Pattern Transformation Triggered by Deformation

T. Mullin*

Manchester Centre for Nonlinear Dynamics, University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom

S. Deschanel, K. Bertoldi, and M. C. Boyce

Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307, USA

(Received 2 March 2007; published 22 August 2007)

Periodic elastomeric cellular solids are subjected to uniaxial compression and novel transformations of the patterned structures are found upon reaching a critical value of applied load. The results of a numerical investigation reveal that the pattern switch is triggered by a reversible elastic instability. Excellent quantitative agreement between numerical and experimental results is found and the transformations are found to be remarkably uniform across the samples. It is proposed that the mechanism will also operate at much smaller scales opening the possibility for imprinting complex patterns at the nanoscale or switching photonic and phononic crystals in a controlled way.

DOI: 10.1103/PhysRevLett.99.084301 PACS numbers: 42.70.Qs, 63.20.−e, 64.60.Cn

The creation and use of nano- and micron-scale periodic structures to achieve unique properties is an exciting challenge of modern physics. The inspiration for much of this work comes from the natural world and, in the case of photonic crystals, is provided by the iridescent phenomena in butterflies, beetles, moths, birds, and fish [1–4]. Other unique properties make them suitable as phononic crystals [5] and as tunable hydrophobic surfaces [6]. The ability to synthetically produce periodic structures at the micron and submicron length scales through microfabrication processes [7,8], interference lithography [9–14], as well as thermodynamically driven self-assembly [15,16] has created new opportunities to mimic natural configurations and properties. These structures are generally static in nature or, in some instances, change in a more or less affine manner with deformation or other external stimuli [17,18]. Hence, properties which are dependent on the precise length scale and/or spacing of the periodic features will exhibit a gradual monotonic change with deformation. Here, we reveal the ability to use deformation to trigger dramatic pattern transformations in certain classes of simple periodic structures at relatively small strains. The transformations are found to be a result of an elastic instability, are triggered over a small range of the applied load, and are thus reversible and repeatable.

Cellular solids are characterized by a porous microstructure whereby the nature of the porosity may be either heterogeneous as a result of the foaming process or may be very regular due to specifically engineered structures such as honeycombs. The mechanical behavior of such systems is reasonably well understood [19]. Here, we manufacture two-dimensional latticed cellular structures from elastomeric sheets. The latticed structures reveal novel pattern transformations triggered by the applied load. Although experimental results are presented for structures fabricated at the millimeter length scale, the deformation-induced transformations are equally applicable to the same periodic structures at the micron and nanoscales and thus enable several avenues for new materials with transformative photonic, phononic, and hydrophobic or hydrophilic attributes.

The experiments were performed using elastic specimens with arrangements of holes cut out of the material. The periodic lattice microstructures were water jet cut from 9.4 mm thick sheets of the photoelastic elastomer PSM-4 (Measurements Group) and the samples were nominally 100 mm by 100 mm. The material is birefringent under strain and has been used to study stress and strain pathways in granular materials and structures [20,21]. Each specimen was subjected to uniaxial compression in the plane of the sheet at a constant nominal strain rate of 10^{-4} s^{-1} using a Zwick screw-driven testing machine.

The first sample comprised a microstructure of a 10 by 10 square array of circular holes of 8.67 mm diameter with 9.97 mm center-to-center spacing, vertically and horizontally. The second specimen was a rectangular lattice of 12 by 17 elliptical holes of major axes 10.02 mm and minor axis 5.34 mm, with horizontal and vertical center-to-center spacings of 11.02 mm and 5.99 mm, respectively. The specimens were placed between two close fitting 5 mm thick polymethylmethacrylate (PMMA) sheets to eliminate out-of-plane buckling. Chalk was lightly dusted on the surfaces to help reduce frictional effects. The specimen and PMMA sheets were located on the Zwick platen and an aluminum bar spanning the entire cross section of the specimen was used to uniformly displace the top surface. A photo-flood lamp backlit the apparatus and both still and video images were recorded. In addition, polarizer sheets were used to qualitatively examine the stress distribution within the structure from the induced birefringence in the material. The nominal stress versus nominal strain behavior was recorded automatically.

Numerical simulations of the deformation of the two periodic structures were conducted utilizing nonlinear finite element analysis; the accuracy of the mesh was ascertained through a mesh refinement study. The elastomeric stress-strain behavior was modeled as a nearly incompressible neo-Hookean solid with a shear modulus of 3.25 MPa. The sheet structures were modeled in two manners: (1) considering a representative volume element (RVE) of the domain with appropriate periodic boundary conditions and containing small internal defects to aid the initiation of instabilities and (2) considering the full specimen, thus capturing any small geometry differences at the boundaries as well as any boundary effects on either constraining (top or bottom surfaces) or triggering (free side surfaces) the transformation.

