Coulomb Blockade of Anyons in Quantum Antidots

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Coulomb interaction turns anyonic quasiparticles of a primary quantum Hall liquid with filling factor $\nu = 1/(2m+1)$ into hard-core anyons. We have developed a model of coherent transport of such quasiparticles in systems of multiple antidots by extending the Wigner-Jordan description of 1D Abelian anyons to tunneling problems. We show that the anyonic exchange statistics manifests itself in tunneling conductance even in the absence of quasiparticle exchanges. In particular, it can be seen as a nonvanishing resonant peak associated with quasiparticle tunneling through a line of three antidots.

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Quasiparticles of two-dimensional (2D) electron liquids in the regime of the fractional quantum Hall effect (FQHE) have unusual properties of fractional charge [1] and fractional exchange statistics [2,3]. The fractional charge was observed in experiments on antidot tunneling [4] and shot-noise measurements [5,6]. The situation with fractional statistics is so far less certain even in the case of the Abelian statistics, which is the subject of this work. Although the recent experiments [7] demonstrating unusual flux periodicity of conductance of a quasiparticle interferometer can be interpreted as a manifestation of the fractional statistics [8,9], this interpretation is not universally accepted [10,11]. There is a number of theoretical proposals (see, e.g., [12,13]) suggesting tunnel structures where the statistics should manifest itself through noise properties. Partly due to complexity of noise measurements, such experiments have not been performed successfully up to now. In this work, we show that coherent quasiparticle dynamics in multiantidot structures should provide clear signatures of the exchange statistics in dc transport. Most notably, in tunneling through a line of three antidots, fractional statistics leads to a nonvanishing peak of the tunnel conductance which would vanish for integer statistics.

These effects rely on the ability of quantum antidots to localize individual quasiparticles of the QH liquids [4,14,15]. The resulting transport phenomena in antidots are very similar to those associated with the Coulomb blockade [16] in tunneling of individual electrons in dots. For instance, similarly to a quantum dot [17], the linear conductance of one antidot shows periodic oscillations with each period corresponding to the addition of one quasiparticle [4,14,15,18,19]. Recently, we have developed a theory of such Coulomb-blockade-type tunneling for a double-antidot system [20], where quasiparticle exchange statistics does not affect the transport. The goal of this work is to extend this theory to antidot structures where the statistics does affect the conductance. The two simplest structures with this property consist of three antidots and have quasi-1D geometries with either periodic or open boundary conditions (Fig. 1). A technical issue that needed to be resolved to calculate the tunnel conductance is that the anyonic field operators defined through the Wigner-Jordan transformation [21–24] are not fully sufficient in the situations of tunneling. As we show below, to obtain correct matrix elements for anyon tunneling, one needs to keep track of the appropriate boundary conditions of the wave functions which are not accounted for directly in the field operators.

Specifically, we consider the antidots coupled by tunneling among themselves and to two opposite edges of the quantum Hall liquid (Fig. 1). The edges play the role of the quasiparticle reservoirs with the transport voltage V applied between them. We assume that the antidot-edge coupling is weak and can be treated as a perturbation. Quasiparticle transport through the antidots is governed then by the kinetic equation similar to that for Coulomb-blockade transport through quantum dots with discrete energy spectra [25]. Coherent quasiparticle dynamics requires that the relaxation rate Γ_d created by direct Coulomb antidot-edge coupling is weak. This condition should be satisfied if the edge-state confinement is sufficiently strong [20]. The requirement on the confinement is less stringent in the case of the antidot line [Fig. 1(b)], in

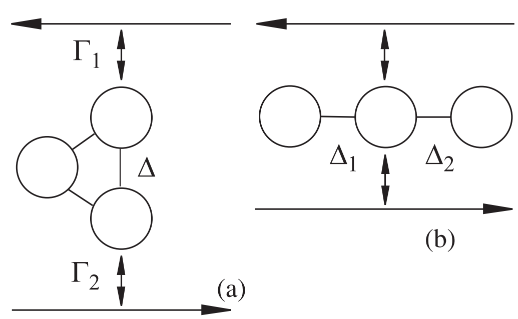

FIG. 1. Tunneling of anyonic quasiparticles between opposite edges of an FQHE liquid through quasi-1D triple-antidot systems: (a) loop, (b) open interval. Quasiparticles tunnel between the edges and the antidots with rates $\Gamma_{1,2}$. The antidots are coupled coherently by tunnel amplitudes Δ.

