Why There is Something Rather than Nothing: Cosmological Constant from Summing over Everything in Lorentzian Quantum Gravity

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(Received 9 April 2007; published 16 August 2007)

The density matrix of the Universe for the microcanonical ensemble in quantum cosmology describes an equipartition in the physical phase space of the theory (sum over everything), but in terms of the observable spacetime geometry this ensemble is peaked about the set of recently obtained cosmological instantons limited to a bounded range of the cosmological constant. This suggests the mechanism of constraining the landscape of string vacua and a possible solution to the dark energy problem in the form of the quasiequilibrium decay of the microcanonical state of the Universe.

DOI: 10.1103/PhysRevLett.99.071301 PACS numbers: 04.60.Gw, 04.62.+v, 98.80.Bp, 98.80.Qc

Euclidean quantum gravity (EQG) is a lame duck in modern particle physics and cosmology. After its summit in the early and late 1980s (in the form of the cosmological wave function proposals [1,2] and baby universes boom [3]) the interest in this theory gradually declined, especially, in a cosmological context, where the problem of quantum initial conditions was superseded by the concept of stochastic inflation [4]. EQG could not stand the burden of indefiniteness of the Euclidean gravitational action [5] and the cosmology debate of the tunneling vs no-boundary proposals [6].

Thus, a recently suggested EQG density matrix of the Universe [7] is hardly believed to be a viable candidate for the initial state of the Universe, even though it avoids the infrared catastrophe of small cosmological constant Λ, generates an ensemble of universes in the limited range of Λ, and suggests a strong selection mechanism for the landscape of string vacua [7,8]. Here we want to justify this result by deriving it from first principles of *Lorentzian* quantum gravity applied to a microcanonical ensemble of closed cosmological models.

Thermal properties in quantum cosmology [9] are incorporated by a mixed physical state, which is *dynamically* more preferable than a pure state of the Hartle-Hawking type. This follows from the path integral for the EQG statistical sum [7,8]. It can be cast into the form of the integral over a minisuperspace of the lapse function $N(\tau)$ and scale factor $a(\tau)$ of spatially closed Friedmann-Robertson-Walker (FRW) metric $ds^2 = N^2(\tau)d\tau^2 + a^2(\tau)d^2\Omega^{(3)}$,

$$e^{-\Gamma} = \int_{\text{periodic}} D[a, N] e^{-\Gamma_E[a,N]}, \tag{1}$$

$$e^{-\Gamma_E[a,N]} = \int_{\text{periodic}} D\Phi(x) e^{-S_E[a,N;\Phi(x)]}. \tag{2}$$

Here $\Gamma_E[a, N]$ is the Euclidean effective action of all inhomogeneous "matter" fields which include also metric perturbations on minisuperspace background $\Phi(x) = (\phi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \dots)$. $S_E[a, N; \Phi(x)]$ is the classical Eucidean action, and the integration runs over periodic fields on the Euclidean spacetime with a compactified time τ (of $S^1 \times S^3$ topology).

For free matter fields $\Phi(x)$ conformally coupled to gravity (which are assumed to be dominating in the system) the effective action is exactly calculable [7]: $\Gamma_E[a, N] = \int d\tau N \mathcal{L}(a, a') + F(\eta)$, $a' \equiv da/Nd\tau$. Here $N\mathcal{L}(a, a')$ is the effective Lagrangian of its local part including the classical Einstein term (with the cosmological constant $\Lambda = 3H^2$) and the contribution of the conformal anomaly of quantum fields and their vacuum (Casimir) energy,

$$\mathcal{L}(a, a') = -aa'^2 - a + H^2a^3 + B\left(\frac{a'^2}{a} - \frac{a'^4}{6a} + \frac{1}{2a}\right). \tag{3}$$

$F(\eta)$ is the free energy of their quasiequilibrium excitations with the temperature given by the inverse of the conformal time $\eta = \int d\tau N/a$. This is a typical boson or fermion sum $F(\eta) = \pm\sum_\omega \ln(1 \mp e^{-\omega\eta})$ over field oscillators with energies ω on a unit 3-sphere. We work in units of $m_P = (3\pi/4G)^{1/2}$, and B is the constant determined by the coefficient of the Gauss-Bonnet term in the overall conformal anomaly of all fields $\Phi(x)$.

