

## Effect of Orbital Angular Momentum on Valence-Quark Helicity Distributions

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We study the quark helicity distributions at large  $x$  in perturbative QCD, taking into account contributions from the valence Fock states of the nucleon which have nonzero orbital angular momentum. We find that the quark orbital angular momentum contributes a large logarithm to the negative helicity quark distributions in addition to its power behavior, scaling as  $(1-x)^5 \log^2(1-x)$  in the limit of  $x \rightarrow 1$ . Our analysis shows that the ratio of the polarized over unpolarized down quark distributions,  $\Delta d/d$ , will still approach 1 in this limit. By comparing with the experimental data, we find that this ratio should cross zero at  $x \approx 0.75$ .

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*Introduction.*—Power-counting rules for the large- $x$  parton distributions in hadrons were derived more than three decades ago based on perturbative quantum chromodynamics (pQCD) combined with an  $S$ -wave quark model of hadrons [1–4]. The basic argument is that when the valence quark carries nearly all of the longitudinal momentum of the hadron, the relevant QCD configurations in the hadronic wave function become far off-shell and can be treated in pQCD. A generic factorization procedure has recently been used to justify the power-counting rule by relating the parton distributions at large- $x$  to the quark distribution amplitudes of hadrons [5]. The power-counting rule has also been generalized to sea quarks, gluons, helicity-dependent distributions [6,7], and generalized parton distributions [8].

The pQCD diagrams associated with the leading Fock state of the proton wave function predict that the positive helicity (quark spin aligned with the proton spin) quark distribution  $q^+(x)$  scales as  $(1-x)^3$ , ( $x$  is the longitudinal momentum fraction of the nucleon carried by the struck quark and is identical to the Bjorken  $x_B$  in the leading twist approximation), whereas the negative helicity (quark spin antialigned with the proton spin) quark distribution  $q^-(x)$  is suppressed by  $(1-x)^2$  relative to the positive helicity one, scaling as  $(1-x)^5$  at large  $x$  [3]. The direct consequence of these power laws for the quark distributions is that the ratio of polarized quark distribution  $\Delta q(x) = q^+(x) - q^-(x)$  over the unpolarized quark distribution  $q(x) = q^+(x) + q^-(x)$  approaches 1 in the limit  $x \rightarrow 1$ ; i.e., at large  $x$ ,  $q^+$  dominates over  $q^-$ . When this prediction is compared to the experimental data [9–12], it is interesting to observe that, for the up quark the ratio increases with  $x$ , and seems to approach 1 at large  $x$ . However, the ratio for the down quark is still far below 1, and remains negative for a wide range of  $x \leq 0.6$  [9]. This discrepancy has stimulated much theoretical interest.

In this Letter we will reexamine the large- $x$  quark helicity distributions in the perturbative QCD framework [3,4]. We work in light-cone gauge, where there is no ghost

contributions, and orbital angular momentum is physical [13]. We will take into account the contributions from not only the leading light-cone Fock state expansion of the nucleon wave function with zero quark orbital angular momentum ( $L_z = 0$ ), but also the valence Fock states with nonzero quark orbital angular momentum ( $L_z \neq 0$ ). These contributions are naturally required to obtain a non-zero anomalous magnetic moment for nucleons [14] and are also present in the wave function solutions in the AdS/CFT correspondence approach [15]. We find that for the negative helicity distribution  $q^-$ , there exist large logarithmic enhancements from the  $|L_z| = 1$  Fock states. With this large logarithmic modification, we can explain the discrepancy between the power-counting rule and experimental data.

*Analysis of the large- $x$  behavior of the quark helicity distributions.*—In the  $x \rightarrow 1$  regime we can use perturbative QCD to analyze the parton distributions, and the contributions depend on the number of spectator partons in the Fock state wave function of the hadron. For example, the valence-quark distribution of the nucleon is dominated by the three-quark Fock states, which consists of zero orbital angular momentum ( $L_z = 0$ ) and nonzero orbital angular momentum ( $L_z \neq 0$ ) components [16]. In the following discussion, we will consider the contributions from both components.

