Photon Bubbles and Ion Acceleration in a Plasma Dominated by the Radiation Pressure of an Electromagnetic Pulse

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The stability of a thin plasma foil accelerated by the radiation pressure of a high intensity electromagnetic (e.m.) pulse is investigated analytically and with particle in cell numerical simulations. It is shown that the onset of a Rayleigh-Taylor-like instability can lead to transverse bunching of the foil and to broadening of the energy spectrum of fast ions. The use of a properly tailored e.m. pulse with a sharp intensity rise can stabilize the foil acceleration.

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Radiation pressure is an effective mechanism of momentum transfer to charged particles that has attracted a great deal of attention over a long time [1] in a wide variety of physical conditions ranging from stellar structures and radiation generated winds (see, e.g., Refs. [2,3]), to the formation of "photon bubbles" in very hot stars and accretion disks [4], to particle acceleration in the laboratory [5,6], and in high-energy astrophysical environments [7]. Radiation pressure arises from the "coherent" interaction of the radiation with the particles in the medium. In an electron-ion plasma, it acts mostly on the lighter particles, the electrons, with a force that is quadratic in the wave field amplitude [8]. Ions are accelerated by the charge separation field caused by the electrons pushed by the radiation pressure. This collective acceleration mechanism is very efficient [9] when the number of ions inside the electron cloud is much smaller than that of the electrons.

The electric fields produced by the interaction of ultrashort and ultraintense laser pulses with a thin target make it possible to obtain multi-MeV, high-density, highly collimated proton and ion beams (see Refs. [10] and references therein) of extremely short duration, in the subpicosecond range. Such laser pulses may also open up the possibility of exploring high-energy astrophysical phenomena, such as the formation of photon bubbles [11] in the laboratory.

Different regimes of plasma ion acceleration have been discussed in the literature; see, e.g., Ref. [12] and references therein. In Ref. [5], a regime where the ion acceleration in a plasma is directly due to the radiation pressure of the electromagnetic (e.m.) pulse has been identified. In this radiation pressure dominant acceleration (RPDA) regime, the ions move forward with almost the same velocity as the electrons and thus have a kinetic energy well above that of the electrons. In contrast to the other regimes, this acceleration process is highly efficient, and the ion energy per nucleon is proportional to the e.m. pulse energy. This acceleration mechanism can be illustrated by considering a thin, dense plasma foil, made of electrons and protons, pushed by an ultraintense laser pulse in conditions where the radiation cannot propagate through the foil, while the electron and the proton layers move together and can be regarded as forming a (perfectly reflecting) relativistic plasma mirror copropagating with the laser pulse. The frequency of the reflected e.m. wave is reduced [13] by $(1 - v/c)/(1 + v/c) \approx 1/4\gamma^2$, with v the mirror velocity and $\gamma = (1 - v^2/c^2)^{-1/2}$. Thus, the plasma mirror is accelerated and acquires from the laser the energy $(1 - 1/4\gamma^2)\mathcal{E}$ where $\mathcal{E}$ is the incident laser pulse energy in the laboratory frame (LF). For large values of γ, when the mirror moves at relativistic speeds, practically all the e.m. pulse energy is transferred to the mirror, essentially in the form of proton kinetic energy. This high conversion efficiency may open up a wide range of applications and be exploited, e.g., in the design of proton dump facilities for spallation sources or for the production of large fluxes of neutrinos [14].

Both in the astrophysical and in the laser plasma contexts, the onset of Rayleigh-Taylor (R-T) type instabilities affects the interaction of the plasma with the radiation pressure which may eventually dig through the plasma and make it porous to the radiation or, in the case of a plasma foil accelerated by a laser pulse, may tear it into clumps [15] and broaden the energy spectrum of the fast ions. In this Letter, we investigate analytically the stability of a plasma foil against long wavelength perturbations in the ultra relativistic conditions that are of interest for the RPDA regime. We find that proper tailoring of the pulse amplitude can allow for stable foil acceleration. In addition, with the help of two-dimensional (2D) Particle in Cell (PIC) simulations, we show that the nonlinear development of the instability leads to the formation of high-density, high-energy plasma clumps.

