Proton Heating via Nonresonant Scattering Off Intrinsic Alfvénic Turbulence

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A novel yet unsophisticated theory is proposed to show that low-β protons can be efficiently heated by enhanced Alfvén waves. The present research is motivated by a plasma physics issue relevant to the explanation of hot stellar coronas observed with x-ray telescopes. The efficient heating is attributed to nonresonant wave-particle scattering that tends to randomize proton motion in directions transverse to the ambient magnetic field.

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In astrophysics, the origin of many types of exceedingly hot stellar coronas observed with x-ray telescopes since the mid 1970s [1–3] is still a hotly pursued research topic that baffles many scientists. Before the discovery of Capella [4,5] and other late-type stars [3], the Sun was the only star known to have a hot corona. Among the possible heating scenarios that might result in the creation of hot coronas, is the intuitive belief held by many researchers that Alfvén waves may play a key role. The main reason is that Alfvén waves might be ubiquitously generated in stellar environments. The solar corona is a prime example. The controversial theoretical difficulty with this picture is that plasmas in stellar coronas have very low-β values (the plasma "β" is the ratio of thermal pressure to magnetic pressure), and, consequently, conventional concept of plasma heating based on cyclotron resonance cannot be applied to protons.

First, in the present study we do not consider nonlinear particle dynamics [6], which is often supposed to be instrumental for the study of stochastic motion of particles. Instead, we use a kinetic-theory approach that generally deals with many-particle systems statistically and inherently includes the stochastic process of interest. We pay special attention to the case in which Alfvénic turbulence is present. In our theory we impose the quasilinear approximation, which was first proposed in the early 1960s [7–12] and is now well known in plasma physics literature [13–16]. Second, the majority of the published quasilinear theories place emphasis on resonant wave-particle interactions, although in general, quasilinear theory also includes nonresonant wave-particle processes as well. In fact, several reviews [13,14] have shown that nonresonant wave-particle interaction processes may play important roles in the stabilization of classic firehose and loss-cone instabilities. Third, for magnetized plasmas the validity of the theory requires that the wave energy density is sufficiently lower than the energy density of the ambient magnetic field so that the particle motion can be approximately described as unperturbed by the wave fields. In other words, the typical wave amplitude is considered to be much lower than the ambient magnetic field magnitude.

To proceed with the discussion let us consider that large amplitude Alfvén waves, as observed in the solar wind [17,18], exist and propagate predominantly along the ambient magnetic field. Moreover, these waves may be treated as if they were intrinsic. The intrinsic Alfvén waves are assumed to have a broad spectrum that is spatially homogeneous and varies slowly in time. The turbulence is attributed to either a remote source or is generated by a small population of energetic ions. The objective of the present work is to examine how the background thermal protons, rather than minor heavy ions, react to Alfvén waves without cyclotron resonance. The quasilinear kinetic equation relevant to the present discussion is well known, and, for example, given in Ref. [14]

$$\frac{\partial F}{\partial t} = \frac{e^2}{2m_p^2}\int dk E_k^2\left[\left(1-\frac{kv_\parallel}{\omega_k}\right)\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}v_\perp + \frac{kv_\perp}{\omega_k}\frac{\partial}{\partial v_\parallel}\right] \times \frac{\gamma_k}{(\omega_k \pm \Omega_p - kv_\parallel)^2+\gamma_k^2} \times\left[\left(1-\frac{kv_\parallel}{\omega_k}\right)\frac{\partial}{\partial v_\perp}+\frac{kv_\perp}{\omega_k}\frac{\partial}{\partial v_\parallel}\right]F, \tag{1}$$

where $F(v_\perp, v_\parallel, t)$ is the proton distribution function; $\Omega_p = eB/m_pc$ is the proton cyclotron frequency (m_p is the proton mass, e is the unit electric charge, c represents speed of light *in vacuo*); E_k is the electric field associated with the intrinsic Alfvén wave with wave number k; $\omega_k = kv_A$ stands for the Alfvén wave dispersion relation and $v_A = B/\sqrt{4\pi n_p m_p}$ stands for the Alfvén speed, n_p being the proton number density; γ_k represents the imaginary part of the complex frequency, i.e., the temporal growth rate of the Alfvén turbulence, and $v_\parallel$ and $v_\perp$ stand for components of the velocity vector parallel and transverse to the ambient magnetic field, respectively.