In the original power-counting analysis of the quark helicity distributions [6], only the contributions from the leading Fock state with  $L_z = 0$  were taken into account. In Figs. 1(a) and 1(b) we show the typical diagrams which contribute to the positive (a) and negative (b) helicity distributions at large  $x$ . In terms of the leading order quark distribution amplitude which corresponds to the  $L_z = 0$  Fock state expansion of the proton wave function [4], these quark helicity distributions can be written as

$$q(x)|_{x \rightarrow 1} = \int [dy_i][dy'_i] \Phi(y_i) \Phi'(y'_i) \mathcal{H}(y_i, y'_i; (1-x)), \quad (1)$$

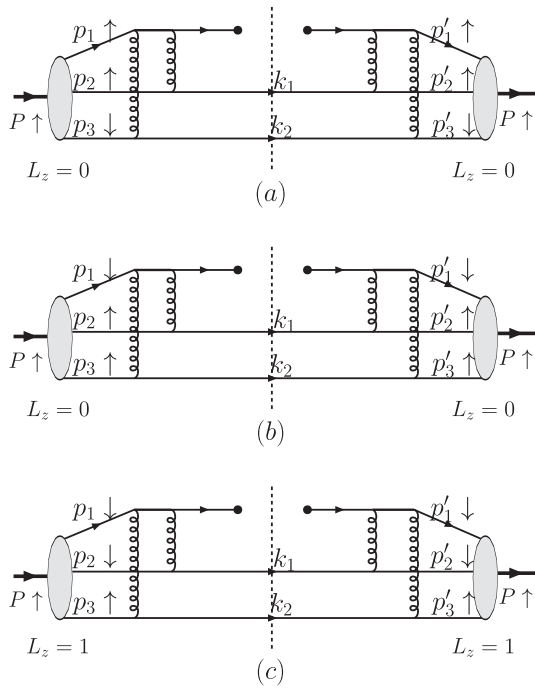


FIG. 1. Examples of Feynman diagrams which contribute to the  $q^\pm$  quark distributions at large  $x$ : (a) for  $q^+$  with power contribution of  $(1-x)^3$ ; (b) for  $q^-$  with  $(1-x)^5$ ; (c) for  $q^-$  with  $(1-x)^5 \log^2(1-x)$ . The wave functions at the left and right sides of the cut line in (a) and (b) represent the leading Fock state expansion with zero quark orbital angular momentum, whereas those for (c) represent the valence Fock state with one-unit of quark orbital angular momentum.

where the integration measure  $[dy_i]$  is defined as  $[dy_i] = dy_1 dy_2 dy_3 \delta(1 - y_1 - y_2 - y_3)$ , and the  $y_i$  are the light-cone momentum fractions of the proton carried by the quarks in the light-front wave functions, i.e.,  $p_i = y_i P$  and  $p'_i = y'_i P$  in Fig. 1. Here,  $\Phi$  and  $\Phi'$  represent the quark distribution amplitudes of the proton at the left and right sides of the cut line, respectively.  $\mathcal{H}$  represents the hard part of the amplitude which depends on  $y_i$  and  $y'_i$ , and  $(1-x)$  as well.

The large- $x$  behavior of the hard factor  $\mathcal{H}$  can be evaluated from the partonic scattering amplitudes as shown by the Feynman diagrams in Fig. 1. The wave functions corresponding to Figs. 1(a) and 1(b) have zero quark orbital angular momentum, and thus the total quark spin will be equal to the proton spin. If the struck quark spin is the same as the proton spin (for the positive helicity distribution  $q^+$ ), the two spectator quarks will be in the spin-0 configuration, whereas for the negative helicity distribution  $q^-$  with the quark spin opposite to the proton spin, the two spectator quarks will be in the spin-1 configuration. It has been known for a long time that the hard partonic part  $\mathcal{H}$  with a spin-1 configuration for the two spectators will be suppressed by  $(1-x)^2$  relative to that with spin-0 configuration [3,7,17]. That is also the reason why the negative helicity distribution is suppressed by

$(1-x)^2$  relative to the positive helicity distribution from this contribution.