Experimental and numerical results for the nominal stress versus nominal strain behavior of the square array of circular holes, up to a compressive strain of 0.10, are shown in Fig. 1. The behavior is characterized by an initial linear elastic behavior with a sudden departure from linearity to a plateau stress. The departure from linearity is a result of a sudden transformation in the periodic pattern as shown in the snapshots of deformed configurations at different strains (Fig. 2). Figure 1 also provides a direct comparison between the numerical and experimental stress-strain results. There is excellent quantitative agreement between the full model calculation and the experiment. During the first loading in the experiment, the instability is abrupt and characterized by the sharp load drop seen in Fig. 1; subsequent experimental cycles do not exhibit the load drop and coincide more directly with the full model simulation result.

The experimental and numerical images of the deformed configurations are shown in Fig. 2. A gradual and homogeneous compression of the circular holes pattern during the linear elastic range of the deformation is replaced by a transformation to a strikingly different pattern of alternating mutually orthogonal ellipses above a nominal strain of 0.04. This corresponds to the plateau region immediately after the departure from linearity. The pattern transformation is a result of the compressive loading of the vertical interhole ligaments which undergo a buckling instability, thereby triggering the change to the new configuration. This is a truly elastic event and is completely reversible

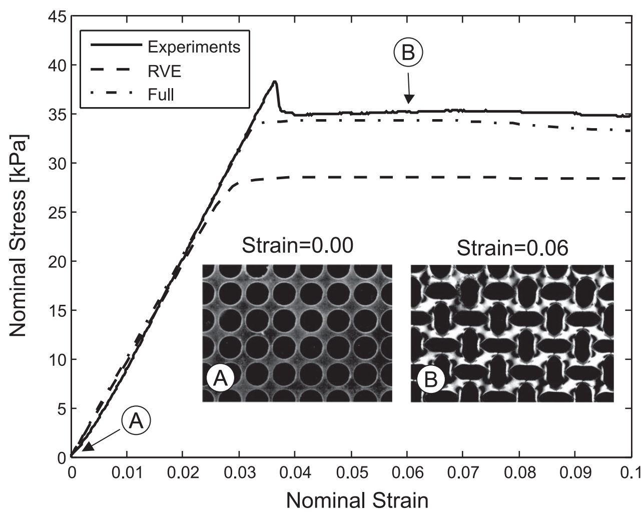

FIG. 1. Nominal stress vs nominal strain curves for the square array showing experimental and computational (full model and RVE model) results. The departure from linearity is the result of an elastic buckling instability in the microstructure that triggers a pattern transformation.