In most quasilinear theories that emphasize resonant interactions, the factor $\gamma_k[(\omega_k \pm \Omega_p - kv_\parallel)^2 + \gamma_k^2]^{-1}$ is approximated by $\pi\delta(\omega_k \pm \Omega_p - kv_\parallel)$ by taking the limit $\gamma_k \to 0$. However, for Alfvén waves in low-β plasmas the inequalities, $\Omega_p \gg \omega_k \gg kv_\parallel$ and $\Omega_p \gg \gamma_k$ are satisfied. This means that the customary treatment is not applicable but instead we have

$$\frac{\gamma_k}{(\omega_k \pm \Omega_p - kv_\parallel)^2 + \gamma_k^2} \approx \frac{\gamma_k}{\Omega_p^2}. \tag{2}$$

Making use of the above, Eq. (1) can be rewritten as

$$\frac{\partial F}{\partial t} = \frac{1}{2n_p m_p}\int dk\left(\frac{\partial}{\partial t}\frac{B_k^2}{8\pi}\right)\left[\left(1 - \frac{kv_\parallel}{\omega_k}\right)\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}v_\perp + \frac{kv_\perp}{\omega_k}\frac{\partial}{\partial v_\parallel}\right]\left[\left(1 - \frac{kv_\parallel}{\omega_k}\right)\frac{\partial}{\partial v_\perp} + \frac{kv_\perp}{\omega_k}\frac{\partial}{\partial v_\parallel}\right]F. \tag{3}$$

In deriving the above result we have made use of the relations $B_k^2 = (c/v_A)^2 E_k^2$ and $\gamma_k B_k^2 = \partial B_k^2/2\partial t$. Here B_k designates the spectral wave magnetic field.

Since we are mainly interested in low-β plasmas, Eq. (3) may be further simplified to the following form

$$\frac{\partial F}{\partial t} = \frac{1}{2n_p m_p}\int dk\left(\frac{\partial}{\partial t}\frac{B_k^2}{8\pi}\right)\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(v_\perp\frac{\partial F}{\partial v_\perp}\right),$$

or equivalently,

$$\frac{\partial F}{\partial \eta} = \frac{1}{u_\perp}\frac{\partial}{\partial u_\perp}\left(u_\perp\frac{\partial F}{\partial u_\perp}\right), \tag{4}$$

where we have introduced dimensionless variables $u_\perp \equiv v_\perp(2T_0/m_p)^{-1/2}$, and

$$\eta = \frac{1}{4n_p T_0}\int dk\frac{B_k^2}{8\pi} \equiv \frac{W_B}{4n_p T_0}, \tag{5}$$

In Eq. (5) $W_B = \int dk B_k^2/(8\pi)$ represents the energy density associated with the wave magnetic field and T_0 stands for the proton temperature in the absence of the Alfvén waves. The rationale for introducing the dimensionless variable η is that we assume that the velocity distribution F depends on time t only through the wave energy density $W_B(t)$. Let us study the situation where the protons possess a Maxwellian distribution before they encounter the wave fields. We are thus interested in the solution to Eq. (4) where the initial condition is given by

$$F(u_\perp, u_\parallel, \eta = 0) = \frac{1}{\pi^{3/2}}\exp(-u_\perp^2 - u_\parallel^2). \tag{6}$$

In fact, the general solution to Eq. (4) may be formally expressed in terms of the Green's function and an arbitrary initial distribution,

$$F(u_\perp, u_\parallel, \eta) = \int_0^\infty du'_\perp u'_\perp G(u_\perp, u'_\perp, \eta)F(u'_\perp, u_\parallel, 0), \tag{7}$$

where the Green's function is given by [19]

$$G(u_\perp, u'_\perp, \eta) = \frac{1}{2\eta}I_0\left(\frac{u_\perp u'_\perp}{2\eta}\right)e^{-(u_\perp^2 + u_\perp'^2)/(4\eta)}, \tag{8}$$

and where $I_0(x)$ is the modified Bessel function of the first kind of zeroth order. For the special case when the initial distribution is given by a Maxwellian (6), we may obtain a self-similar solution [20],

$$F(u_\perp, u_\parallel, \eta) = \frac{1}{1 + 4\eta}\exp\left(-\frac{u_\perp^2}{1 + 4\eta} - u_\parallel^2\right). \tag{9}$$

Clearly the perpendicular temperature is enhanced so that after heating it becomes

$$T_\perp = T_0 + \frac{W_B}{n_p} = T_0 + \frac{1}{n_p}\int dk\frac{B_k^2}{8\pi}. \tag{10}$$

