Direct Measurements of Fractional Quantum Hall Effect Gaps

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We measure the chemical potential jump across the fractional gap in the low-temperature limit in the two-dimensional electron system of GaAs/AlGaAs single heterojunctions. In the fully spin-polarized regime, the gap for filling factor $\nu = 1/3$ increases *linearly* with the magnetic field and is coincident with that for $\nu = 2/3$, reflecting the electron-hole symmetry in the spin-split Landau level. In low magnetic fields, at the ground-state spin transition for $\nu = 2/3$, a correlated behavior of the $\nu = 1/3$ and $\nu = 2/3$ gaps is observed.

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Plateaus of the Hall resistance corresponding to zeros in the longitudinal resistance at fractional filling factors of Landau levels in two-dimensional (2D) electron systems, known as the fractional quantum Hall effect [1], are believed to be caused by electron-electron interactions (for a recent review, see Ref. [2]). Two theoretical approaches to the phenomenon have been formulated over the years. One approach is based on a trial wave function of the ground state [3], and the other exploits an introduction of composite fermions to reduce the problem to a single-particle one [4,5]. The fractional gap is predicted to be determined by the Coulomb interaction in the form $e^2/\varepsilon l_B$ [where ε is the dielectric constant and $l_B = (\hbar c/eB)^{1/2}$ is the magnetic length], which leads to a square-root dependence of the gap on magnetic field B. Observation of such a behavior would confirm the predicted gap origin.

Attempts to experimentally estimate the fractional gap value yielded similar results, at least, in high magnetic fields [6–11]. Still, the expected dependence of the gap on magnetic field has not been either confirmed or rejected. The problems with experimental verification are as follows. Standard measurements of activation energy at the longitudinal resistance minima allow one to determine the mobility gap [6,7] which may be different from the gap in the spectrum. The data for the gap obtained by thermodynamic measurements depended strongly on temperature [8]; for this reason, the magnetic field dependence of the gap may be distorted.

In this Letter, we report measurements of the chemical potential jump across the fractional gap at filling factor $\nu = 1/3$ and $\nu = 2/3$ in the 2D electron system in GaAs/AlGaAs single heterojunctions using a magnetocapacitance technique. We find that the gap $\Delta\mu_e$ increases with decreasing temperature, and in the low-temperature limit it saturates and becomes independent of temperature. In magnetic fields above ≈ 5 T, the limiting value $\Delta\mu_e^0$ for $\nu = 1/3$ is described with good accuracy by a linear increase of the gap with magnetic field and is practically coincident with the gap for $\nu = 2/3$. In lower magnetic fields, the minimum of the gap $\Delta\mu_e^0$ at $\nu = 2/3$ occurring at a critical field of ≈ 4 T, which corresponds to ground-state spin transition [12–15], is accompanied with a change in the behavior of that at $\nu = 1/3$. The correlation between the magnetic field dependences of both gaps indicates the presence of a spin transition for the $\nu = 1/3$ gap. The linear dependence of the fractional gap on magnetic field as well as the electron-hole symmetry in the spin-split Landau level are the case in the completely spin-polarized regime. Since the interaction-enhanced gaps in the integer quantum Hall effect in different 2D electron systems also increase linearly with magnetic field [16,17], the linear law seems robust. This indicates that electron-electron interactions in 2D should be treated in a less straightforward way.

Measurements were made in an Oxford dilution refrigerator with a base temperature of ≈ 30 mK on remotely doped GaAs/AlGaAs single heterojunctions with a low-temperature mobility $\approx 4 \times 10^6$ cm^2/V s at electron density 9×10^{10} cm^{-2}. Samples had the quasi-Corbino geometry with areas 27×10^4 (sample 1) and 2.1×10^4 μm^2 (sample 2). The depth of the 2D electron layer was 200 nm. A metallic gate was deposited onto the surface of the sample, which allowed variation of the electron density by applying a dc bias between the gate and the 2D electrons. The gate voltage was modulated with a small ac voltage of 2.5 mV at frequencies in the range 0.1–2.5 Hz, and both the imaginary and real components of the current were measured with high precision ($\sim 10^{-16}$ A) using a current-voltage converter and a lock-in amplifier. Smallness of the real current component as well as proportionality of the imaginary current component to the excitation frequency ensure that we reach the low-frequency limit and the measured magnetocapacitance is not distorted by lateral transport effects. A dip in the magnetocapacitance in the quantum Hall effect is directly related to a jump of the chemical potential across a corresponding gap in the spectrum of the 2D electron system [18]:

$$\frac{1}{C} = \frac{1}{C_0} + \frac{1}{Ae^2 dn_s/d\mu}, \quad (1)$$

where C_0 is the geometric capacitance between the gate and the 2D electrons, A is the sample area, and the derivative $dn_s/d\mu$ of the electron density over the chemical potential is the thermodynamic density of states.

A magnetocapacitance trace C as a function of gate voltage V_g is displayed in Fig. 1(a) for a magnetic field of 9 T. Narrow minima in C accompanied at their edges by local maxima are seen at filling factor $\nu \equiv n_s hc/eB = 1/3$ and $\nu = 2/3$. Near the filling factor $\nu = 1/2$, the capacitance C in the range of magnetic fields studied reaches its high-field value determined by the geometric capacitance C_0 (dashed line). We have verified that the obtained C_0 corresponds to the value calculated using Eq. (1) from the zero-field capacitance and the density of states $m/\pi\hbar^2$ (where $m = 0.067m_e$ and m_e is the free electron mass). The geometric capacitance C_0 increases with electron density as the 2D electrons are forced closer to the interface. The chemical potential jump $\Delta\mu_e$ for electrons at fractional filling factor can be determined by integrating the magnetocapacitance over the dip:

$$\Delta\mu_e = \frac{Ae^2}{C_0}\int_{\text{dip}} \frac{C_0 - C}{C} dn_s = \frac{e}{C_0}\int_{\text{dip}} (C_0 - C) dV_g. \quad (2)$$

As seen from Fig. 1(b), the difference $\delta C = C - C_0$ versus ν is nearly symmetric about filling factor $\nu = 1/2$. Maxima in $\delta C > 0$ observed near $\nu = 0$ and $\nu = 1$ are related to the so-called negative thermodynamic compressibility [19,20]. The effect is caused by the intralevel interactions between quasiparticles which provide a negative contribution of order $-(e^2/\varepsilon l_B)\{\nu\}^{1/2}$ to the chemical potential (here $\{\nu\}$ is the deviation of the filling factor from the nearest integer). This leads to an upward shift of the features at fractional filling factors which is more pronounced for $\nu = 2/3$ and for magnetic fields below $\approx$ 10 T. To allow for the shift, we replace the value C_0 in the integrand of Eq. (2) by a reference curve C_{ref} that is obtained by extrapolating the $C(\nu)$ dependence from above and below the fractional-ν feature. Based on the above contribution $d\mu/dn_s \propto \{\nu\}^{-1/2}$ in the spirit of Ref. [8], one can describe the experimental $\delta C(\nu)$ excluding the fractional-ν structures [the dotted line in Fig. 1(b)]. This naturally gives an extrapolation curve C_{ref}. Importantly, we have verified that the determined $\Delta\mu_e$ is not sensitive to the particular extrapolation law. Note that unlike penetrating field technique [8], our method is not able to yield the thermodynamic compressibility in absolute units; however, both techniques are equally good for determining gaps in the spectrum.