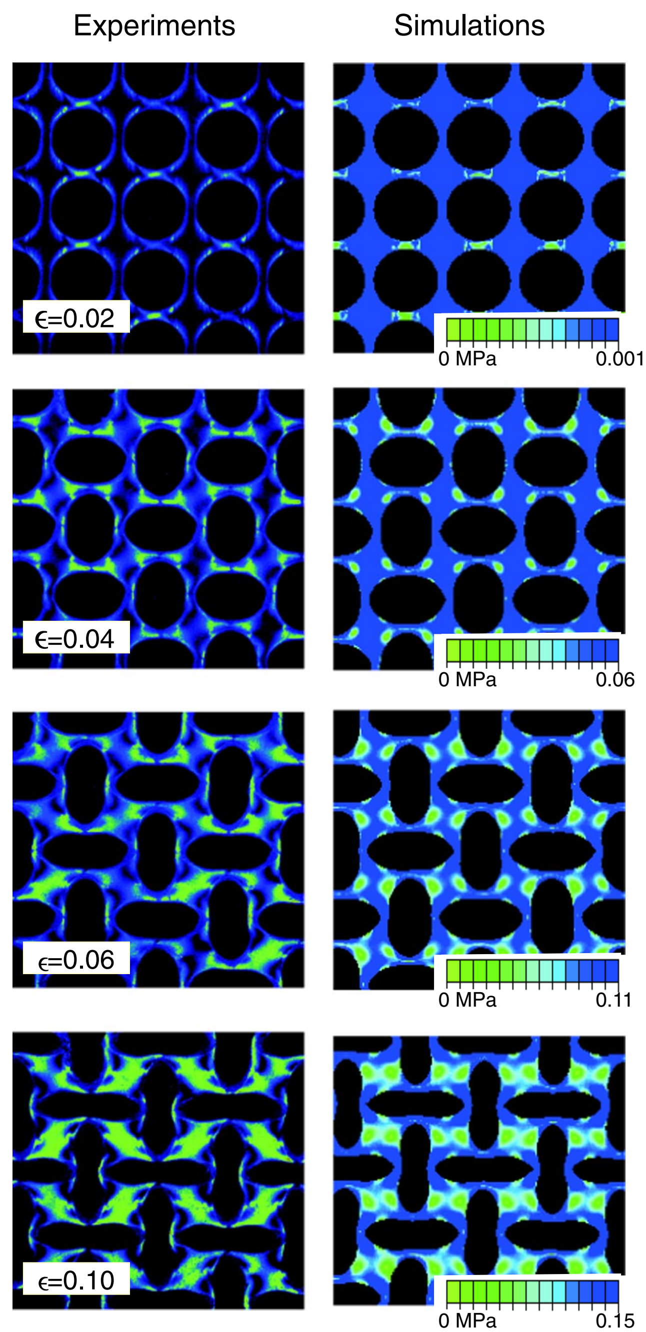

FIG. 2 (color online). Experimental (left) and numerical (right) images of the square lattice at different levels of macroscopic strain: 2%, 4%, 6%, and 10%. The experimental samples were viewed through cross polars and the colors give a qualitative indicator of the regions of localized stress which provide a useful comparison with the calculated principal stress levels.

upon unloading (and occurs again upon reloading). Once formed, the new pattern becomes further accentuated in shape (the major axis of each ellipse lengthens and the minor axis of each ellipse shortens) with increasing macroscopic compressive strain as seen in the deformed images at strains of 0.06 and 0.10. The color contours seen in both the experiment and simulation results track the difference in the in-plane principal stress levels.

After transformation to the new configuration, much of the macroscopic deformation is observed to be accommodated by the rotation of the four matrix domains diagonally bridging neighboring holes; these domains experience negligible strain but undergo large rotations. This local behavior has also been observed to be a deformation mechanism in thermoplastic vulcanizates [22].

The RVE results exhibit an earlier departure from linearity than the full model since the former does not capture the influence of the boundary conditions (on the top, bottom, and side surfaces) on the initiation of the instability. However, the RVE results do capture the deformation-triggered transformation to the new pattern. Hence, the RVE results indicate that the transformation event does not rely on a finite size lattice with triggering from boundary edge conditions and, hence, will also occur in very large arrays.

Interestingly, in a recent model of Triantafyllidis *et al.* [23], the ‘‘failure surface’’ of two-dimensional two-phase periodic structures was determined in biaxial strain space. The onset of failure is taken to be a result of local structural instabilities as well as corresponding loss of ellipticity in the macroscopic homogenized tangent stiffness tensor of the composite material. The onset of failure is taken as the point at which instability initiates and deformation localizes in the material. This study is pertinent to the present work in its framework and ability to assess the onset of ‘‘failure’’ which corresponds to, in our study, the onset of the phase transformation. Here, we emphasize not only the onset of the transfiguration, but also the uniformity and robustness of the metamorphosis which occurs throughout the structure (as demonstrated in our experiments and simulations). In other words, the instability does not localize deformation over a few repeating cells or rows, but instead results in a completely homogeneous pattern transformation throughout the structure. The transformed structure is then further accentuated with continuing deformation and this occurs at constant load—a characteristic of a ‘‘superelastic’’ deformation-induced phase transforming solid such as shape memory alloys. The entire process is reversible and repeatable. This basic result encourages pursuit of other such periodic elastomeric structures where next we study regular patterns of elliptical holes.

The nominal stress-strain behavior for the rectangular array of elliptical holes up to a strain of 0.06 is shown in Fig. 3 with corresponding deformation images shown in Fig. 4. There is excellent agreement between the full model predictions and the experimental results. The stress-strain