which antidot quasiparticles move along the edge, suppressing the antidot-edge coupling at low frequencies. We also assume that all quasiparticle energies on the antidots, tunnel amplitudes Δ, temperature T, and Coulomb interaction energies U between quasiparticles on different antidots are much smaller than the energy gap Δ^* for excitations on each antidot. This condition ensures that the state of each antidot is characterized completely by the occupation number n of its relevant quantized energy level. In any given range of the backgate voltage or magnetic field (which produces the overall shift of the antidot energies—see, e.g., [4,14,15]), there can be at most one quasiparticle on each antidot, $n = 0, 1$. This "hard-core" property of the quasiparticles means that they behave as fermions in terms of their occupation factors, despite the anyonic exchange statistics. All these assumptions can be summarized as: $\Gamma_d, \Gamma_j \ll \Delta, U, T \ll \Delta^*$.

Under these conditions, the antidot tunneling is dominated by the antidot energies. The quasi-1D geometry of the antidot systems we consider makes it possible to introduce the quasiparticle "coordinate" x numbering successive antidots; e.g., $x = -1, 0, 1$ for systems in Fig. 1. The quasiparticle Hamiltonian can be then written as

$$H = \sum_x [\epsilon_x n_x - (\Delta_x \xi_{x+1}^\dagger \xi_x + \text{H.c.})] + \sum_{x<y} U_{x,y} n_x n_y, \quad (1)$$

where ϵ_x are the energies of the relevant localized states on the antidots (taken relative to the common chemical potential of the edges at $V = 0$), Δ_x is the tunnel coupling between them, $U_{x,y}$ is the quasiparticle Coulomb repulsion, and $n_x \equiv \xi_x^\dagger \xi_x$. The quasiparticle operators $\xi_x^\dagger$, ξ_x in (1) can be viewed as the Klein factors left in the standard operators for the edge-state quasiparticles when all the edge magnetoplasmon modes are suppressed by the gap Δ^*. Characteristics of such Klein factors depend on the geometry of a specific tunneling problem; nontrivial examples can be found in [12,13,26,27]. In the Hamiltonian (1), ξ_x describe the hard-core anyons with exchange statistics $\pi\nu$. Wigner-Jordan transformation expresses them through the Fermi operators c_x [21]:

$$\xi_x = e^{i\pi(\nu-1)\sum_{z<x} n_z} c_x, \qquad \xi_y \xi_x = \xi_x \xi_y e^{i\pi\nu sgn(x-y)}, \quad (2)$$

with similar relations for $\xi^\dagger$.

Anyonic statistics creates an effective interaction between the quasiparticles which can be understood as the Aharonov-Bohm (AB) interaction between a flux tube "attached" to one of the particles and the charge carried by another. In general, this interaction can be masked by the direct Coulomb interaction $U_{x,y}$. In the *antidot loop* [Fig. 1(a)], however, $U_{x,y}$ is constant, $U_{x,y} = U$, and the interaction term in (1) reduces to $Un(n-1)/2$, with $n = \sum_x n_x$—the total number of the quasiparticles on the antidots. In this case, the Coulomb interaction contributes to the energy separation between the group of states with different n, but does not affect the level structure for given n. The hard-core property of quasiparticles limits n to the interval [0, 3]. For $n = 0$ and $n = 3$, the system has the "empty" and "completely filled" state with respective energies $E_0 = 0$, $E_3 = \sum_x \epsilon_x + 3U$. The spectrum E_{1k} of the three $n = 1$ states $|1k\rangle = \sum_x \phi_k(x) \xi_x^\dagger |0\rangle$ is obtained as usual from (1). In the uniform case $\epsilon_x = \epsilon$, $\Delta_x = \Delta$, with an external AB phase φ, one has $\phi_k(x) = e^{ikx}/L^{1/2}$ and

$$E_{1k} = \epsilon - \Delta \cos k, \qquad k = (2\pi m + \varphi)/L, \quad (3)$$

where $m = 0, 1, 2$, and the loop length is $L = 3$.