Semiclassically, the integral (1) is dominated by the saddle points—solutions of the Friedmann equation

$$\frac{a'^2}{a^2} + B\left(\frac{1}{2}\frac{a'^4}{a^4} - \frac{a'^2}{a^4}\right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4}, \tag{4}$$

modified by the quantum B-term and the radiation term C/a^4, with the constant C satisfying the bootstrap equation $C = B/2 + dF(\eta)/d\eta$. Such solutions represent garland-type instantons which exist only in the limited range $0 < \Lambda_{\min} < \Lambda < 3m_P^2/2B$ [7,8] and eliminate the infrared catastrophe of $\Lambda = 0$. The period of these quasithermal instantons is not a freely specifiable parameter, but as a function of Λ follows from this bootstrap. Therefore, this is not a canonical ensemble.

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Contrary to this construction we suggest the density matrix as the canonical path integral of *Lorentzian* quantum gravity. Its kernel in the representation of 3-metrics and matter fields, denoted below as q, reads

$$\rho(q_+, q_-) = e^{\Gamma} \int_{q(t_\pm)=q_\pm} D[q, p, N] e^{i \int_{t_-}^{t_+} dt(p\dot{q} - N^\mu H_\mu)}, \tag{5}$$

where the integration runs over histories of phase-space variables $(q(t), p(t))$ interpolating between $q_\pm$ at $t_\pm$ and the Lagrange multipliers of the gravitational constraints $H_\mu = H_\mu(q, p)$—lapse and shift functions $N(t) = N^\mu(t)$. The measure $D[q, p, N]$ includes the gauge-fixing factor of the delta function $\delta[\chi] = \prod_t \prod_\mu \delta(\chi^\mu)$ of gauge conditions χ^μ and the relevant ghost factor [10,11] (condensed index μ includes also continuous spatial labels). It is important that the integration range of N^μ,

$$-\infty < N < +\infty, \tag{6}$$

generates in the integrand the delta-functions of the constraints $\delta(H) = \prod_\mu \delta(H_\mu)$. As a consequence, the kernel (5) satisfies the set of Wheeler-DeWitt equations

$$\hat{H}_\mu(q, \partial/i\partial q)\rho(q, q') = 0, \tag{7}$$

and the density matrix (5) can be regarded as an operator delta-function of these constraints

$$\hat{\rho} \sim \text{“}\prod_\mu \delta(\hat{H}_\mu)\text{”}. \tag{8}$$

This expression should not be understood literally because the multiple delta function here is not uniquely defined, for the operators $\hat{H}_\mu$ do not commute and form an open algebra. Moreover, exact operator realization $\hat{H}_\mu$ is not known except the first two orders of a semiclassical $\hbar$-expansion [12]. Fortunately, we do not need a precise form of these constraints, because we will proceed with their path-integral solutions adjusted to the semiclassical perturbation theory.

The very essence of our proposal is the interpretation of (5) and (8) as the density matrix of a *microcanonical* ensemble in spatially closed quantum cosmology. A simplest analogy is the density matrix of an unconstrained system having a conserved Hamiltonian $\hat{H}$ in the microcanonical state with a fixed energy E, $\hat{\rho} \sim \delta(\hat{H} - E)$. A major distinction of (8) from this case is that spatially closed cosmology does not have freely specifiable constants of motion like the energy or other global charges. Rather it has as constants of motion the Hamiltonian and momentum constraints H_μ, all having a particular value—zero. Therefore, the expression (8) can be considered as a most general and natural candidate for the quantum state of the *closed* Universe. Below we confirm this fact by showing that in the physical sector the corresponding statistical sum is a uniformly distributed (with a unit weight) integral over the entire phase space of true physical degrees of freedom. Thus, this is the sum over everything. However, in terms of the observable quantities, like spacetime geometry, this distribution turns out to be nontrivially peaked around a particular set of universes. Semiclassically this distribution is given by the EQG density matrix and the saddle-point instantons of the above type [7].