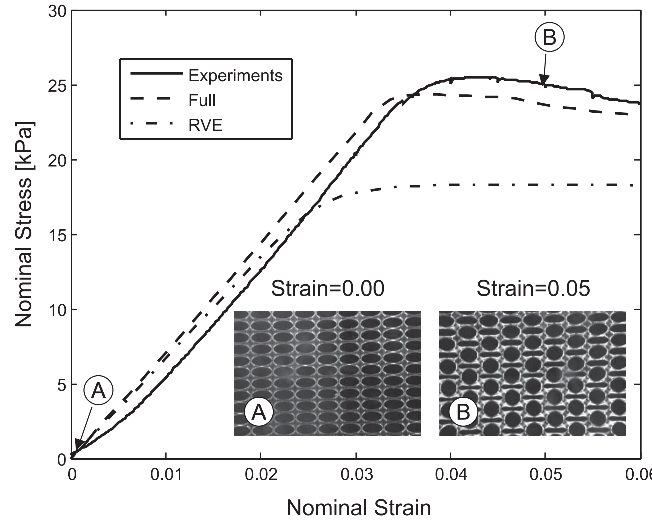

FIG. 3. Experimental and numerical results for the nominal stress-nominal strain behavior of the rectangular latticed structure of elliptical holes. The departure from linearity indicates the onset of the pattern transformation.

behavior is observed to be linear elastic with homogeneous compression up to a strain of 0.03. The stress then plateaus as a result of the pattern transformation event. Again the transformation is a result of a local buckling instability in

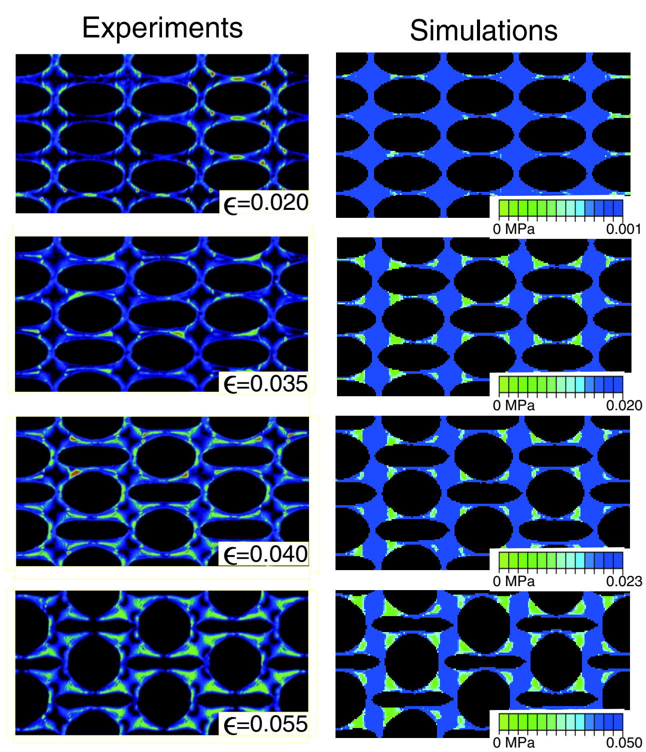

FIG. 4 (color online). Experimental (left) and numerical (right) images of the rectangular lattice of elliptical holes at macroscopic strains of 1.5%, 2%, 2.5%, and 3.5%. The color patterns in the experiment provide a qualitative image of the stress patterns and correlate well with the calculated principal stress patterns.

the ligaments. In this case, the array of identical ellipses is transformed to an alternating array of high and low aspect ratio ellipses where the aspect ratio contrast increases with increasing macroscopic strain and the low aspect ratio ellipses become nearly circular. This behavior is again reversible and repeatable.

We have uncovered surprising novel pattern transformation processes in elastomeric cellular solids using the simple application of a unidirectional load. The effect has been demonstrated at millimeter length scales (suitable for microwaves) but should also persist at the micro (infrared) or even nanoscales. There is considerable current interest in the development of photonic crystals using periodic lattices of the type investigated here. The inspiration for much of this work comes from the brilliant coloration of birds and insects [1–4]. The optical phenomena which pertain to the natural effects are diverse and mimicking the full range of behavior synthetically remains a formidable challenge. Propagation through structures and interference and diffraction from them at various levels are all likely to be important [24–26]. The manufacture of photonic crystals, on the other hand, usually involves the construction of relatively simple periodic structures [7,8]. Our work indicates the exciting prospect of a technological advance by imprinting complex patterns during the fabrication process using a minimum number of developmental steps. For example, this could be done by heating a stiff substrate above its glass transition temperature, applying a load to trigger the transformation, and then freezing the pattern in by cooling. This technique could be taken one step further whereby a reheating step would recover the initial pattern (due to the shape memory effect in amorphous polymers). For example, the elliptical patterns thus formed from the initial array of circular holes are of significant interest in photonic crystals as they have a marked effect on the transmission of polarized light [27,28].

