Washboard Road: The Dynamics of Granular Ripples Formed by Rolling Wheels

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We report laboratory experiments on rippled granular surfaces formed under rolling wheels. Ripples appear above a critical speed and drift slowly in the driving direction. Ripples coarsen as they saturate and exhibit ripple creation and destruction events. All of these effects are captured qualitatively by 2D soft-particle simulations in which a disk rolls over smaller disks in a periodic box. The simulations show that compaction and segregation are inessential to the ripple phenomenon. We describe a simplified scaling model which gives some insight into the mechanism of the instability.

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Ripples which spontaneously appear due to the action of rolling wheels on unpaved roads bedevil transportation worldwide, especially in developing countries [1]. This effect, known as *corrugated* or *washboard* road, can severely limit the usefulness of unsurfaced roads. More generally, the appearance of ripples on a granular surface under tangential stress is reminiscent of other sorts of wind- and water-driven ripples [2], and of dune formation [3]. This resemblance suggests that this problem, which is well discussed in the engineering literature [1,4–8], might benefit from a simple, physics-oriented approach.

Engineering models of washboard formation range from coupled, damped pendulum models [5,6] to full continuum simulations of the deformable road surface [8]. Numerous experimental studies have been undertaken, from laboratory-scale rigs [4–6] to full-scale road tests [7]. In all cases, however, the engineering goal was to understand all the complexities of the system and to mitigate or eliminate the effect. In contrast, we aim to understand the simplest system that exhibits washboard road and to study it as a nonlinear, pattern forming instability. Only a few theoretical studies of this kind have appeared in the physics literature [9,10]. In addition to carrying out well instrumented laboratory-scale experiments, we present here the first application of soft-particle discrete element method (DEM) simulations to this problem. We also present a simple theoretical treatment in order to gain some insight into the fundamental mechanism of the instability and to understand the scaling and important dimensionless groups.

The experimental setup is shown in Fig. 1. The road consists of a deep layer of sand arranged on the circumference of a 1 m diameter rotating table. We used natural, rough sand with a grain diameter of $300 \pm 100\ \mu$m and the bed was typically 50 mm deep. A 100 mm diameter, 20 mm thick hard rubber wheel was attached to a 330 mm long arm in the form of a lever. The wheel rolled freely on the sand bed as the table rotated at a constant speed. No torque was applied to the wheel other than that produced by its contact with the bed. The table was typically rotated at about 0.6 Hz which corresponds to a horizontal velocity of the wheel $v \approx 2$ m/s.

A potentiometer was attached to the arm to record its angle, and its output was digitized. Simultaneously, a commercial laser displacement profiler [11] was used to record the bed shape. Thus, we could measure the wheel position and ripple shape with a vertical accuracy of 1 mm. Data acquisition was triggered by an optical sensor fixed to the table, and several thousand vertical positions could be measured in one table rotation. During a typical run, the arm inclination remained confined between 70° and 90°, so that the wheel motion was almost vertical. The large angle implies that the natural pendulum frequency of the arm plays no role, and the wheel is merely restored by falling under its own weight. The finite circumference of the table effectively imposes periodic boundary conditions on the ripple pattern, so that its wavelength is quantized

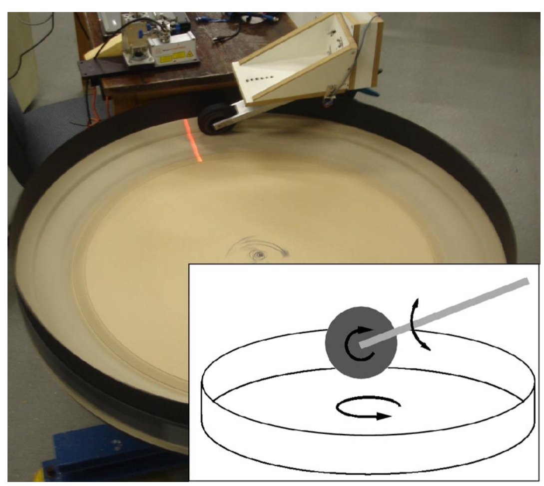

FIG. 1 (color online). Experimental setup. A bed of natural sand is laid on the circumference of a rotating table (1 m in diam and rotation rate between 0.2 and 0.8 Hz). A hard rubber wheel attached to an arm is free to bounce and roll on the granular bed.

since a fully developed washboard pattern contains an integer number of ripples around the table.