The equation of motion of an element of area $|d\sigma|$ of a perfectly reflecting mirror can be written in the LF as $d\mathbf{p}/dt = P d\sigma$, where $\mathbf{p}$ is the momentum of the mirror element, $d\sigma$ is normal to the mirror surface, and P is the Lorentz invariant radiation pressure. We assume that the mirror acceleration and curvature are small on the radiation oscillation period and wavelength, that each element of the mirror is directly illuminated by the e.m. pulse, and that no secondary reflections occur. In addition, for the sake of geometrical simplicity, we refer to a 2D configuration where the mirror velocity and the mirror normal vector remain coplanar (in the x-y plane, x being the direction of propagation of the e.m. pulse). The radiation pressure P is given in terms of the amplitude of the electric field E_M of the incident e.m. pulse and of the pulse incidence angle θ_M in the CMF by $P = (E_M^2/2\pi)\cos^2\theta_M$, where $E_M^2 = (\omega_M^2/\omega_0^2)E_0^2$ with $\omega_M^2/\omega_0^2 = (1-\beta\cos\phi)^2/(1-\beta^2)$, the subscript 0 denotes quantities in the LF and ϕ the angle the mirror velocity β makes with the x-axis in the laboratory frame. The angle θ_M vanishes when the incidence angle θ_0 in the LF vanishes and $\phi = 0, \pi$, but is a fast increasing function of γ for $\theta_0 \neq 0$, or $\phi \neq 0, \pi$, [16]. The equation of motion of a mirror element of unit length along z and uniform density n_0 in the LF is

$$\frac{\partial p_x}{\partial t} = \frac{P}{n_0 l_0}\frac{\partial y}{\partial s}, \qquad \frac{\partial p_y}{\partial t} = -\frac{P}{n_0 l_0}\frac{\partial x}{\partial s}, \tag{1}$$

with $\partial x/\partial t = \beta_x c$ and $\partial y/\partial t = \beta_y c$. Here, $p_{x,y} = m_i c\beta_{x,y}/(1-\beta^2)^{1/2}$ are the spatial components of the momentum 4-vector of the mirror element, l_0 is the mirror thickness, and m_i is the ion mass. Lagrangian coordinates, x_0 and y_0, have been adopted such that $x, y = x, y(x_0, y_0, t)$ and $ds = (dx_0^2 + dy_0^2)^{1/2}$. In the nonrelativistic limit and for constant P, Eqs. (1) coincide with Ott's equations [17] (see also [18]) for the motion of a thin foil.

Assuming that the unperturbed mirror moves along the x-axis, i.e., that the initial conditions correspond to a flat mirror along y_0 so that $dx_0 = 0$, $dy_0 = ds$, and $\theta_M = 0$, we write Eqs. (1) as

$$\frac{dp_x^0}{dt} = \frac{E_0^2}{2\pi n_0 l_0}\frac{m_i c\gamma_0 - p_x^0}{m_i c\gamma_0 + p_x^0}, \tag{2}$$

where p_x^0 is the unperturbed x component of momentum and depends on the variable t only and $m_i^2c^2\gamma_0^2 = m_i^2c^2 + (p_x^0)^2$. Equation (2) has been analyzed in Refs. [5,14]. The electric field of the e.m. pulse at the mirror position $x(t)$ depends on time as $E_0 = E_0[t - x(t)/c]$. We introduce the phase of the wave $\psi = \omega_0[t - x^0(t)/c]$, at the unperturbed mirror position $x^0(t)$ as a new independent variable. Differentiating with respect to time, we obtain

$$\frac{d\psi}{dt} = \omega_0\frac{m_i c\gamma_0 - p_x^0}{m_i c\gamma_0}. \tag{3}$$