An equally exciting prospect is that the pattern can be switched on and off by the application of an external force. It is known that external pressure applied to the wings of beetles can in itself suppress iridescence [29]. In the case considered here the pattern transformation event is nonlinear in nature and thus very small changes in macroscopic strain produce a sudden transformation in the pattern. Hence we have the distinct possibility of fabricating an optical switch which could be used, for example, to direct light in a controlled manner.

T. M. acknowledges the support of the EPSRC and the hospitality of MIT (T. Peacock) where this work was carried out.

*tom.mullin@manchester.ac.uk

[1] A. R. Parker, R. C. McPhedran, D. R. McKenzie, L. C. Botten, and N. A. P. Nicorovici, Nature (London) **409**, 36 (2001).
[2] P. Vukusic and J. R. Sambles, Nature (London) **424**, 852 (2003).
[3] R. O. Prum, T. Quinn, and R. H. J. Torres, Exp. Biol. **209**, 748 (2006).
[4] D. J. Kemp, P. Vukusic, and R. L. Rutowski, Functional Ecology **20**, 282 (2006).
[5] S. X. Yang, J. H. Page, Z. Y. Liu, M. L. Cowan, C. T. Chan, and P. Sheng, Phys. Rev. Lett. **93**, 024301 (2004).
[6] A. R. Parker and C. R. Lawrence, Nature (London) **414**, 33 (2001).
[7] S. Kinoshita and S. Yoshioka, Chem. Phys. Chem. **6**, 1442 (2005).
[8] M. Campbell, D. N. Sharp, M. T. Harrison, R. G. Denning, and A. J. Turberfield, Nature (London) **404**, 53 (2000).
[9] K. Hennessy, C. Reese, A. Badolato, C. F. Wang, A. Imamoglu, P. M. Petroff, and E. J. Hu, J. Vac. Sci. Technol. B **21**, 2918 (2003).
[10] L. Thylen, M. Qiu, and S. Anand, Chem. Phys. Chem. **5**, 1268 (2004).
[11] C. K. Ullal, M. Maldovan, E. L. Thomas, G. Chen, Y. J. Han, and S. Yang, Appl. Phys. Lett. **84**, 5434 (2004).
[12] J. Y. Cheng, A. M. Mayes, and C. A. Ross, Nat. Mater. **3**, 823 (2004).
[13] J. H. Jang, C. K. Ullal, V. V. Tsukruk, and E. L. Thomas, Nano Lett. **6**, 740 (2006).
[14] T. Choi, J. H. Jang, C. K. Ullal, M. C. LeMieux, V. V. Tsukruk, and E. L. Thomas, Adv. Funct. Mater. **16**, 1324 (2006).
[15] L. Zhai, M. C. Berg, F. C. Cebeci, Y. Kim, J. M. Milwid, M. F. Rubner, and R. E. Cohen, Nano Lett. **6**, 1213 (2006).
[16] E. V. Shevchenko, D. V. Talapin, N. A. Kotov, S. O'Brien, and C. B. Murray, Nature (London) **439**, 55 (2006).
[17] W. Park and J. B. Lee, Appl. Phys. Lett. **85**, 4845 (2004).
[18] M. Aschwanden and A. Stemmer, Opt. Lett. **31**, 2610 (2006).
[19] L. J. Gibson and M. F. Ashby, *Cellular Solids: Structure and Properties* (Cambridge University Press, Cambridge, England, 1997).
[20] T. S. Majmudar and R. P. Behringer, Nature (London) **435**, 1079 (2005).
[21] I. Zuriguel, T. Mullin, and J. M. Rotter, Phys. Rev. Lett. **98**, 028001 (2007).
[22] M. C. Boyce, S. Socrate, K. Kear, O. Yeh, and K. J. Shaw, J. Mech. Phys. Solids **49**, 1323 (2001).
[23] N. Triantafyllidis, M. D. Nestorovic, and M. W. Schraad, J. Appl. Mech. **73**, 505 (2006).
[24] H. Ghiradella, J. Morphol. **142**, 395 (1974).
[25] H. Ghiradella, Appl. Opt. **30**, 3492 (1991).
[26] H. Ghiradella, Microsc. Res. Tech. **27**, 429 (1994).
[27] J. Elliott, I. I. Smolyaninov, N. I. Zheludev, and A. V. Zayats, Phys. Rev. B **70**, 233403 (2004).
[28] J. B. Masson and G. Gallot, Phys. Rev. B **73**, 121401 (2006).
[29] A. Mallock, Proc. R. Soc. A **85**, 598 (1911).