We also investigated the washboard formation in 2D DEM simulations. The simulation considers individual deformable disks, rotating and colliding with one another, subject to contact friction and gravity [12]. We used the following physical parameters in the simulation: particle diameter 8 mm, mass 0.16 g, spring constant 40 $\mathrm{kN\,m^{-1}}$, coefficients of restitution 0.5 and friction 0.3. In the simulation, the wheel was treated like any other disk but its density was $1/5$ that of the other disks and its diameter was 12.5 times larger. A constant horizontal velocity v was imposed on the wheel, but it was free to rotate and move vertically. The disks were made slightly polydisperse ($\pm 20\%$ in diameter) in order to avoid crystallization. In order to mimic the experimental setup, the simulation was made periodic in the horizontal direction. 25 000 small disks were initialized at random positions in the box, and then allowed to fall under gravity to settle into the bed, resulting in a layer 20 diam thick and 1500 diam long. The simulations were typically run for 500 passages of the wheel.

Such 2D simulations cannot be expected to reproduce the experimental data quantitatively. The simulated grains are softer than natural sand and idealized as spherical, while the ratio of the diameter of the wheel to that of the grains is necessarily much larger in the experiment than in the simulation. In spite of these simplifications, the qualitative results of the DEM simulations offer unique insights into the mechanism of the instability.

The wheel initially rolls smoothly on the flat granular surface. After a few tens of passes, if the velocity is high enough, a small localized ripple starts forming at some position. New ripples rapidly grow from that location, downstream from the initial position. Ripples then grow in height and eventually spread over the whole circumference of the table (in the experiments) or the whole length of the periodic box (in the simulation). A typical pattern is shown in Fig. 2 which displays both experimental and simulation data. There is an excellent qualitative agreement between the experiment and simulation.

The ripples are strongly asymmetrical with the steeper face close to the angle of repose. Individual ripples tend to be separated by flat regions. The height of ripples ranges from a few millimeters up to 50 mm as the wavelength (or pitch) ranges from 50 to 500 mm. Figure 2 also shows the trajectory of the wheel. For large v, the wheel becomes airborne near the crest of each ripple. At lower v, however, a clear washboard pattern can form while the wheel remains constantly in contact with the bed.

As is observed on actual roads [1], we found a critical value of the driving velocity v_c below which the bed remains flat. We find experimentally that $v_c \simeq 1.5\ \mathrm{m\,s^{-1}}$, a value quite similar to that found for real roads. The exact nature of the bifurcation to the rippled state remains unclear, however. The bed becomes extremely sensitive to small, but finite, perturbations for v near v_c, but it is

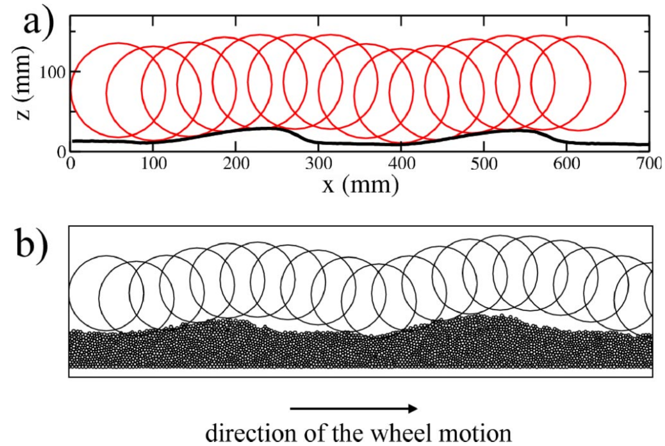

FIG. 2 (color online). Bed profile and wheel trajectory for typical washboard patterns obtained experimentally (a) and numerically (b).

difficult to establish whether v_c represents the onset of a linear instability to infinitesimal perturbations, or whether there is any velocity hysteresis near v_c. Most previous studies [1,4–8] have only examined the case of large, artificial initial perturbations and $v \gg v_c$.