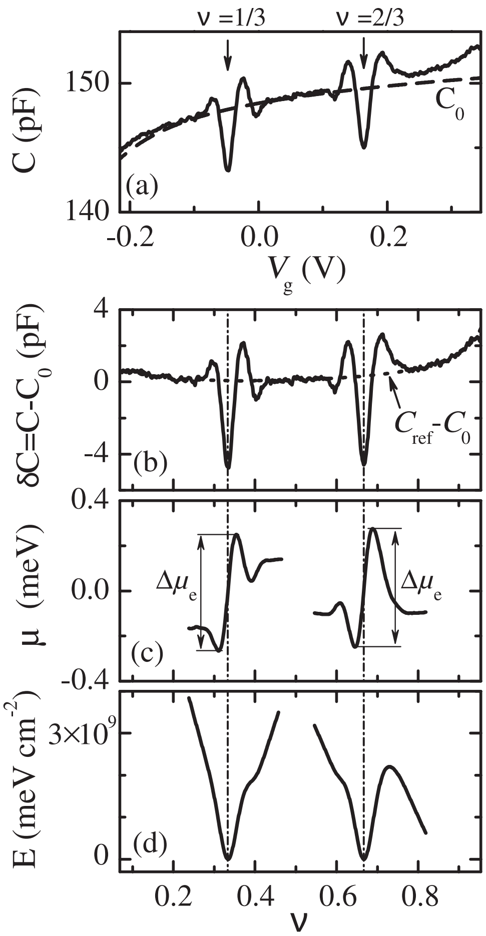

FIG. 1. (a) Magnetocapacitance as a function of gate voltage in sample 1 in a magnetic field of 9 T at a temperature of 0.18 K. Also shown by a dashed line is the geometric capacitance C_0. (b) The difference $C - C_0$ as a function of filling factor. The dotted line corresponds to the reference curve C_{ref}. (c) The chemical potential near $\nu = 1/3$ and $\nu = 2/3$ obtained by integrating the magnetocapacitance; see text. (d) The energy of the 2D electron system near $\nu = 1/3$ and $\nu = 2/3$ obtained by integrating the chemical potential. The zero level in (c) and (d) corresponds to the fractional ν.

It is easy to determine the behavior of the chemical potential when the filling factor traverses the fractional gap, as shown in Fig. 1(c). The chemical potential jump corresponds to a cusp on the dependence of the energy E of the 2D electron system on filling factor [2], shown in Fig. 1(d) for illustration. The difference between the μ values well above and well below the jump is smaller than $\Delta\mu_e$, as determined by the local maxima at the edges of the dip in C. This can be caused by both the gap closing and the interaction effect similar to that mentioned above for integer filling factor. The magnetocapacitance data do not allow one to distinguish whether or not the fractional gap closes when the Fermi level lies outside the gap.

In Fig. 2, we show the temperature dependence of the gap for $\nu = 1/3$ and $\nu = 2/3$. As the temperature is decreased, the value $\Delta\mu_e$ increases and, in the limit of

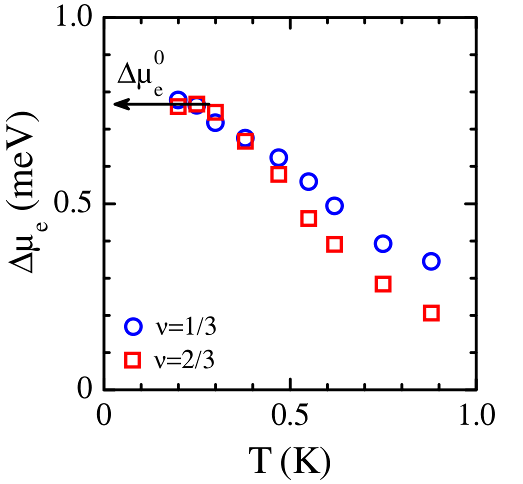

FIG. 2 (color online). Dependence of the fractional gap on temperature in sample 2 at $B = 11.5$ T. The value $\Delta\mu_e^0$ is indicated.

low temperatures, the gap saturates and becomes independent of temperature. It is the saturated low-temperature value $\Delta\mu_e^0$ that will be studied in the following as a function of the magnetic field. Note that the rapid decay of the gap with temperature at $T \ll \Delta\mu_e^0/k_B$ may indicate a phase transition [21].