The far-off-shell propagators in the partonic Feynman diagrams are the dominant contributions to the power-counting of  $(1-x)$  at large  $x$ . One of the gluon propagators in Fig. 1 behaves as

$$\frac{1}{(p_3 - k_2)^2} = \frac{1}{2p_3 \cdot k_2} \approx -\frac{1}{\langle k_{1\perp}^2 \rangle} \frac{1-x}{y_3} \quad (2)$$

at large  $x$ . In the above expression, we have omitted all higher order terms suppressed by  $(1-x)$ . Here,  $\langle k_{1\perp}^2 \rangle \sim \langle k_{2\perp}^2 \rangle$ , represents a typical transverse momentum scale for the spectator system. Each propagator will provide a suppression factor of  $(1-x)$ . Counting the hard propagators in Fig. 1(a), we find the total suppression factor is

$$\sim \frac{(1-x)^8}{y_2 y_3 (1-y_2) y'_2 y'_3 (1-y'_2)}. \quad (3)$$

We notice that the above expression does not introduce additional dependence on  $(1-x)$  upon integration over  $y_i$  and  $y'_i$ , assuming that the leading twist distribution amplitudes  $\Phi$  and  $\Phi'$  are proportional to  $y_1 y_2 y_3$  and  $y'_1 y'_2 y'_3$  [4], respectively. Combining these results with the power behavior for the other parts of the partonic scattering amplitudes and the phase space integral, we find the positive helicity distribution  $q^+$  scales as  $(1-x)^3$ , whereas the negative helicity distribution  $q^-$  scales as  $(1-x)^5$  [3,4,6,7].

In the above analysis, we only considered the contributions from the leading Fock state of the proton with zero quark orbital angular momentum. In general, the contributions from the higher Fock states and the valence Fock states with nonzero quark orbital angular momentum will introduce additional suppression in  $(1-x)$  [4,7]. However, the nonzero-quark-orbital-angular-momentum Fock state can provide large logarithmic enhancement to the helicity-flip amplitudes. For example, it was found that the quark orbital angular momentum contributes a large logarithmic term to the nucleon's helicity-flip Pauli form factor  $F_2(Q^2)$ , which leads to the scaling behavior  $F_2(Q^2) \sim \log^2(Q^2/\Lambda^2)/Q^6$  at  $Q^2 \rightarrow \infty$  [18,19]. This is consistent with recent experimental data from JLab [20]. In the following, we will study the quark orbital angular momentum contribution to the  $q^-$  quark distribution which is also associated with the helicity-flip amplitude. The corresponding contributions to the positive helicity distribution are always power suppressed [7].

In Fig. 1(c), we show an example of a contribution from the  $L_z = 1$  Fock state of the proton. Because the quark orbital angular momentum contributes one unit of the proton spin, we will have difference between the total quark spin and the proton spin. If the two spectator quarks are in the spin-0 configuration, this will enhance the power-counting in the hard factor  $\mathcal{H}$ . On the other hand, in order to get a nonzero contribution, we have to perform the intrinsic transverse momentum expansion for the hard

partonic scattering amplitudes [19], which will introduce an additional suppression factor in  $(1-x)$  [7]. For example, one of the contributions from the diagram shown in Fig. 1(c) to the negative helicity distribution will be proportional to

$$q^-(x) \propto \int (p_1^x + ip_1^y)(p_1^x - ip_1^y) \tilde{\psi}^{(3)}(y_i, p_{i\perp}) \tilde{\psi}^{(3)}(y'_i, p'_{i\perp}) \times T_H(y_i, p_{i\perp}; y'_i, p'_{i\perp}), \quad (4)$$

where  $\tilde{\psi}^{(3)}$  is a light-front wave function amplitude for the  $L_z = 1$  Fock state of the proton [16]. The intrinsic transverse momentum expansion is essential to get the nonzero contributions. Otherwise, the integral over the transverse momenta  $p_{i\perp}$  and  $p'_{i\perp}$  will vanish because of the explicit factors  $p_1^x + ip_1^y$  and  $p_1^x - ip_1^y$  in the above equation. One intrinsic transverse momentum expansion comes from the propagator we mentioned above,

$$\frac{1}{(p_3 - k_2)^2} = \frac{1}{(y_3 P - k_2 + p_{3\perp})^2} \approx \frac{\beta(1-x)}{y_3 k_{2\perp}^2} \left( 1 - \frac{\beta(1-x)}{y_3 k_{2\perp}^2} 2p_{3\perp} \cdot k_{2\perp} \right), \quad (5)$$