Washboard patterns exist over a wide range of parameters. We varied the bed thickness, the grain size and shape, the wheel size, shape, and mass. As long as the bed thickness was sufficient to supply enough material for the ripples, in all cases the results were qualitatively identical. All our observations are broadly consistent with previous engineering studies [1,4–8], but have the advantage of a simpler suspension system and more modern instrumentation for data acquisition. Perhaps surprisingly, the size and shape of the grains has no effect whatsoever on either the wavelength or the amplitude of the ripples. In experiments, we tried two different natural sands with $d = 300 \pm 100\ \mu\mathrm{m}$ and 3.0 ± 0.8 mm, and also replaced the sand with long grain rice. In simulations, we halved the grain size and doubled the bed thickness for the same size wheel. The ripple patterns were identical up to small statistical fluctuations. The mass of the wheel and its suspension strongly affects the pattern. Heavier wheels produce larger amplitude ripples with shorter wavelengths. Wheel diameter, however, seems to be unimportant, and this was tested by also using a nonrotating square "wheel." Varying the diameter of the wheel, keeping its mass constant in the simulation, leaves the pattern unchanged. The insensitivity of the pattern to the wheel diameter and grain size raises interesting questions about the scaling of the washboard and are incorporated into the model discussed below.

We find that both the amplitude and wavelength of the ripples grow initially. They remain roughly proportional to one another as they evolve toward a saturated value which scales with the kinematic length scale v^2/g. Thus, as they spread around the table, their amplitude and wavelength increase and merging events take place. Figure 3 is a typical space-time plot obtained experimentally with

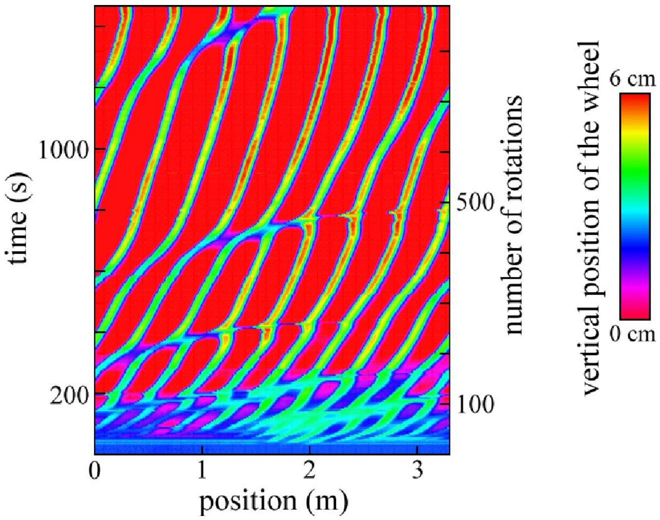

FIG. 3 (color online). An experimental space-time plot of the vertical position of the wheel for $v = 2\ \mathrm{ms}^{-1}$.

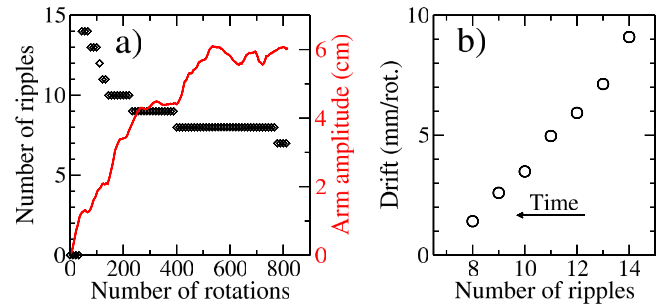

FIG. 4 (color online). Experimental ripple dynamics from Fig. 3. (a) Time evolution of the number of ripples (diamonds) and of the amplitude of the ripples (solid line). (b) Drift velocity in millimeters per rotation, as a function of the number of ripples.