Using the variable ψ and the normalized fluence of the e.m. pulse $w(\psi) = \int_0^\psi [R(\psi')/\lambda_0]d\psi'$, with $R(\psi) = E_0^2(\psi)/(m_i n_0 l_0 \omega_0^2)$, and $\lambda_0 = 2\pi c/\omega_0$, the solution of Eq. (2) with the initial condition $p_x^0(0) = 0$ is

$$p_x^0(\psi) = m_i c\frac{w(\psi)[w(\psi)+2]}{2[w(\psi)+1]}, \tag{4}$$

while from Eq. (3), we obtain that t and ψ are related by $\psi + \int_0^\psi [w(\psi') + w^2(\psi')/2]d\psi' = \omega_0 t$. For a constant amplitude e.m. pulse, when $R = R_0$, these expressions reduce [14] to $w(\psi) = (R_0/\lambda_0)\psi$ and to

$$\psi + (R_0/\lambda_0)\psi^2/2 + (R_0/\lambda_0)^2\psi^3/6 = \omega_0 t, \tag{5}$$

with $p_x^0 \approx m_i c(R_0/\lambda_0)\omega_0 t$ for $t \ll \omega_0^{-1}(\lambda_0/R_0)$ and $p_x^0 \approx m_i c(3R_0\omega_0 t/2\lambda_0)^{1/3}$ for $t \gg \omega_0^{-1}(\lambda_0/R_0)$. These relationships are obtained under the assumption that the charge separation electric field $E_\parallel$ which accelerates the ions is sufficiently large so that $E_\parallel = 2\pi n_0 e l_0 > (E_0^2/2\pi n_0 l_0) \times [(m_i c\gamma_0 - p_x^0)/(m_i c\gamma_0 + p_x^0)]$. This condition can be rewritten in terms of the dimensionless laser pulse amplitude $a_0 = eE_0/m_e c\omega_0$ and of the dimensionless parameter $\epsilon_0 = 2\pi n_0 e^2 l_0/m_e\omega_0 c$ as $a_0 < \epsilon_0[(m_i c\gamma_0 + p_x^0)/(m_i c\gamma_0 - p_x^0)]^{1/2}$ and is equivalent to the full opacity condition for a thin overdense plasma foil in the CMF. In the opposite limit, the ions are accelerated by the electric field $E_\parallel$ until $p_x^0 \sim m_i c(2a_0/\epsilon_0)^{1/2}$ when they enter the RPDA regime.

Now we investigate the linear stability of the accelerated mirror with respect to perturbations $x^1(y_0, \psi)$, $y^1(y_0, \psi)$ that bend the plasma foil. Linearizing Eqs. (1) around the solution given by Eq. (4), we obtain

$$\frac{\partial}{\partial\psi}\left[\frac{p_x^0(\psi)}{m_i c}\frac{\partial x^1}{\partial\psi}\right] = \frac{R(\psi)}{2\pi}\frac{\partial y^1}{\partial y_0}, \tag{6}$$

$$\frac{\partial}{\partial\psi}\left[\frac{m_i c}{p_x^0(\psi)}\frac{\partial y^1}{\partial\psi}\right] = -\frac{R(\psi)}{2\pi}\frac{\partial x^1}{\partial y_0}. \tag{7}$$

Here, we retain only the leading terms in the ultrarelativistic limit $p_x^0/m_i c \gg 1$ for the foil motion and neglect a term proportional to $(\partial R/\partial\psi)x^1/\lambda_0$ on the right hand side of Eq. (6). We look for WKB solutions of the form