where  $\beta$  is the longitudinal momentum fraction of the spectator carried by  $k_2$ , and we have kept the linear dependence on  $p_{3\perp}$  in the above expansion. Only this linear term will contribute when integrating over  $p_{i\perp}$ :  $\int k_{2\perp} \cdot p_{3\perp} (p_1^x + ip_1^y) \tilde{\psi}^{(3)} \propto (k_2^x + ik_2^y) y_3 \Phi_4(y_1, y_2, y_3)$ , where  $\Phi_4$  is one of the twist-4 quark distribution amplitudes of the proton [19,21]. From the above expansion, we find that this term will introduce additional factor of  $(1-x)/y_3$  in the hard factor. Similarly, because of the  $p_1^x - ip_1^y$  factor in Eq. (4), we have to do the expansion in intrinsic transverse momentum associated with the wave function at the right side of the cut line, and again the expansion of the gluon propagator with momentum of  $p_3 - k_2$  will introduce another suppression factor of  $(1-x)/y'_3$  in the hard factor. Thus the total suppression factor from the above two expansions will be  $(1-x)^2/y_3 y'_3$ , which gives the same power-counting contribution to  $q^-$  as that from the leading Fock state with  $L_z = 0$  in the above.

We thus find the contributions from  $L_z = 1$  Fock state of the proton do not change the power counting for the  $q^-$  quark distribution at large  $x$ . However, the additional factor  $1/y_3 y'_3$  from the intrinsic transverse momentum expansions will lead to a large logarithm when integrating over  $y_i$  and  $y'_i$ . This is because, combining the above two factors with all other factors from the propagators shown in Eq. (3), the total dependence on  $y_i$  and  $y'_i$  for the hard factor will be

$$\sim \frac{1}{y_2 y_3^2 (1-y_2) y'_2 y_3'^2 (1-y'_2)}, \quad (6)$$

where we have  $y_3^2$  and  $y_3'^2$  in the denominator. On the other hand, we expect the twist-4 quark distribution amplitude to have the following behavior at the end point region:

$y_3 \Phi_4(y_1, y_2, y_3) \propto y_1 y_2 y_3$  and  $y'_3 \Phi_4(y'_1, y'_2, y'_3) \propto y'_1 y'_2 y'_3$  [21]. Thus we will have logarithmic divergences for the integrations over  $y_3$  and  $y'_3$ , for which we can regularize in terms of  $\log(1-x)$  as indicated in the above propagator expansion. This will lead to a double logarithmic contribution  $\log^2(1-x)$  in addition to the power term  $(1-x)^5$  to the  $q^-$  quark distribution at large  $x$ .

Similarly, the contributions from higher orbital angular momentum Fock states can be analyzed following the above method. However, these contributions are more suppressed in the limit of  $x \rightarrow 1$ . For example, the  $L_z = 2$  Fock state contributes to  $q^-$  distribution as  $(1-x)^9 \ln^4(1-x)$ , which is  $(1-x)^4 \ln^2(1-x)$  suppressed relative to the above  $L_z = 1$  contribution.

In summary, for the negative helicity distribution  $q^-$ , the leading Fock state with zero quark orbital angular momentum  $L_z = 0$  contributes to a power term  $(1-x)^5$ , whereas the valence Fock state with  $|L_z| = 1$  contributes to a double logarithmic enhanced term  $(1-x)^5 \log^2(1-x)$ . So, in the limit  $x \rightarrow 1$ , the  $q^-$  distribution will be dominated by the contributions from  $L_z = 1$  Fock state of the proton, scaling as  $(1-x)^5 \log^2(1-x)$ . In the intermediate  $x$  range, the subleading terms can also be important. For example in Ref. [6], the quark helicity distributions were parameterized by the leading and subleading power terms and fit to the experimental data. This was later updated to account for the latest data in Ref. [22]. Thus, as a first step towards a comprehensive phenomenology, we follow the parameterizations for  $q^+$  and  $q^-$  in Ref. [6] by adding the newly discovered double logarithms enhanced contributions,

$$\begin{aligned} u^+(x) &= \frac{1}{x^\alpha} [A_u(1-x)^3 + B_u(1-x)^4], \\ d^+(x) &= \frac{1}{x^\alpha} [A_d(1-x)^3 + B_d(1-x)^4], \\ u^-(x) &= \frac{1}{x^\alpha} [C_u(1-x)^5 + C'_u(1-x)^5 \log^2(1-x) \\ &\quad + D_u(1-x)^6], \\ d^-(x) &= \frac{1}{x^\alpha} [C_d(1-x)^5 + C'_d(1-x)^5 \log^2(1-x) \\ &\quad + D_d(1-x)^6], \end{aligned} \quad (7)$$

where the additional two parameters  $C'_u$  and  $C'_d$  come from the logarithmic modifications to the  $q^-$  quark distribution at large  $x$ , and all other parameters refer to [6]. In the following, we will fit to the current experimental data at large  $x$  region with the above parameterizations for the valence up and down quarks.