We now turn to Eq. (3) and define perpendicular and parallel temperatures,

$$T_\perp = \frac{m_p}{2}\int d\mathbf{v}v_\perp^2 F, \qquad T_\parallel = m_p\int d\mathbf{v}v_\parallel^2 F.$$

Then by taking the appropriate velocity moments of Eq. (3), we obtain

$$\begin{aligned}\frac{\partial T_\perp}{\partial t} &= \frac{1}{n_p}\frac{\partial W_B}{\partial t}\left(1 + \frac{T_\parallel - T_\perp}{m_p v_A^2}\right),\\ \frac{\partial T_\parallel}{\partial t} &= \frac{2}{n_p}\frac{\partial W_B}{\partial t}\frac{T_\perp - T_\parallel}{m_p v_A^2}.\end{aligned} \tag{11}$$

The above results also support the recent test-particle simulation of proton heating by Alfvén waves which is demonstrated in the paper by Wang *et al.* [21]. However, we remark that since the wave energy density is considered to be small in comparison with the energy density of the ambient magnetic field, the terms on the right-hand side of Eq. (11) inversely proportional to v_A^2 are small. This is because

$$\left|\frac{T_\perp - T_\parallel}{m_p v_A^2}\right| \sim \frac{W_B}{n_p m_p v_A^2} \equiv \frac{B_W^2}{B_0^2} \ll 1,$$

where B_W denotes the typical wave amplitude.

To understand the physics of the perpendicular heating a few remarks may be pertinent: One can readily see from the kinetic Eq. (3) or (4) that in the presence of intrinsic Alfvén turbulence the average magnetic moment of the protons is no longer conserved. The heating mechanism suggested in the present discussion differs from the earlier theoretical studies carried out by [22–26] based on nonlinear dynamics. In our theory we suggest that stochastic diffusion due to the turbulent wave fields may result in heating. Microscopically each proton is pitch-angle scattered by the wave field in the wave frame in which the scattering process itself does not change the particle energy. However, since the waves are propagating, the protons gain energy in the laboratory (or plasma) frame. It is implicitly assumed that the intrinsic Alfvén waves have an infinite reservoir of energy.

We should also comment on two other points. First, the anisotropy created by the perpendicular heating would make the plasma unstable to excitation of proton cyclotron waves via cyclotron resonance, e.g., Ref. [27]. One of the well-known consequences of these waves is that they tend to heat protons in the parallel direction by consuming the free energy available with the perpendicular temperature. While this process is in progress, the intrinsic Alfvén waves keep heating the protons to its optimal level, which is dictated by the intrinsic wave energy density, in the perpendicular direction. As a result, it is expected that eventually the proton distribution would evolve into a state that is nearly isotropic so that the proton cyclotron instability is no longer significant.

If we denote the initial temperature by T_0, as before, then we anticipate that the protons may achieve a final quasi-isotropic temperature,

$$T \simeq T_0 + \frac{W_B}{n_p} = T_0 + \int dk \frac{B_k^2}{8\pi n_p} \equiv T_0\left(1 + \frac{1}{\beta_p}\frac{B_W^2}{B_0^2}\right),$$

where β_p denotes the proton β and B_W^2/B_0^2 is the ratio of wave-field energy density to that of the ambient field. In the low solar corona, a β value of $\beta_p \sim 10^{-3}$ or even lower is not uncommon, while $B_W^2/B_0^2 \sim 10^{-2}$ may be typical. This means that $T/T_0 \sim 10$.

Second, one may generalize the preceding discussion to Alfvén-proton cyclotron waves with frequencies higher than those of Alfvén waves and close to proton cyclotron frequency, but still satisfying the condition

$$|\Omega_p - \omega_k| \gg k v_\parallel.$$

In this case, the quantity η [Eq. (5)] may be readily redefined as

$$\eta = \frac{\Omega_p^2}{4 n_p T_0} \int dk \frac{B_k^2}{8\pi(\Omega_p - \omega_k)^2}.$$

As a result, the heating process is expected to be even more effective.

In the solar physics literature numerous authors have discussed the hot corona. However, most of these discussions are mainly concerned with the outer corona high above the chromosphere. Interested readers may refer to Refs. [28–36], for instance. Here we must point out that the formation of the transition region, where the steep temperature gradient (from several thousand degrees to a couple of million degrees) exists, is still an issue to be resolved, although some recent models are proposed based on ion cyclotron resonance [36]. All existing theories proposed to discuss the heating of the solar corona are based on the notion that ion cyclotron resonance is essential. In this regard, the heating process, which involves Alfvénic turbulence and low-β plasmas discussed in this Letter is new and very relevant to the study of stellar coronas.

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