Anyonic statistics can be seen in the $n = 2$ states, $|2l\rangle = (1/\sqrt{2})\sum_{xy} \psi_l(x, y) \xi_y^\dagger \xi_x^\dagger |0\rangle$. The fermion-anyon relation (2) suggests that the stationary two-quasiparticle wave functions should coincide up to the exchange phase with that for free fermions:

$$\psi_l(x, y) = \frac{e^{i\pi(1-\nu)\text{sgn}(x-y)/2}}{\sqrt{2}} \det\begin{pmatrix} \phi_q(x) & \phi_q(y) \\ \phi_p(x) & \phi_p(y) \end{pmatrix}. \quad (4)$$

Here ϕs are the single-particle eigenstates of the Hamiltonian (1). [The states (4) are numbered with the index l of the third "unoccupied" eigenstate of (1) complementary to the two occupied ones q, p.] The boundary conditions for the ϕs are affected by the exchange phase in Eq. (4). To find them, we temporarily assume for clarity that coordinates x, y are continuous and lie in the interval $[0, L]$. Subsequent discretization does not change anything substantive in this discussion. The 1D hard-core particles are impenetrable and can be exchanged only by moving one of them, say, x, around the loop from $x = y + 0$ to $x = y - 0$ [Fig. 2(a)]. Since the loop is imbedded in the underlying 2D system, such an exchange means that the wave function acquires the phase factor $e^{i\pi\nu}$, in which the sign of ν is fixed by the properties of the 2D system, e.g., the direction of magnetic field in the case of FQHE liquid. Next, if the second particle is moved similarly, from $y = x + 0$ to $y = x - 0$, the wave function changes in the same way, for a total factor $e^{i2\pi\nu}$. Equation (4) shows that only one of these changes can agree with the 1D form of the exchange phase. As a result, the wave function (4) satisfies different boundary conditions in x and y:

$$\psi_l(L, y) = \psi_l(0, y)e^{i\varphi}, \qquad \psi_l(x, L) = \psi_l(x, 0)e^{i(\varphi+2\pi\nu)}. \quad (5)$$

Conditions (5) on the wave function (4) mean that the single-particle functions ϕ in (4) satisfy the boundary

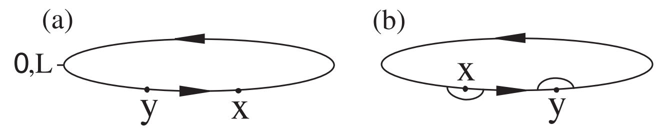

FIG. 2. Exchanges of hard-core anyons on a 1D loop: (a) real exchanges by transfer along the loop embedded in a 2D system and (b) formal exchanges describing the assumed boundary conditions (5) of the wave function.

condition that correspond to the effective AB phase $\varphi' = \varphi + \pi - \pi\nu$, i.e., the addition of an extra quasiparticle to the loop changed the AB phase by $\pi - \pi\nu$, where $-\pi\nu$ comes from the exchange statistics and π from the hard-core condition. This gives the energies of the two-quasiparticle states (4) as $U + E_{1q} + E_{1p}$, where, if the loop is uniform, the single-particle energies are given by Eq. (3) with $\varphi \to \varphi'$. In this case, $\sum_k E_{1k} = 0$, and the energies E_{2l} of the two-quasiparticle states are

$$E_{2l} = 2\epsilon + U - \Delta \cos l, \tag{6}$$

where $l = (2\pi m' + \varphi - \pi\nu)/3$, and $m' = 0, 1, 2$.

One of the consequences of this discussion is that the sign of ν in the 1D exchange phases of Eqs. (2) and (4) can be chosen arbitrarily for a given fixed sign of the 2D exchange phase. Reversing this sign only exchanges the character of the boundary conditions (5) between x and y. This fact has a simple interpretation. Although the 1D hard-core anyons cannot be exchanged directly, formally, coordinates x and y in Eq. (4) are independent and one needs to define how they move past each other at the point $x = y$. Depending on whether the x particle moves around y from below or [as in Fig. 2(b)] from above, its trajectory does or does not encircle the y particle, and the boundary condition for x is or is not affected by the statistical phase. The choice made for x immediately implies the opposite choice for y [Fig. 2(b)], accounting for different boundary conditions (5). This interpretation shows that in calculation of any matrix elements, the participating wave functions should be taken to have the same boundary conditions. While this requirement is natural for processes with the same number of anyons, it is less evident for tunneling that changes the number of anyons. Indeed, the most basic, tunnel Hamiltonian, description of tunneling into the point z of the system leads to the states