From the normalization of the density matrix in the physical Hilbert space we have

$$1 = \mathrm{Tr}_{\rm phys}\hat{\rho} = \int dq\, \mu(q, \partial/i\partial q)\rho(q, q')|_{q'=q}$$
$$= e^{\Gamma} \int_{\rm periodic} D[q, p, N] e^{i\int dt(p\dot{q} - N^\mu H_\mu)}. \tag{9}$$

Here, in view of the coincidence limit $q' = q$, the integration runs over periodic histories $q(t)$, and $\mu(q, \partial/i\partial q) = \hat{\mu}$ is the measure which distinguishes the physical inner product in the space of solutions of the Wheeler-DeWitt equations $(\psi_1|\psi_2) = \langle\psi_1|\hat{\mu}|\psi_2\rangle$ from that of the space of square-integrable functions, $\langle\psi_1|\psi_2\rangle = \int dq \psi_1^* \psi_2$. This measure includes the delta function of unitary gauge conditions $\chi^\mu = \chi^\mu(q, p)$ and an operator factor incorporating the relevant ghost determinant [12].

On the other hand, in terms of the physical phase space variables the Faddeev-Popov path integral equals [10,11]

$$\int D[q, p, N] e^{i\int dt(p\dot{q} - N^\mu H_\mu)}$$
$$= \int Dq_{\rm phys} Dp_{\rm phys} e^{i\int dt(p_{\rm phys}\dot{q}_{\rm phys} - H_{\rm phys}(t))}$$
$$= \mathrm{Tr}_{\rm phys}(Te^{-i\int dt \hat{H}_{\rm phys}(t)}), \tag{10}$$

where T denotes the chronological ordering. The physical Hamiltonian and its operator realization $\hat{H}_{\rm phys}(t)$ are nonvanishing here only in unitary gauges explicitly depending on time [12], $\chi^\mu(q, p, t)$. In static gauges, $\partial_t \chi^\mu = 0$, they vanish, because the full Hamiltonian in closed cosmology is a combination of constraints.

The path integral (10) is gauge independent on-shell [10,11] and coincides with that in the static gauge. Therefore, from Eqs. (9) and (10) with $\hat{H}_{\rm phys} = 0$, the statistical sum of our microcanonical ensemble equals

$$e^{-\Gamma} = \mathrm{Tr}_{\rm phys}\mathbf{I}_{\rm phys} = \int dq_{\rm phys} dp_{\rm phys}$$
$$= \text{sum over everything}. \tag{11}$$

Here $\mathbf{I}_{\rm phys} = \delta(q_{\rm phys} - q'_{\rm phys})$ is a unit operator in the physical Hilbert space, whose kernel when represented as a Fourier integral yields extra momentum integration (2π-factor included into $dp_{\rm phys}$). This sum over everything (as a counterpart to the concept of creation from “anything” in [13]), not weighted by any nontrivial density of states, is a result of general covariance and the closed nature of the Universe lacking any freely specifiable constants of motion. The Liouville integral over entire *physical* phase space, whose structure and topology is not known, is very nontrivial. However, below we show that semiclassically it is determined by EQG methods and supported by instantons of [7] spanning a bounded range of the cosmological constant.

Integration over momenta in (9) yields a Lagrangian path integral with a relevant measure and action

$$e^{-\Gamma} = \int D[q, N] e^{iS_L[q,N]}. \qquad (12)$$

As in (9) integration runs over periodic fields (not indicated explicitly but assumed everywhere below) even despite the Lorentzian signature of the underlying spacetime. Similarly to the procedure of [7,8] leading to (1) and (2), we decompose $[q, N]$ into a minisuperspace $[a_L(t), N_L(t)]$ and the matter $\Phi_L(x)$ variables, the subscript L indicating their Lorentzian nature. With a relevant decomposition of the measure $D[q, N] = D[a_L, N_L] D\Phi_L(x)$, the microcanonical sum reads

$$e^{-\Gamma} = \int D[a_L, N_L] e^{i\Gamma_L[a_L,N_L]}, \qquad (13)$$

$$e^{i\Gamma_L[a_L,N_L]} = \int D\Phi_L(x) e^{iS_L[a_L,N_L;\Phi_L(x)]}, \qquad (14)$$

where $\Gamma_L[a_L, N_L]$ is a Lorentzian effective action. The stationary point of (13) is a solution of the effective equation $\delta\Gamma_L/\delta N_L(t) = 0$. In the gauge $N_L = 1$ it reads as a Lorentzian version of the Euclidean Eq. (4) and the bootstrap equation for the radiation constant C with the Wick rotated $\tau = it$, $a(\tau) = a_L(t)$, $\eta = i\int dt/a_L(t)$. However, with these identifications C turns out to be purely imaginary [in view of the complex nature of the free energy $F(i\int dt/a_L)$]. Therefore, no periodic solutions exist in spacetime with a *real* Lorentzian metric.