$$y^1(y_0, \psi) \propto \exp[\Phi(\psi) - iky_0], \tag{8}$$

with $\Phi(\psi) = \int_0^\psi \Gamma(\psi')d\psi'$ and growth rate $\Gamma \gg 1$. We find $\Gamma(\psi) = [kR(\psi)/2\pi]^{1/2}$, with $x^1 \sim -iy^1(m_i c/p_x^0)$. For a constant amplitude pulse, using Eq. (5) in the form $\psi^3 \sim 6\omega_0 t(\lambda_0/R_0)^2$, we obtain $y^1 \propto \exp[(t/\tau_r)^{1/3} - iky_0]$, where $\tau_r = \omega_0^{-1}(2\pi)^{3/2}R_0^{1/2}/(6k^{3/2}\lambda_0^2)$ is proportional to the square root of the ratio between the radiation pressure and the ion mass. Thus, in the ultrarelativistic limit where at constant t the phase ψ decreases with increasing pulse intensity as $R_0^{-2/3}$, the perturbation grows faster with increasing ion mass and decreasing radiation pressure. This is due to the fact that, for given laser pulse amplitude, a light-ion foil is accelerated faster and moves with a velocity closer to the speed of light than a heavy-ion foil. On the contrary, in the nonrelativistic limit, the perturbation grows, in agreement with Ref. [17], as $x^1, y^1 \propto \exp[t/\tau - iky_0]s$, with $\tau = \omega_0^{-1}(2\pi/kR_0)^{1/2}$. In the ultrarelativistic limit, the instability develops more slowly with time than in the nonrelativistic case: $t^{1/3}$ instead of t. In addition, the nonrelativistic time τ is inversely proportional to the square root of the radiation pressure. On the other hand, if we express y^1 in terms of the unperturbed momentum p_x^0, in both limits, we find an exponential growth of the form $y^1(y_0, p_x^0) \propto \exp[\kappa p_x^0/(m_i c) - iky_0]$, where $\kappa = (k\lambda_0)^{1/2}/(2\pi R_0/\lambda_0)^{1/2} \propto (k/l_0)^{1/2}(m_i/m_e)^{1/2}(\omega_{\text{pe},0}/\omega_0)a_0^{-1}$, with $\omega_{\text{pe},0}^2 = 4\pi n_0 e^2/m_e$. This exponential growth of the perturbation with the unperturbed momentum for a constant amplitude pulse can be effectively stopped by tailoring the shape of the e.m. pulse. We refer to the ultrarelativistic limit. From Eq. (4), which in this limit takes the form $p_x^0(\psi) = m_i c \int_0^\psi [R(\psi')/2\lambda_0]d\psi'$, and from Eq. (8), we see that the stability condition can be formulated as follows: it is possible to choose the dependence of the e.m. pressure $R(\psi)$ on the phase ψ such that as ψ reaches a given value ψ_m, either finite or equal to infinity, the ion momentum $p_x^0(\psi)$ grows, formally to infinity, while $\Phi(\psi)$ remains finite. As an example, we can take $R(\psi)$ of the form shown in Fig. 1, $R(\psi) = R_0(1 - \psi/\psi_m)^{-\alpha}\chi(\psi_1 - \psi)$, with $1 < \alpha < 2$, $\chi(x) = 1$ for $x > 0$, and $\chi(x) = 0$ for $x < 0$ and $\psi_m > \psi_1$ so as to keep the pulse fluence (energy per unit surface) finite. In this case, the maximum value of the ion momentum $p_x^0/(m_i c) \approx R_0\psi_m(1 - \psi_1/\psi_m)^{1-\alpha}/[2\lambda_0(\alpha - 1)]$ tends to infinity for $\psi_1 \to \psi_m$, while $\Phi(\psi_m) = (2kR_0/\pi)^{1/2}\psi_m[1 - (1 - \psi_1/\psi_m)^{1-\alpha/2}]/(2 - \alpha)$ remains finite. The time shape of the pulse can be obtained by inserting $\psi = \psi(t)$ from Eq. (3): for $\alpha < 3/2$, the acceleration time is finite. The PIC simulations presented in Ref. [5] show a stable phase of the RPDA of protons where a portion of the foil, with the size of the pulse focal spot, is pushed forward by a superintense e.m. pulse. The wavelength of the reflected radiation is substantially larger than that of the incident pulse, as consistent with the light reflection from the comoving relativistic mirror with the e.m. energy transformation into the kinetic energy of the plasma foil. At this stage, the initially planar plasma slab is transformed into a "cocoon" inside which the laser pulse is almost confined. The protons at the front of the cocoon are accelerated up to energies in the multi-GeV range at a rate that agrees with the $t^{1/3}$ scaling predicted by the analytical model. The proton energy spectrum has a narrow feature corresponding to a quasimonoenergetic beam, but part of it extends over a larger energy interval.