*Phenomenological applications.*—In order to demonstrate the importance of the new scaling behavior for the negative helicity distributions for the valence up and down quarks, we analyze the latest experimental data from SLAC, HERMES, and Jefferson Lab, including Hall A and Hall B data [9–12]. We will keep the original fit values for other parameters [22] except the two new parameters:

$C'_u$  and  $C'_d$ . We only use the experimental data in the large- $x$  region, i.e.,  $x > 0.3$ , where the sea contribution is not significant. We perform our fit at a fixed  $Q^2 = 4 \text{ GeV}^2$ , and all the experimental data are evolved to this scale by using the GRSV parameterization [23] for the polarized and unpolarized quark distributions. The evolution introduces some theoretical uncertainties.

From our fit, we find the following values for  $C'_u$  and  $C'_d$ ,

$$C'_u = 0.493 \pm 0.249, \quad C'_d = 1.592 \pm 0.378. \quad (8)$$

The minimum of the functional  $\chi^2$  is achieved at  $\chi^2 = 11.4$  and  $\chi^2/\text{DOF} = 1.14$ . We further notice that the additional two terms in Eq. (7) do not change significantly the sum rules for the up and down quarks, such as the Bjorken and momentum sum rule, which are essential for constraining the parameters in Refs. [6,22].

In Fig. 2, we show the above fit. We plot the ratios of the polarized quark distributions  $\Delta q$  over the unpolarized quark distributions  $q$  as functions of  $x$  for both up and down quarks, compared with the experimental data. From these comparisons, we find that the ratio for the up quark  $\Delta u/u$  can still be described by the parameterization based on the original power-counting rule for  $u^+$  and  $u^-$  [22]. However, for the down quark we have to take into account a large contribution from the newly discovered term for the negative helicity distribution  $d^-$ ; the difference between our result and the original parameterization [22] becomes significant at large  $x$ . The analysis of the anomalous magnetic moment and generalized parton distributions of nucleons also indicates significant contributions from the orbital angular momenta of up and down quarks [24]. This is in qualitative agreement with our fitting results, taking into account the large error bar for  $C'_u$ . A precision determination of these contributions shall be obtained by further development for a consistent set of parameters for

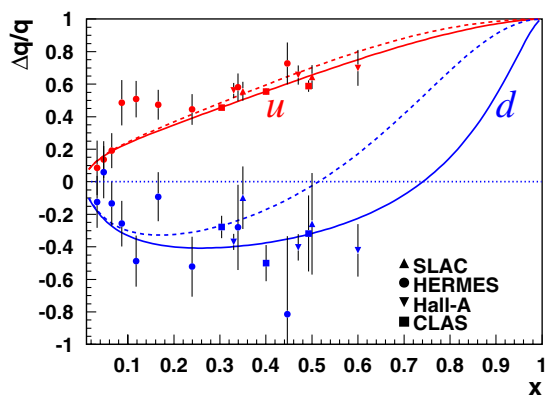


FIG. 2 (color online). Comparison of the quark helicity distributions Eq. (7) with the experimental data, plotted as functions of  $x$  for up (the upper curves) and down (the lower curves) quarks. The circles are for HERMES data [11], the triangles up for SLAC [12], the triangles down for JLab Hall A data [9], the filled squares for CLAS [10]. The dashed curves are the predictions from [22], and the solid ones are our fit results (only the large- $x$  ( $> 0.3$ ) experimental data were used in the fit).

Eq. (7) from next-to-leading-order QCD analysis of both polarized and unpolarized data over the full range in  $x$  [22].

Another important prediction of our fit is that the ratio of  $\Delta d/d$  will approach 1 at extremely large  $x$ , and it will cross zero at  $x \approx 0.75$ . It will be interesting to check this prediction in future experiments, such as the 12 GeV upgrade of Jefferson Lab [25].

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