On the contrary, such solutions exist in the Euclidean spacetime. Alternatively, the latter can be obtained with the time variable unchanged $t = \tau$, $a_L(t) = a(\tau)$, but with the Wick rotated lapse function

$$N_L = -iN, \qquad iS_L[a_L, N_L; \phi_L] = -S_E[a, N; \Phi]. \quad (15)$$

In the gauge $N = 1$ ($N_L = -i$) these solutions exactly coincide with the instantons of [7]. The corresponding saddle points of (13) can be attained by deforming the integration contour (6) of N_L into the complex plane to pass through the point $N_L = -i$ and relabeling the real Lorentzian t with the Euclidean τ. In terms of the Euclidean $N(\tau)$, $a(\tau)$ and $\Phi(x)$ the integrals (13) and (14) take the form of the path integrals (1) and (2) in EQG,

$$i\Gamma_L[a_L, N_L] = -\Gamma_E[a, N]. \qquad (16)$$

However, the integration contour for the Euclidean $N(\tau)$ runs from $-i\infty$ to $+i\infty$ through the saddle point $N = 1$. This is the source of the conformal rotation in Euclidean quantum gravity, which is called to resolve the problem of unboundedness of the gravitational action and effectively renders the instantons a thermal nature, even though they originate from the microcanonical ensemble. This mechanism implements the justification of EQG from the canonical quantization of gravity [14] (see also [15] for the black hole context).

To show this we calculate (1) in the one-loop approximation with the measure inherited from the canonical path integral (5) $D[a, N] = DaDN\mu[a, N]\delta[\chi]\mathrm{Det}Q$. Here $\mu[a, N]$ is a local measure determined by the Lagrangian $N\mathcal{L}(a, a')$, (3), in the local part of $\Gamma_E[a, N]$,

$$\mu_{1-\text{loop}} = \prod_\tau \left(\frac{\partial^2(N\mathcal{L})}{\partial\dot{a}\partial\dot{a}}\right)^{1/2} = \prod_\tau \left(\frac{D}{Na^2a'^2}\right)^{1/2}, \qquad D = aa'^2(a^2 - B + Ba'^2). \qquad (17)$$

The factor $\delta[\chi]\mathrm{Det}Q$ contains a gauge condition $\chi = \chi(a, N)$ fixing the one-dimensional diffeomorphism, $\tau \to \bar\tau = \tau - f/N$, which for infinitesimal $f = f(\tau)$ has the form $\Delta^f N \equiv \bar N(\tau) - N(\tau) = \dot f$, $\Delta^f a \equiv \bar a(\tau) - a(\tau) = \dot a f/N$, and the ghost operator $Q = Q(d/d\tau)$ is determined by the gauge transformation $\Delta^f\chi = Q(d/d\tau)f(\tau)$.

The conformal mode σ of the perturbations $\delta a = \sigma a_0$ and $\delta N = \sigma N_0$ on the saddle-point background (labeled below by zero, $a = a_0 + \delta a$, $N = N_0 + \delta N$) originates from imposing the background gauge $\chi(a, N) = \delta N - (N_0/a_0)\delta a$. In this gauge $Q = a(d/d\tau)a^{-1}$, and the quadratic part of Γ_E takes the form [16]

$$\delta^2_\sigma\Gamma_E = -\frac{3\pi m_P^2}{2}\int d\tau N D\left[\left(\frac{\sigma}{a'}\right)'\right]^2, \qquad (18)$$

where D is given by (17). As is known from [7] for the background instantons $a_0^2(\tau) \geq a_-^2 > B$ (a_- is the turning point with the smallest value of $a_0(\tau)$), so that $D > 0$ everywhere except the turning points where D degenerates to zero. Therefore $\delta^2_\sigma\Gamma_E < 0$ for real σ, but the Gaussian integration runs along the imaginary axes and yields the functional determinant of the positive operator—the kernel of the quadratic form (18)

$$e^{-\Gamma_{1-\text{loop}}} = e^{-\Gamma_0}\mathrm{Det}Q_0 \int D\sigma\left(\prod_\tau D/a'^2\right)^{1/2} e^{-(1/2)\delta^2_\sigma\Gamma_E}$$
$$= e^{-\Gamma_0}\mathrm{Det}\left(\frac{d}{d\tau}\right)\left[\mathrm{Det}\left(-\frac{1}{\sqrt{D}}\frac{d}{d\tau}D\frac{d}{d\tau}\frac{1}{\sqrt{D}}\right)\right]^{-1/2}.$$

In view of periodic boundary conditions for both operators their determinants cancel each other (their zero modes to be eliminated because they correspond to the conformal Killing symmetry of FRW instantons) [16]. Therefore, the contribution of the conformal mode reduces to the selection of instantons with a fixed time period, effectively endowing them with a thermal nature.