$v = 2\ \mathrm{ms}^{-1}$. Our ripples always travel forward, whereas ripples on real roads, with driven wheels, have sometimes been observed to travel in both directions [1]. Figure 4(a) shows the ripple evolution for the run shown in Fig. 3. As the ripple amplitude saturates, the number of ripples drops from 14 to 7. The amplitude increases abruptly each time a ripple disappears. As the velocity is decreased, the ripples can split and the amplitude and wavelength then decreases, but this is not always observed. Figure 4(b) shows that the ripple drift velocity slows significantly as the wavelength increases, as one would expect since the volume of individual ripples increases while the flux remains roughly constant. The DEM simulation qualitatively reproduces all of the behavior shown in Figs. 3 and 4.

Using DEM simulations, we can examine aspects of the internal structure of the ripples which are difficult to access experimentally. Figure 5(a) shows that the slightly polydisperse grains remain well mixed inside the ripple. This demonstrates that size segregation is not crucial to the formation of the ripples, although it is almost certainly present on real roads [1].

The local packing fraction of a pile can be computed from its Voronoï tessellation. This algorithm partitions space into cells corresponding to individual particles. The area of each disk divided by the area of its cell can be interpreted as a local packing fraction. This is shown in Fig. 5(b). The grains located at the edge of the pile have an infinitely large, open Voronoï cell for which no packing fraction can be defined. The internal grains show no ripple-related structure in their packing, although the overall packing is denser than the initial state of the simulation. Thus, we conclude that varying compaction of the grains is also not essential to the formation of ripples. This contradicts a recent model [10], in which compaction played a central role.

Figure 5(c) shows the displacement of the grains caused by one pass of the wheel. The configuration is the state before the pass, and the color shows the horizontal distance those grains are about to travel. The displacement is localized to the crest of the ripples, which move on a static bed. Although individual particle displacements can be relatively large (up to 12 diam), it typically takes 200–300 passes of the wheel for a ripple to travel a distance of one wavelength.

While the dynamics of the wheel and its suspension are simple, the behavior of granular materials is poorly understood. There is no continuum theory that can be reliably applied to this problem, though phenomenological models exist which can be implemented with large-scale finite-element codes [8]. On the other hand, existing simplified models are either based on compaction [10], which we have shown to be unimportant, or on rather *ad hoc* automata [9].

We now consider a model aimed at elucidating the physical scales of the ripple problem. Stresses in a non-cohesive material are related to inertial forces or gravitational forces, so the only dimensional parameters are the 2D bed density ρ (i.e., the 3D density multiplied by the width of the wheel) and the acceleration due to gravity g. The experiments and simulations show that the particle size is not important, if it is small enough compared to the wheel and ripples. The primary dimensionless parameter

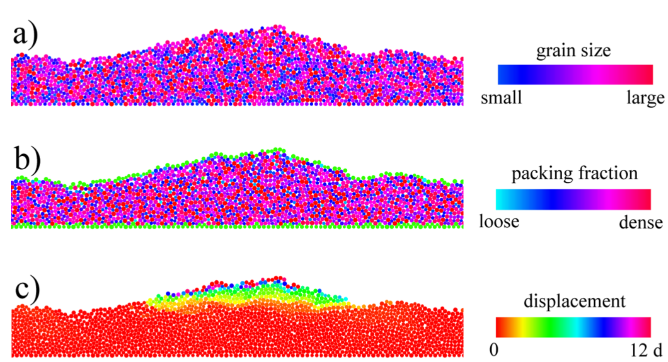

FIG. 5 (color online). The internal structure of the ripples, using DEM simulation. (a) Grain size, (b) local packing fraction, (c) displacement between two consecutive passes of the wheel.

