive our models from loop corrections nor to survey generic candidates here. Instead, we will show that natural nonlocal operators such as the inverse d'Alembertian can explain the time lag between $z \sim 2300$ and the onset of acceleration at redshift $z \sim 0.7$, without recourse to large parameters. Large numbers come in our models precisely from the long time lags themselves, a mechanism reminiscent of some old ideas of Dirac.

*Nonlocal triggers.*—For simplicity, we deal with homogeneous, isotropic and spatially flat geometries

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x} \cdot d\mathbf{x}. \tag{1}$$

These correspond to the following Hubble and deceleration parameters

$$H(t) \equiv \frac{\dot{a}}{a}, \qquad q(t) \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}, \tag{2}$$

and to Ricci scalar

$$R = 6(1-q)H^2 \tag{3}$$

(our conventions are $R \equiv g^{\mu\nu} R_{\mu\nu}$ and $R_{\mu\nu} \equiv \partial_\rho \Gamma^\rho_{\nu\mu} + \ldots$). For much of cosmic history, $a(t)$ grows as a power of time

$$a(t) \sim t^s \Rightarrow H(t) = \frac{s}{t}, \qquad q(t) = \frac{1-s}{s}. \tag{4}$$

Perfect radiation dominance corresponds to $s = \frac{1}{2}$ and perfect matter dominance to $s = \frac{2}{3}$. The Ricci scalar of course vanishes for $s = \frac{1}{2}$ and is positive for $s = \frac{2}{3}$. It is the lowest dimension curvature invariant and the only simple curvature invariant to vanish at finite $s$, so we concentrate here on $R$-based models.

We seek the inverse of some differential operator to provide the required time lag between the transition from radiation dominance to matter dominance at $t_{\rm eq} \sim 10^5$ years. The simplest choice is the scalar wave operator, suggested also by the fact that, for our background (1), dynamical gravitons obey the scalar wave equation [13] with

$$\Box \equiv \frac{1}{\sqrt{-g}} \partial_\rho (\sqrt{-g} g^{\rho\sigma} \partial_\sigma) \rightarrow -\frac{1}{a^3} \frac{d}{dt}\left(a^3 \frac{d}{dt}\right). \tag{5}$$

Acting on any function of time $f(t)$, its retarded inverse reduces to simple integrations:

$$\left[\frac{1}{\Box} f\right](t) \equiv \mathcal{G}[f](t) = -\int_0^t dt' \frac{1}{a^3(t')} \int_0^{t'} dt'' a^3(t'') f(t''). \tag{6}$$

If we make the simplifying (and numerically justified) assumption that the power changes from $s = \frac{1}{2}$ to some other value at $t = t_{\rm eq}$, the integrals in (6) are easily carried out for our choice of $f = R$

$$\mathcal{G}[R](t)|_s = -\frac{6s(2s-1)}{(3s-1)} \times \left\{ \ln\left(\frac{t}{t_{\rm eq}}\right) - \frac{1}{3s-1} + \frac{1}{3s-1}\left(\frac{t_{\rm eq}}{t}\right)^{3s-1} \right\}. \quad (7)$$

For the matter dominance value of $s = 2/3$, and at the present time of $t_0 \sim 10^{10}$ years, this yields

$$\mathcal{G}[R](t_0)|_{s=\frac{2}{3}} \simeq -14.0. \quad (8)$$

If we think of correcting the field equations by this term (apart from small additions that enforce conservation, and whose form we will shortly exhibit) times the Einstein tensor, this result already illustrates how nonlocality allows simple time evolution to generate large numbers without fine-tuning.

Much larger values can be obtained through other operators, for example, the Paneitz operator arising in the context of conformal anomalies [14]. When specialized to our geometry (1), it takes the form

$$\frac{1}{\sqrt{-g}}\Delta_P \equiv \Box^2 + 2D_\mu\left(R^{\mu\nu} - \frac{1}{3}g^{\mu\nu}R\right)D_\nu \rightarrow \frac{1}{a^3}\frac{d}{dt}\left(a\frac{d}{dt}a\frac{d}{dt}a\frac{d}{dt}\right). \quad (9)$$

One gets about $10^6$ from the dimensionless combination of the inverse of this operator acting on $R^2$.

*Specific models.*—Here, we evaluate the consequences of the simplest alteration of the Einstein action,

$$\Delta\mathcal{L} \equiv \frac{1}{16\pi G}R\sqrt{-g}f(\mathcal{G}[R]). \quad (10)$$

[One could modify the cosmological term in a similar way, but that turns out to require fine-tuning to delay the onset of acceleration sufficiently.]