$$\xi_z^\dagger |1k\rangle = (1/\sqrt{2}) \sum_{xy} \psi_k(x, y) \xi_y^\dagger \xi_x^\dagger |0\rangle,$$
$$\psi_k(x, y) = [\phi_k(x)\delta_{y,z} - e^{i\pi(1-\nu)\mathrm{sgn}(x-y)} \delta_{x,z} \phi_k(y)]/\sqrt{2}. \tag{7}$$

One can see that Eq. (7) automatically implies specific choice of the boundary conditions which corresponds to the tunneling anyon not being encircled by anyons already in the system. This means that in the calculation of the tunnel matrix elements with the states (4), one should always pair the coordinate of the tunneling anyon with the discontinuous one in (5). Then, the tunnel matrix elements are obtained as

$$\langle 2l|\xi_z^\dagger|1k\rangle = \sqrt{2} \sum_x \psi_l^*(x, z)\phi_k(x). \tag{8}$$

For instance, in the case of uniform loop with states (3) and (6), we get up to an irrelevant phase factor

$$\langle 2l|\xi_z^\dagger|1k\rangle = (2/3)\cos[(k - l)/2]. \tag{9}$$

Specific anyonic interaction between quasiparticles can be seen in the fact that the matrix elements (9) do not vanish for any pair of indices k, l. In the fermionic case $\nu = 1$, one of the elements (9) always vanishes for any given k, since the two-particle state after tunneling necessarily has one particle in the original single-particle state. By contrast, the tunneling anyon can shift existing particle out of its state.

The matrix elements involving empty or fully occupied states coincide with those for fermions. Taken together with Eqs. (8) and (9) for transitions between the partially filled states, they determine the rates $\Gamma_j(E) = \gamma_j f_\nu(E)|\langle\xi_z^\dagger\rangle|^2$ of tunneling between the jth edge and the antidots, where γ_j is the overall magnitude of the tunneling rate, and

$$f_\nu(E) = (2\pi T/\omega_c)^{\nu-1}|\Gamma(\nu/2 + iE/2\pi T)|^2 e^{-E/2T}/2\pi\Gamma(\nu)$$

is its energy dependence associated with the Luttinger-liquid correlations in the edges [28]. Here $\Gamma(z)$ is the gamma function and ω_c is the cutoff energy of the edge excitations. The rates $\Gamma_j(E)$ can be used in the standard kinetic equation to calculate the conductance of the antidot system [20]. Anyonic statistics of quasiparticles affects the position and amplitude of the conductance peaks through the shift of the energy levels by quasiparticle tunneling [described, e.g., by Eq. (6)] and through the kinetic effects caused by the anyonic features in the matrix elements (8). In the case of the antidot loop [Fig. 1(a)]; however, effects of statistics are masked by the external AB flux φ through the loop. Since the area of practical antidots is much larger than the internal area of the loop, φ is essentially random and cannot be controlled by external magnetic field on the relevant scale of one period of conductance oscillations. Below, we present the results for the similar case of a *line of antidots* [Fig. 1(b)], the conductance of which is insensitive to the AB flux and shows effects of statistics in the tunneling matrix elements.

As before, the quasiparticle Hamiltonian is given by Eq. (1). In this geometry, the interaction energy $U_1 \equiv U_{1,0} = U_{0,-1}$ between the nearest-neighbor antidots is in general different from the interaction $U_2 \equiv U_{1,-1}$ between the quasiparticles at the ends. The localization energies on the antidots can be written as $\epsilon_j = \epsilon + x\delta + 2\lambda|x|$. We consider first the unbiased line, $\delta = 0$. At low temperatures, $T \ll \Delta, U$, only the ground states of n quasiparticles with energies E_n participate in transport: $E_0 = 0$, $E_1 = \epsilon + \lambda - \omega$, $E_2 = 2\epsilon + 3\lambda - \bar{\omega} + (U_a + U_b)/2$, and $E_3 = 3\epsilon + 2U_a + U_b + 4\lambda$, where $\omega = (\Delta_1^2 + \Delta_2^2 + \lambda^2)^{1/2}$ and $\bar{\omega}$ is given by the same expression with λ replaced by $\bar{\lambda} = \lambda - (U_1 - U_2)/2$. In this regime, the linear conductance G consists of three peaks, with each peak associated with addition of one more quasiparticle to the antidots,