As suggested in [7,8,17] these instantons serve as initial conditions for inflationary universes which evolve according to the Lorentzian version of Eq. (4) and, at late stages, have two branches of a cosmological acceleration with Hubble scales $H_{\pm}^2 = (m_P^2/B)[1 \pm (1 - 2BH^2)^{1/2}]$. If the initial $\Lambda = 3H^2$ is a composite inflaton field decaying at the end of inflation, then one of the branches undergoes acceleration with $H_+^2 = 2m_P^2/B$. This is determined by the new quantum gravity scale suggested in [8]—the upper bound of the instanton Λ-range, $\Lambda_{\max} = 3m_P^2/2B$. To match the model with inflation and the dark energy phenomenon, one needs B of the order of the inflation scale in the very early Universe and $B \sim 10^{120}$ now, so that this parameter should effectively be a growing function of time.

This picture seems to fit into string theory at its low-energy field-theoretic level. Then, with a bounded range of Λ, it might constrain the landscape of string vacua [7,8]. Moreover, string theory has a qualitative mechanism to promote the constant B to the level of a moduli variable indefinitely growing with the evolving size $R(t)$ of extra dimensions. Indeed B as a coefficient in the overall conformal anomaly of four-dimensional quantum fields basically counts their number N, $B \sim N$. In the Kaluza-Klein (KK) theory and string theory the effective four-dimensional fields arise as KK and winding modes with the masses [18]

$$m_{n,w}^2 = \frac{n^2}{R^2} + \frac{w^2}{\alpha'^2} R^2 \tag{19}$$

(enumerated by the KK and winding numbers), which break their conformal symmetry. These modes remain approximately conformally invariant as long as their masses are much smaller than the spacetime curvature, $m_{n,w}^2 \ll H_+^2 \sim m_P^2/N$. Therefore, the number of conformally invariant modes changes with R. Simple estimates show that for pure KK modes ($w = 0$, $n \leq N$) their number grows with R as $N \sim (m_P R)^{2/3}$, whereas for pure winding modes ($n = 0$, $w \leq N$) their number grows with the decreasing R as $N \sim (m_P \alpha'/R)^{2/3}$. Thus, the effect of indefinitely growing B is possible for both scenarios with expanding or contracting extra dimensions. In the first case this is the growing tower of superhorizon KK modes which make the horizon scale $H_+ = m_P\sqrt{2/B} \sim m_P/(m_P R)^{1/3}$ indefinitely decreasing with $R \to \infty$. In the second case this is the tower of superhorizon winding modes which make this acceleration scale decrease with the decreasing R as $H_+ \sim m_P(R/m_P\alpha')^{1/3}$. This effect is flexible enough to accommodate the present day acceleration scale (though, by the price of fine-tuning an enormous coefficient of expansion or contraction of R). This gives a new dark energy mechanism driven by the conformal anomaly and transcending the inflationary and matter-domination stages starting with the state of the microcanonical distribution.

To summarize, within a minimum set of assumptions [the equipartition in the physical phase space (11)], we have the mechanism of generating a limited range of the positive cosmological constant which is likely to constrain the landscape of string vacua and get the full evolution of the Universe as a quasiequilibrium decay of its initial microcanonical state. Thus, contrary to anticipations of Sidney Coleman that "there is nothing rather than something" [3], one can say that something (rather than nothing) comes from everything.

The author thanks O. Andreev, C. Deffayet, A. Kamenshchik, J. Khoury, A. Linde, V. Mukhanov, H. Tye, A. Tseytlin, I. Tyutin, and B. Voronov for thought provoking discussions. The work was supported by the RFBR Grant No. 05-02-17661, LSS 4401.2006.2 Grant, and SFB 375 Grant at the Ludwig-Maximilians University in Munich.

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