In Fig. 3, we show how the $\nu = 1/3$ and $\nu = 2/3$ gap $\Delta\mu_e^0$ changes with magnetic field. Above $B \approx 5$ T, the data for $\nu = 1/3$ are best described by a linear increase of the gap with magnetic field [22] and are practically coincident with the data for $\nu = 2/3$. In lower magnetic fields, the value $\Delta\mu_e^0$ for $\nu = 2/3$ is a minimum [inset of Fig. 3(a)], which corresponds to spin transition in the ground state [12–15]. The occurrence of the spin transition has been double-checked by analyzing the $C(V_g)$ curves [inset of Fig. 3(b)]. Just below $B = 4$ T, in the close vicinity of the transition, a double-minimum structure in the capacitance at filling factor $\nu = 2/3$ is observed. This structure is conspicuous at the lowest temperatures, as caused by the resistive contribution to the minima in C, and is similar to the structures observed at the magnetic transitions in the integer [23–25] and $\nu = 2/3$ fractional quantum Hall effect [13,14,26]. As seen from Fig. 3(a), the minimum of the $\nu = 2/3$ gap is accompanied with a deviation at $B = B_c \approx 4$ T in the magnetic field dependence of the $\nu = 1/3$ gap from the linear behavior, the lowest-B data for the latter gap being comparable to the Zeeman energy in bulk GaAs (dashed line). It is worth noting that the data obtained on both samples in the magnetic field range where they overlap are very similar; measurements on sample 2 in magnetic fields below 4 T were hampered by the integer quantum Hall effect in Corbino-like contacts.

Since in high magnetic fields the value of fractional gap determined in the experiment is similar to the previously obtained results [6–8], it is unlikely to be strongly influenced by the residual disorder in the 2D electron system.

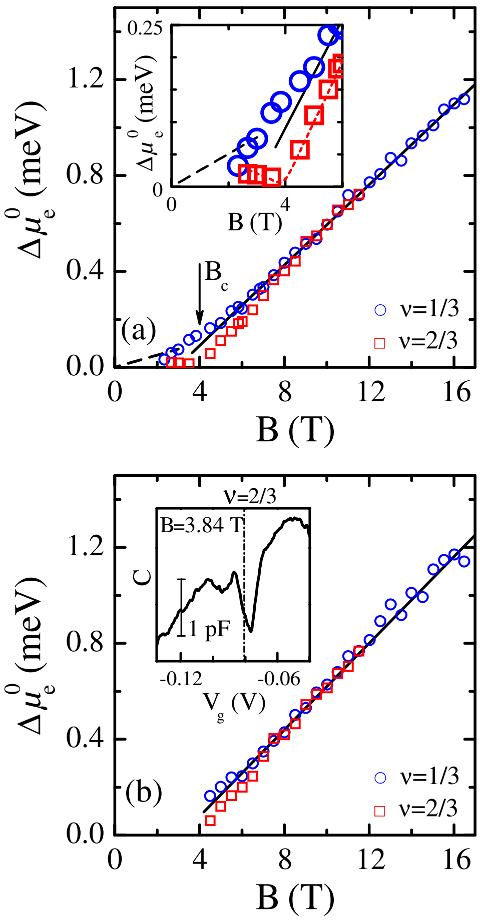

FIG. 3 (color online). (a) Change of the fractional gap $\Delta\mu_e^0$ with magnetic field for sample 1. The solid line is a linear fit to the high-field data for the $\nu = 1/3$ gap. The change in the behavior of the $\nu = 1/3$ gap is marked by the arrow. The dashed line corresponds to the Zeeman energy in bulk GaAs. A close-up view on the low-field region is displayed in the inset. The dotted line is a guide to the eye. (b) The same as in (a) for sample 2. Inset: Magnetocapacitance versus gate voltage in sample 1 at $T \approx 30$ mK.