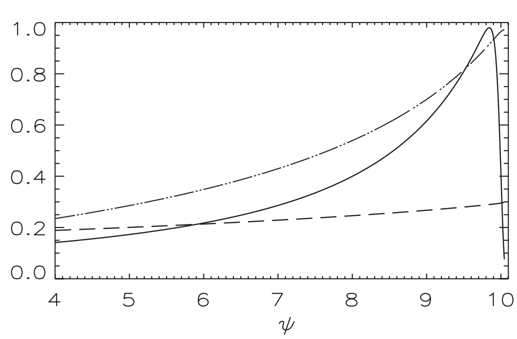

FIG. 1. Profiles of $R(\psi)$ (solid line), of $p_x^0(\psi)$ (dash-dotted line), and of $\Phi(\psi)$ (dashed line), for $\psi_m/\psi_1 = 1.3$ and $\alpha = 1.5$.

In order to investigate the onset and the nonlinear evolution of the instability of the foil, we have performed a series of numerical simulations with heavier ions (aiming at a faster growth rate), using the 2D version of the PIC e.m. relativistic code REMP, see Ref. [19]. The size of the computation box is $95\lambda \times 40\lambda$ with a mesh of 80 cells per λ. Such a high spatial resolution is required because the interaction of a superintense e.m. pulse with an overdense plasma slab is accompanied by strong plasma compression. The total number of quasiparticles in the plasma region is equal to 2×10^6. A thin plasma slab, of width 20λ and thickness 0.5λ, is localized at $x = 20\lambda$. The plasma is made of fully ionized aluminum ions with $Z = 13$; the ion to electron mass ratio is 26.98×1836. The electron density corresponds to the ratio $\omega_{\text{pe}}/\omega = 13$. An s-polarized pulse with electric field along the z-axis is initialized in vacuum at the left-hand side of the plasma slab. The pulse has a "Gaussian" envelope given by $a_0 \exp(-x^2/2l_x^2 - y^2/2l_y^2)$, with $l_x = 40\lambda$, $l_y = 20\lambda$, and $R_{\max}/\lambda_0 \sim 2 \times 10^{-2}$. Its dimensionless amplitude, $a_0 = 320$, corresponds for $\lambda = 1\ \mu$m to the intensity $I = 1.37 \times 10^{23}\ \text{W/cm}^2$, close to the value that is expected for the recently proposed superpower lasers such as *HiPER* and *ELI* [20]. The boundary condition are periodic along the y-axis and absorbing along the x-axis for both the e.m. radiation and the quasiparticles.

The results of these simulations are shown in Figs. 2 and 3 at $t = 75$, 87.5. The distribution in the x-y plane of the ion density and of the z-component of the electric field E_z are shown in frames (a) and (b); frame (c) shows the ion phase plane, (p_x, x), and frame (d) the energy spectrum of the aluminum ions. In Fig. 2(a), we see the typical initial stage of the R-T instability: the instability develops with a

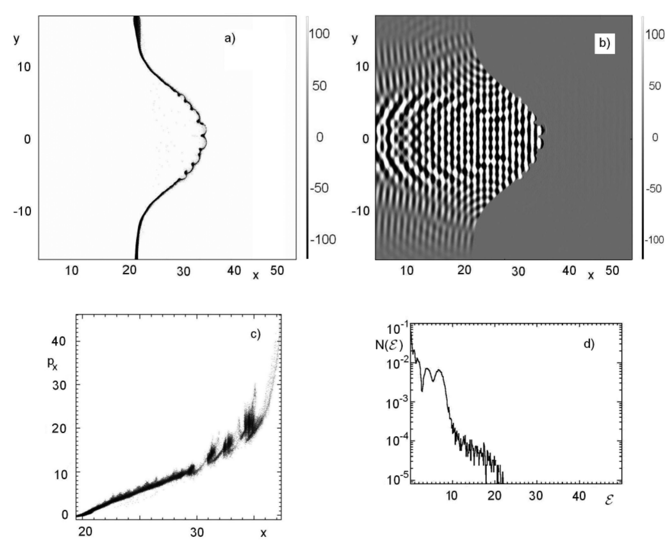

FIG. 2. (a) Ion density distribution in the x-y plane; (b) distribution of the electric field E_z; (c) ion phase plane (p_x, x); (d) ion energy spectrum at $t = 75$. The wavelength λ of the incident radiation and its period $2\pi/\omega_0$ are chosen as units of length and time. The ion momentum and kinetic energy are given in GeV/c and in GeV, respectively.