$$G = \frac{(e\nu)^2}{T} \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} \frac{a_n f_\nu(E_{n+1} - E_n)}{1 + \exp[-(E_{n+1} - E_n)/T]}, \tag{10}$$

where $a_n \equiv |\langle n + 1|\xi_0^\dagger|n\rangle|^2$. The amplitudes a_0, a_2 are effectively single particle, and thus, independent of

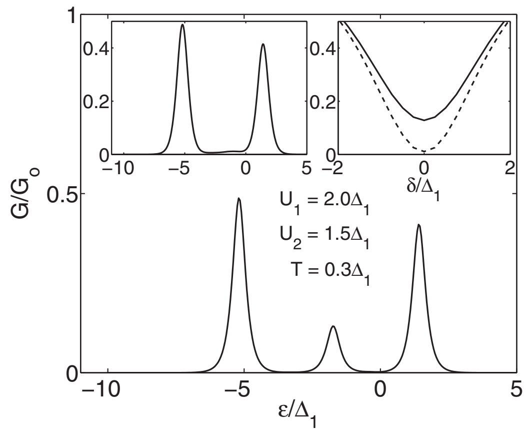

FIG. 3. Linear conductance G of the antidot line in a $\nu = 1/3$ FQHE liquid [Fig. 1(b)] as a function of the common antidot energy ϵ relative to the edges. In contrast to electrons ($\nu = 1$, left inset), tunneling of quasiparticles with fractional exchange statistics produces nonvanishing conductance peak associated with transition between the ground states of one and two quasiparticles. The maximum of this peak is shown in the right inset ($\nu = 1/3$—solid line, $\nu = 1$—dashed line) as a function of the bias δ. The curves are plotted for $\Delta_1 = \Delta_2$, $\lambda = 0$, $\gamma_1 = \gamma_2$; conductance is normalized to $G_0 = (e\nu)^2\Gamma_1(0)/\Delta_1$.

the exchange statistics: $a_0 = (\omega + \lambda)/2\omega$, and $a_2 = (\bar{\omega} - \bar{\lambda})/2\bar{\omega}$. By contrast, the amplitude a_1 of the transition from one to two quasiparticles is multiparticle and is found from Eqs. (4) and (8) to be strongly statistics dependent,

$$a_1 = \frac{\Delta_1^2\Delta_2^2}{(\omega + \lambda)\omega(\bar{\omega} - \bar{\lambda})\bar{\omega}}\cos^2(\pi\nu/2). \tag{11}$$

In particular, a_1 vanishes for electron tunneling ($\nu = 1$), but is nonvanishing in the case of fractional statistics, e.g., for $\nu = 1/3$, when $\cos^2(\pi\nu/2) = 3/4$. This is illustrated in Fig. 3 which shows the conductance G obtained by direct solution of the full kinetic equation for tunneling through the antidots. Qualitatively, the vanishing amplitude a_1 for electrons can be understood as a result of destructive interference between the two terms in the two-particle wave function which correspond to different ordering of the added/existing electron on the antidot line. The opposite signs of these two terms lead to vanishing overlap with the single-particle state in the tunnel matrix element. Fractional statistics of quasiparticles makes this destructive interference incomplete. Finite bias $\delta \neq 0$ along the line suppresses this interference making the effect of the statistics smaller. One can still distinguish the fractional statistics by looking at the dependence of the amplitude of the middle peak of conductance on the bias δ shown in the right inset in Fig. 3.

In conclusion, we have developed a model of coherent transport of anyonic quasiparticles in systems of multiple antidots. In antidot loops, addition of individual quasiparticles shifts the quasiparticle energy spectrum by adding statistical flux to the loop. In the case without loops, energy levels are insensitive to quasiparticle statistics, but the statistics still manifests itself in the tunneling rates and hence dc conductance of the antidot system.

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