Bearing in mind the above-mentioned problems with the data interpretation of transport experiments, one can crudely compare the gap $\Delta\mu_e^0$ obtained by thermodynamic measurements with the recent results of activation energy measurements in similar samples at $\nu = 1/3$ [7]. The double activation energy Δ_{act} is approximately equal to $\Delta\mu_e^0$ at $B \lesssim 8$ T, whereas in higher magnetic fields its increase with B weakens compared to that of $\Delta\mu_e^0(B)$. Although the ratio $\Delta\mu_e^0/\Delta_{act}$ reaches the factor of about 1.7 in the highest accessible magnetic fields, it is still far from the theoretically expected value $\Delta\mu_e^0/\Delta_{act} = 3$ [5,27]. The discrepancy may be due to particularities of the activation energy method.

We would like to emphasize that the thermodynamic measurements allow us to study the variation of the ground-state energy of the 2D electron system as the density of quasiparticles is varied at filling factors around

$\nu = 1/3$ and $\nu = 2/3$; the fractional gap is equal to $\Delta\mu_e(\nu) = dE/dn_s|_{\nu+0} - dE/dn_s|_{\nu-0}$. As inferred from the correlated behavior of the gaps at $\nu = 1/3$ and $\nu = 2/3$ in low magnetic fields [Fig. 3(a)], the change at $B = B_c$ in the dependence of the $\nu = 1/3$ gap on magnetic field is connected with the expected change of the spin of the "excited" state [2,28]; i.e., the 2D electron system at filling factors just above $\nu = 1/3$ should be fully spin polarized at $B > B_c$, while in the opposite case of $B < B_c$, the $\nu = 1/3$ gap is expected to be of Zeeman origin. The regime of small Zeeman energies was studied in Ref. [29] where the pressure-induced vanishing of the g factor/Zeeman energy in $B \gtrsim 5$ T was found to be responsible for the decrease of the $\nu = 1/3$ gap. Note that the lowest-B behavior may be more sophisticated because the gap can in principle collapse due to disorder in the 2D electron system. In the high-field limit, we are interested in here, the gaps at $\nu = 1/3$ and $\nu = 2/3$ are coincident with each other reflecting the electron-hole symmetry in the spin-split Landau level (Figs. 1 and 3), and the electron spins are completely polarized both at ν just above 1/3 and at ν just above 2/3. We note that the observed electron-hole symmetry is basically a property of a low-disordered 2D electron system.

It is important that the linear dependence of the fractional gap on magnetic field is observed in the fully spin-polarized regime. This linear behavior is concurrent with that of the interaction-enhanced gaps in the integer quantum Hall effect [16,17] and, therefore, the linear law seems robust, being valid for different gaps of many-body origin in different 2D electron systems with different interaction and disorder strengths. This points to a failure of the straightforward way of treating electron-electron interactions in 2D which leads to a square-root dependence of the gap on magnetic field. There is a noteworthy distinction between the interaction-enhanced gaps in the integer quantum Hall effect and the fractional gap: while in the former case the exchange energy is involved due to a change of the (iso)spin index, the exchange energy contribution to the fractional gap in the fully spin-polarized regime is not expected. Most likely, electron-electron correlations in 2D systems should be considered more carefully for the sufficient theory to be developed.

In summary, we have studied the variation of the ground-state energy of the 2D electron system in GaAs/AlGaAs single heterojunctions with changing filling factor around $\nu = 1/3$ and $\nu = 2/3$ and determined the fractional gap in the limit of low temperatures. In low magnetic fields, the minimum of the $\nu = 2/3$ gap, which corresponds to ground-state spin transition, is found to be accompanied with a change in the behavior of the $\nu = 1/3$ gap. The correlation between the dependences of both gaps on magnetic field indicates the presence of a spin transition for the $\nu = 1/3$ gap. In high magnetic fields, in the completely spin-polarized regime, the gap for $\nu = 1/3$ increases linearly with magnetic field and is coincident with that for $\nu = 2/3$, reflecting the electron-hole symmetry in the spin-split Landau level. The straightforward way of treating electron-electron interactions in 2D fails to explain the linear behavior of the fractional gap which is concurrent with that of the interaction-enhanced gaps in the integer quantum Hall effect.

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