growth rate of several tens of laser periods, consistent with the analytical estimate $\omega_0\tau_r < 1$, with the formation of cusps and of multiple bubbles in the plasma density distribution. These are accompanied by a modulation of the e.m. pulse at its front, as seen in Fig. 2(b). In the nonlinear stage, we see the formation of relatively large scale clump bubbles. The ions are accelerated forward, and their momentum spectrum is made up of quasimonoenergetic beamlets, shown in Figs. 2(c) and 2(d), which correspond to the cusp regions, and of a relatively high-energy tail which is formed by the ions at the front of the bubbles. In Fig. 3(a), the fully nonlinear stage of the instability results in the formation of several clumps in the ion density distribution, moving with relativistic velocities, with more diffuse, lower density plasma clouds between them. The e.m. wave partially penetrates through, and partially is scattered by, the clump-plasma layer [see Fig. 3(b)]. The high-energy tail in the ion spectrum [Fig. 3(d)] grows much faster than in the stable case. At later times, because of the mass reduction of the diffuse clouds at the front of the pulse, the maximum ion energy scales linearly with time. The local maxima at relatively lower energy correspond to the plasma clumps. When the instability develops from noise, the clumps in the instability nonlinear stage at different angles, but remain well collimated in the forward direction close to the axis. More collimated clumps can be obtained using shaped laser pulses (e.g., annular pulses with minimum intensity at the axis) and shaped targets. Although the thin foil analytical model does not describe short wavelength effects, such as plasma compressibility, ablation (see [21]), and the internal structure of the accelerated plasma slab, these effects are incorporated in the PIC simulations.

In the relativistic regime, the R-T instability of a plasma foil accelerated by the radiation pressure of the reflected e.m. pulse develops much more slowly than in the non-relativistic regime with a time scale proportional to the square root of the ratio between the radiation pressure and the ion mass. A properly tailored e.m. pulse with a steep intensity rise can stabilize the foil acceleration. Numerical simulations show that the nonlinear development of the instability leads to the formation of high-density, high-energy plasma clumps and to a higher rate of ion acceleration in the regions between the clumps. For the HiPER and ELI laser systems, the ion energy can reach several tens of GeV.

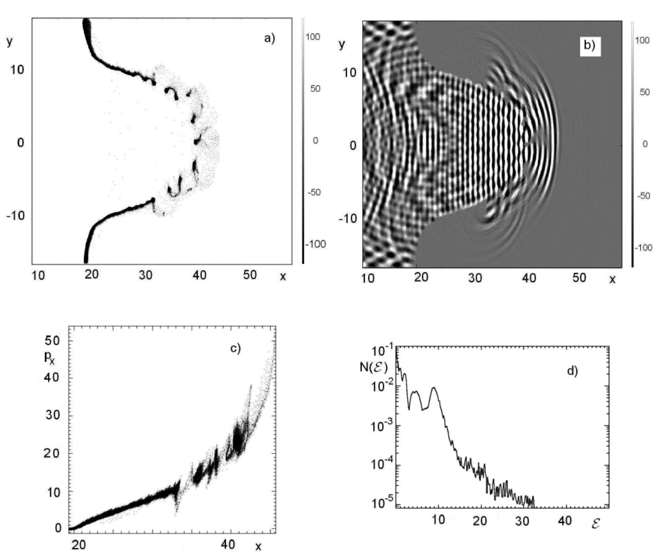

FIG. 3. Same as in Fig. 2 but at $t = 87.5$.

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