needed to describe the material is its approximate angle of repose θ_c. If the wheel is supported by static forces, the penetration of the wheel into the bed is determined by its mass m, which gives rise to a penetration length scale $L_1 = \sqrt{m/\rho}$. There is a secondary dependence on the radius of the wheel R through the ratio R/L_1. The simulations show that this dependence is very weak. The experiments with the nonrotating square wheel, where no such length scale is present, show that this is a secondary effect.

The final significant dimensional parameter is v, the horizontal speed of the wheel. This gives rise to a second penetration length scale $L_2 = mg/\rho v^2$, if the wheel becomes supported by dynamic pressure. The ratio L_1/L_2 behaves like a Froude number $\mathrm{Fr} = (v^2/g)\sqrt{\rho/m}$. This is the only dimensionless group that can be formed from v, m, g, and ρ, thus it should determine the stability of the wheel-bed interaction.

We hypothesize that the instability results from a switch from highly dissipative, plastic behavior at low speeds to weakly dissipative, dynamic supporting forces at high speeds. In the absence of any special spring or dashpot suspension, the saturated ripples involve ballistic trajectories, so the natural length scale for the ripple wavelength λ is the length scale $L = v^2/g$. Then $\lambda/L = f_\lambda(\mathrm{Fr})$, where f_λ is a relatively weak function of Fr. We expect the saturated amplitude of the ripples A to be determined by λ and the angle of repose θ_c, so that $A/L = f_A(\mathrm{Fr})$, with $f_A(\mathrm{Fr}) \propto \tan\theta_c$.

The key element of a continuum theory to describe this system is the function for the volume flux of sand q which will be roughly proportional to zv, where z is the depth the wheel penetrates into the sand. The flux will also depend on the slope angle and should diverge as the surface angle approaches the angle of repose. In addition there will be dependence on the local dimensionless groups zg/v^2 and $Ng/\rho v^4$, where N is the normal force. The penetration depth z will be determined by the forces acting on the wheel. Thus, the key to an accurate theory is to correctly model N. We propose

$$N = \begin{cases} 0 & z < 0 \\ -z\delta\rho v^2 & z > 0, \dot{z} < 0 \\ -z\delta\rho v^2 + z^2\mu\rho g & z > 0, \dot{z} > 0, \end{cases} \qquad (1)$$

where δ and μ are geometric factors that may depend on z/R. This term is effective whether the wheel is moving up or down and is conservative. The second term is completely dissipative and represents the plastic response of the bed. Because of this strong piecewise nonlinearity, the system cannot be treated by linear stability analysis. The mean normal force balances gravity and thus the displacement z on a flat bed will be of the order of L_1/Fr^2. Thus, as Fr increases, the interaction with the bed becomes less dissipative and the wheel can bounce upward from the bed. It is this bounce that drives the instability because it allows a phase lag to develop between the volume flux q and the driving force N, leading to a mutually reinforcing oscillation.

Washboard road will no doubt continue to annoy drivers for as long as there are unpaved roads and wheels to roll over them. We have reexamined this engineering problem from the perspective of basic nonlinear physics. We have argued that the appearance of the ripples at a critical speed should be regarded as an instability of the flat road. Using simplified experiments and DEM simulations, we have shown that neither compaction nor segregation processes are responsible for the instability, contrary to some existing theories. Using very general dimensional analysis arguments, we have identified a candidate for the dimensionless parameter which controls the instability. The piecewise form of the important nonlinearity in the normal force is such that the onset of the instability will not be amenable to standard linear or weakly nonlinear stability analysis techniques. Well above onset, we experimentally observed interesting ripple-merging and coarsening events, which are also reproduced by the DEM simulations. We hope to use these insights to obtain a better understanding of this fascinating example of nonlinear pattern formation.

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