Naively varying a nonlocal action such as (10) would result in advanced Green's functions as well as the retarded ones (6) we desire. However, because conservation only depends on the Green's function being the inverse of a differential operator, one gets causal and conserved equations by simply replacing the advanced Green's functions by the retarded ones [12]. (To derive causal and conserved field equations from quantum field theory, one uses the Schwinger-Keldysh formalism [15]. This will generally result in dependence upon the real part of the propagator, as well as the retarded Green's function, which, if anything, may lead to even stronger effects than those we consider.) The resulting correction to the Einstein tensor is

$$\Delta G_{\mu\nu} = [G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu]\{f(\mathcal{G}[R]) + \mathcal{G}[Rf'(\mathcal{G}[R])]\} + \left[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\right] \times \partial_\rho(\mathcal{G}[R])\partial_\sigma(\mathcal{G}[Rf'(\mathcal{G}[R])]). \quad (11)$$

As promised, it takes the form of a nonlocal distortion of the Einstein tensor, plus additional terms which enforce the Bianchi identity for any $g_{\mu\nu}$. The additional terms involve derivatives, so they are typically small when $f(x)$ varies slowly. Note also that, except for the very special case of $f(x) = -x$, no model of this form can be obtained from integrating out a scalar. Whatever these models' origin, then, they are not scalar-tensor gravities in disguise.

Now note from (7) that $\mathcal{G}[R](t)$ is small for a long time after the onset of matter dominance. During this period, we may think of $\Delta G_{\mu\nu}$ as a perturbation of the stress tensor source, with $\Delta G_{00} = -8\pi G\Delta\rho$ and $g^{ij}\Delta G_{ij} = -24\pi G\Delta p$. Our corrections will tend to induce acceleration if evolution during matter domination carries us to the point where

$$\Delta G_{00} + g^{ij}\Delta G_{ij} = -8\pi G(\Delta\rho + 3\Delta p) > 6qH^2 = \frac{4}{3}\frac{1}{t^2}. \quad (12)$$

Naturally, once our corrections exceed the Einstein range, they are no longer perturbations, and numerical integration of the field equations is required.

One illustrative class of models has

$$f(x) = Ce^{-(3/4)kx}. \quad (13)$$

The resulting modification $\Delta G_{\mu\nu}$ gives

$$\Delta G_{00} + g^{ij}\Delta G_{ij} \simeq \frac{4}{3}\frac{1}{t^2} \times C\left(1 + \frac{3}{4}k\right)(2-3k)\left(\frac{t}{t_{\rm eq}}\right)^k. \quad (14)$$

Note that the right-hand side is positive for $k$ in the range $-\frac{4}{3} < k < +\frac{2}{3}$; actually, the range $0 < k < +\frac{2}{3}$ is needed to make the correction term grow. Our results do depend on two dimensionless coupling constants, $C$ and $k$, but neither need be very different from unity to provide a suitable delay for the onset of acceleration. For example, taking $k = 0.1$ and $C = 0.2$ would result in about the right onset time, in accord with the usual meaning of no fine-tuning as involving parameters $\sim 1$. [Strictly, all models with correction terms linear in $x$ face solar system constraints on their coefficient; this is not an obstacle for our purposes, as it can easily be fixed by more realistic, if analytically more complicated, models.]

As stated earlier, it is possible to construct the scalar potential $V(\phi)$ to support an arbitrary expansion history $a(t)$ obeying $\dot{H} > 0$ [6,7], and a similar construction exists for $f(R)$ theories [5,8]. The same possibility is of course present in our models, and indeed a procedure has recently been worked out for reconstructing the nonlocal distortion function $f(x)$ which would support an arbitrary expansion history [16]. Hence, there are certainly models of the type (10) that fit the supernova data. Nor must one even resort to exotic choices of $f(x)$. As might have been guessed from viewing these models as effective nonlocal distortions of Newton's constant, quiescence at recombination requires that $f(x)$ be small for $x$ near zero, whereas obtaining de Sitter expansion at asymptotically late times requires that $f(x)$ approach $-1$ from above for large, negative $x$. The onset of acceleration is controlled by the range of $x$ at which $f(x)$ becomes of order $-1$.

*Conclusions.*—We have explored the cosmological effects of some very simple nonlocally modified Einstein models inspired by loop corrections. Since their actual derivations from realistic quantum effects are likely to require nonperturbative summations, we regard them as purely phenomenological for now. Their two—equally important—main virtues are (unlike local variants): they naturally incorporate a delayed response to the transition from radiation to matter dominance, yet avoid major fine-tuning. There are of course many other open questions raised by the present proposal, such as finding optimal candidate actions while ensuring that nonlocality has no negative unintended consequences. Some apparent worries, such as (unwanted) solar system effects, are easily allayed. There, $\mathcal{G}[R] \sim GM/(c^2 r)$ is a small number. Although a single power of $\mathcal{G}[R]$ is observable—and constrains Brans-Dicke theory tightly [17]—higher powers, such as occur here, are negligible.

It should also be mentioned that nonlocality may have a positive use in the black hole information problem; see, e.g., [18]: the infalling matter that creates or accretes to a black hole is imprinted on the external geometry through its stress tensor. Nonlocal dependence on the Einstein tensor will retain that information. While $T_{\mu\nu}$ does not completely subsume the matter's internal structure, it is a significant repository thereof; furthermore, $\mathcal{G}$ is singular on null surfaces such as the event horizon.

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